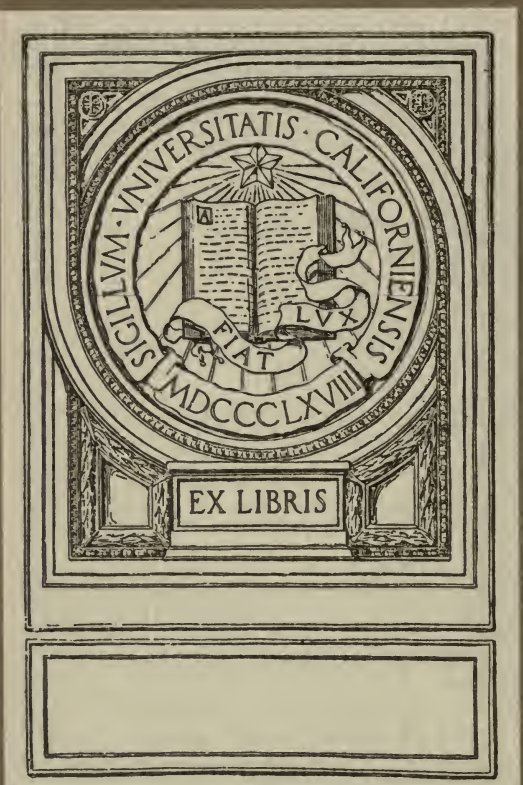
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TREATISE

ON

ANNUITIES;

WITH

NUMEROUS TABLES, BASED ON THE EXPERIENCE

OF THE EQUITABLE SOCIETY

AND ON THE

NORTHAMPTON RATE OF MORTALITY.

BY THE LATE **GRIFFITH DAVIES, F.R.S.,**

ACTUARY TO THE GUARDIAN ASSURANCE COMPANY, AND TO THE REVERSIONARY INTEREST SOCIETY.

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PREFACE.

The Executors of the late Mr. Griffiths Davies, agreeably to suggestions made by several Gentlemen professionally engaged in the business of Life Assurance, as well as from a sense of duty and regard to the memory of the Deceased, have thought it desirable to put into general circulation his larger Treatise on Life Contingencies. For various reasons, this Treatise, as originally designed, was never completed; nevertheless, so much of it as was written, has been in print since the year 1825, and many copies have from time to time been disposed of by the Author. In order to make the Work somewhat more complete than it was left by Mr. Davies, the Executors have prepared a Title-page and a Table of Contents. They have also completed Problem III. on Successive Annuities (page 376), from a manuscript copy (made from the proof sheets) kindly lent them by a friend of the Author, the originals having been lost or mislaid. Table XLIV., showing the value of Policies on Single Lives according to the Northampton Table at 3 per cent., has also been completed, from the Original Table given in the Tract on Life Contingencies published by Mr. Davies in the year 1825—a work which is now out of print and very scarce.

It appears that the "Introductory Observations" were either never completed, or, if finished, portions must have been destroyed, as the whole of the proof sheets struck off are given in a bound copy of the Work which Mr. Davies had; but the Chapter is unfinished, and it has been judged better to leave this part untouched rather than add any fresh matter to it, although a few pages are wanting to complete

the Introduction and make the numbers of the pages run consecutively.

In consequence of the Treatise not having been completed, in accordance with the Author's original design, and formally published, he missed the opportunity which the preparation of a Preface to the Work would have afforded him of explaining the principle upon which the Tables are computed, and the extent of his claim to be considered the author of what is generally styled the "Columnar Method." At the time the Tract was published, it was his intention, as therein stated, to have brought out forthwith his larger Work; and rather than enter into detailed explanation of the New Method in that Tract, he had reserved the expression of his views on the subject until the larger Treatise should have been given to the public. As, however, that opportunity never arrived, the Executors deem it but justice to Mr. Davies to allude to the matter, in now submitting the Work more generally to the profession as well as the public at large.

It is well known that the first Inventor of the Columnar Method in this country was the celebrated Mr. George Barrett, who was a most indefatigable calculator, and who, by his unwearied industry, computed a variety of most extensive Tables on Single and Joint Lives: at the same time, it is apparent to every attentive observer that there exists a wide difference between the new method of calculating Life Annuities and Assurances, as invented by him, and the same method improved and extended by Mr. Davies. In order to explain more fully the difference, it may be stated that Column A, according to Barrett's method, is formed by multiplying the number living at each age by the *improved amount* of £1 for as many years as are equal to the difference between that age and the oldest in the Table of Observation. Column B, opposite any given age, contains the successive additions of all the numbers in Column A *at* and above such age; so that in finding the Annuity at any age, the number in Column B opposite age one year older must be divided by

the number in Column A opposite the given age: whereas the number inserted in Column D, according to Davies' method, is the number living at each age multiplied by the *present value* of £1 due as many years hence as are equal to that age, and the number in Column N opposite any age is the sum of the numbers in Column D at all higher ages. In order to show the value of an increasing Annuity, another column was formed by both Barrett and Davies, containing the successive sums of the numbers inserted in Columns B or N respectively. Mr. Barrett proceeded no further; but Mr. Davies extended the method, and gave two other columns, M and R, for assurances—the first being the sum of the decrements at each age multiplied by the present value of £1 due as many years hence as are equal to that age increased by unity, and the latter column being the successive additions of Column M, and adapted for determining the value of increasing assurances. But the most important distinction between the two methods is, that Mr. Davies' method is much simpler in principle than that of Mr. Barrett, as the columnar numbers given by the latter must be considered more as numerical results of algebraical expressions; whereas in Davies' arrangement it will be found, on reference to age 0, that the number in Column D represents the number of children just born, and those opposite ages 1, 2, 3, 4, &c., to the end of life, the present sums which would be required for the payment of £1 to each survivor of such children at the end of 1, 2, 3, 4, &c. years to the extremity of life; and the sum thereof inserted in Column N, opposite age 0, represents the present fund required to provide the payment of Annuities of £1 each for life to all the children given in Column D at age 0; and from this method very considerable amount of labour is avoided by multiplying the number living at each age by a fraction less than a unit; but by Barrett's method the number living at each age has to be multiplied by the amount of £1 improved for as many years as are equal to the difference between that age and the greatest tabular duration, as already stated, which makes each

product a large multiple of the number living, and which induced Mr. Barrett to introduce a column (x) for the purpose of representing the number of integral figures.

It is somewhat remarkable, that a method of showing the value of Annuities and Assurances, similar to that of Barrett as *improved* and *extended* by Davies, was invented and published in Germany as far back as the year 1785, by Professor John Nicholas Tetens, of Leipsic, which was made known in this country for the first time by Frederick Hendriks, Esq., Actuary to the "Globe Assurance Company," in an article contributed by him and inserted in the first Number of the *Assurance Magazine*. Mr. Barrett's method was first printed in 1813, in an appendix to Baily's *Doctrine of Life Annuities and Assurances;* but there is not the slightest doubt that it was an independent invention, although posterior in date to that of Tetens, as there is not the remotest probability that Barrett ever knew anything of Tetens' work or method. This point has been well sifted by Professor De Morgan, in an account of a "Correspondence between Mr. George Barrett and Mr. Francis Baily," published in No. xv. of the before-named *Journal*. As already observed, Mr. Davies' explanation of his improvement and extension of Barrett's method never appeared; but he frequently expressed his regret that he had missed the opportunity of making it, as he had often felt that there were gentlemen in and out of the Profession who entertained an opinion that he claimed to himself the credit due to Mr. Barrett for the method in question. In the *Companion to the British Almanack for the year* 1840 a paper by Professor De Morgan is inserted, on the "Calculations of Life Contingencies," in which he enters into the comparative merits of Barrett's invention and Davies' *improvement* and *extension* of it. In discussing the subject he states, "This method of Mr. Barrett was rendered still more commodious, and we believe extended, by Mr. Griffith Davies, in his Tables of Life Contingencies (1825), a Work now unfortunately out of print. * * * The great principle

of the method—namely, the formation of Tables by which deferred, temporary, and increasing benefits are as easily calculated as those for the whole life, belongs to Mr. Barrett as much as the invention and construction of Logarithms to Napier. On the other hand, Mr. Griffith Davies, by the alterations presently noted and the separate exhibition of Columns M and R, has increased the utility and extended the power of the method to an extent of which its Inventor had not the least idea, and has all the rest of the claim in the matter which is made for Briggs in the adaptation of Logarithms to practical use." When Mr. Davies saw this statement, he was much pleased with it, and was perfectly satisfied that the Professor had done justice to both Mr. Barrett and himself, as he had set forth the subject in the light in which Mr. Davies had always meant it to be understood.

It is scarcely requisite to add, that one of the principal features of the present Work is the Rate of Mortality it contains, based upon the Experience of the Equitable Society for a period of 57 years from 1768 to 1825, deduced from statements given in the addresses of the late William Morgan, Esq., submitted to the General Courts of Directors of that Society, and in Notes added by him to the latter editions of Dr. Price's *Observations on Reversionary Payments;* and although Mr. Morgan afterwards admitted that he was not at the time aware of the many instances in which there were several policies on the same life, it is satisfactory to find that Mr. Davies' conclusions have not by that circumstance been materially affected, as a close approximation exists between his Table of Mortality and one formed by the present eminent Actuary of the abovementioned important Institution, from facts recorded in the Office Books.

The Executors, whilst deeply regretting that the second part of the Work was not completed by Mr. Davies, so as to make it a comprehensive Treatise on Life *Assurances* as well as Annuities, at the same time believe the Work in its present form, if it were merely on account of its being the only

book which contains a complete set of most useful Actuarial Tables based on the Equitable Experience, to be valuable and deserving the attention of everyone connected with the science of Life Contingencies. It is moreover particularly adapted to students, as it enters into an elaborate investigation of the construction of a Table of Mortality from any given data.

In submitting the Work, with the additions already mentioned, to the Profession and Public at large, at this late period, when so many new improvements have been introduced into the science of Life Annuities and Assurances, the Executors cannot lose sight of the fact that possibly some portions of the Work might have been more applicable and useful had they been formally published at the time when they came from the Author's hands; yet they nevertheless believe that the Book contains much that will always prove valuable to Society generally, but to the Actuary more especially.

UNIVERSITY OF CALIFORNIA

Observations ON LIFE ASSURANCES.

CHAPTER I.

Introductory Observations.

IN addition to the sustaining of Life, the cultivation of his mind, and the gratification of his senses and passions, the various pursuits and inventions of Man are principally directed to the acquiring of property, the protecting of that property from the encroachment of his neighbours, the securing of it from risk or accident, and making it available for himself or his family.

To guard his property from the encroachment of his neighbours, he seeks protection from the Laws and Government of his Country; to secure it from the risk of the Elements, he has recourse to Marine and Fire Insurances; to make it available for himself, so as to mete out the savings of youth and industry for the support of declining years, he may sink his Capital upon a Life Annuity; but to render *certain* his *Contingent Interests* and *Expectations*, and in case of premature death to secure to his family a substitute for its most valuable Capital, the Industry of its Protector, are the legitimate objects of Life Assurance.

Few of these objects can however be realized by man in his individual capacity, as they are only attainable by a number of persons uniting to form a common Fund for their mutual benefit; by this means, as they are alike subject to accident, all have an equal right to claim compensation, although the unfortunate must always gain an advantage from the contributions of his more fortunate companions.

Upon this principle of association the Rates or Premiums for Life Assurance are deduced, grounded upon the improvement of money, and the average duration of human life, so that the loss sustained by the society, through the premature death of one assurer, may be compensated by the greater longevity of another.

Without placing themselves under a kind of voluntary obligation, persons of moderate incomes are often tempted to consider their savings at the year's end so trifling as to be scarcely worth appropriating for the benefit of their families, and are therefore induced to lay out their surplus upon some species of luxury or artificial wants, or in assisting their friends often to their own prejudice; and, with the disadvantage attending the accumulation of small sums, even the most prudent characters thus situated, must, in case of premature death, leave their families to pine upon scanty pittances in a state of society with which

they must be totally unacquainted, and therefore the less able to endure the unavoidable difficulties with which they find themselves suddenly surrounded, by the unexpected loss of their Parents or Protectors.

To this class of persons, as well as to all others whose incomes depend upon the uncertain duration of human existence, Life Assurance is peculiarly adapted. For instance, to a young man in a liberal profession, with a small family dependent on his exertions, anxiously expecting the benefits of increasing reputation, what satisfaction, on a premature death-bed, can equal that arising from a consciousness of having at a trifling sacrifice anticipated the event, and made the produce in some degree available for the objects of his affection and solicitude? or, even in case of his having no family to provide for, it must be some consolation to himself and to his connections, in the event of an early death, to be able to secure the capital consumed in his education for the benefit of a successor, who may have a chance of being more fortunate in making it productive.

Instances are frequently occurring of persons possessing extensive capitals in business or partnership concerns dying, and leaving their families to sink in absolute want before their affairs can be wound up, or the relative shares of the different parties adjusted; whereas, by appropriating but

a small proportion of their annual profits to Life Assurance, such characters may make a certain and available provision for their survivors, independently of the uncertain produce of their concerns, or the caprice of those with whom they may have been connected.

The advantages of Life Assurance may likewise be extended to the higher classes of society, as the life heir to an entailed estate may avail himself of this means to provide for the younger branches of his family, without hazarding the risk of leaving them dependent upon his friends or successors. The next heir to an entailed estate, which, in case of his death happening before that of the present possessor, reverts to another branch of the family, may secure to his survivors an equivalent by assuring a sum payable at his death, in case the present possessor shall outlive him. Thus a person of 20, heir expectant to an estate on the demise of another person aged 60, may secure £1000 to his family in the event of his dying without coming into possession, at an annual premium of about £15.

It would be too great a digression from the object of this work to enter at length into the various advantages of Life Assurance, and its application to the securing of all sorts of contingent interests and expectations: to what has been stated we shall therefore merely add, that the practice, though comparatively of recent invention, has

already been attended with incalculable advantages in this country, by supplying the means of comfortable subsistence to many thousands of families which would otherwise have been left destitute; and when it is considered that the very nature of the practice is calculated to encourage industry, stimulate forethought, and diminish the number of those who may be unable to provide for themselves, Life Assurance properly conducted and generally embraced may be regarded in the light of a National Benefit.

In proportion to the benefits of Life Assurance, and to the extent of capital annually invested therein, is the importance that the public should enjoy these advantages with the least possible sacrifice of income that may be compatible with their *security;* or in other words, upon such rate of premiums as the nature of the risk, when viewed with all its reasonable contingencies would warrant and demand.

The evil of charging excessive premiums cannot however long remain in a country where capital is allowed to flow freely from one channel to another, as the natural effects of competition must necessarily reduce the profits on Life Assurance to the level of that derived from other species of investments; on the contrary, the peculiar nature of the subject renders it extremely dangerous lest the Rates for Life Assurance should be so far reduced as to diminish the

security of those who may select this mode of accumulating their savings for the benefit of their families; for if the premiums charged by societies established for these purposes should by excessive competition be rendered inadequate to the payments of the claims, which sooner or later must come upon them, whatever honour, wealth, or probity the present managers of them may possess, whatever capitals they may boast of, or however prosperous they may appear to go on even for a considerable time, the result must ultimately terminate in litigation, disappointment, and ruin; and instead of a National Benefit, Life Assurance in such a case would inevitably become a National Calamity.

The success of the Equitable Society, and the immense surplus which, by cautious measures and favourable circumstances it acquired, have induced many to over-rate the profits on Life Assurances, and contributed in no small degree to increase the danger lest other institutions, formed at a period less favourable to accumulation, should adopt Rates inadequate to defray the necessary expences of management, after providing for the various contingencies, such as the fluctuation of Interest and an increased mortality, which ought to be regarded for the security of the Assured.

This extraordinary success has of late been most improperly urged as an argument for reducing the Rates for Life Assurance, without

any reference to the peculiar advantages which that institution enjoyed; for it ought to be considered, that the immense surplus it acquired was in a great measure derived from sources which are entirely without the reach of more modern institutions.

By its Deed of Settlement it appears, that the premiums originally charged for each £100 insured, were, on a life of

		Per Annum.
14 for one year,..£1 11 9	For Life,..£2 7 7	
20 .....1 15 6	..... 2 15 4	
30 .....2 4 6	..... 3 12 3	
40 .....3 2 0	..... 4 12 2	
50 .....4 4 8	..... 5 18 4	
60 .....6 4 10	..... 8 5 2	
67 .....7 18 1	.....11 18 8	

Added to which, each Insurer was subject, by the 57th clause, to an extra charge of 15s. Entrance-money for every £100 assured; and by the 65th, to another charge of 10s. or 20s. per cent. Deposit according to the term for which the Assurance was effected, which by the 72nd was made returnable only at the expiration of the Policy. Both these charges were continued for eight years after the establishment of the society, when the latter was annulled, and the former reduced to 5s. per cent. at which sum it still remains.

These charges, however excessive they may appear, particularly on the short period Assurances, were not the only advantages which the Equitable enjoyed; for Mr. Morgan, in the Preface to the

last edition of his work on Assurances, referring to the observations he had made in 1779, seventeen years after the establishment of the society, states, that "the Assurances for the benefit of "surviving families at this period were but few in "comparison with those which were made on the "lives of those improvident persons, who, in the "disposal of their property, seemed to have as "little consideration for their families as for themselves; and as the price of an annuity on a life, "however young, very rarely exceeded seven years "purchase, the Assurances were seldom made for "a longer term, so that a *very small proportion was* "*made on the whole continuance of life*, or with any "other view than to secure a purchaser from the "risk of losing his Annuity."

And in his Address to the General Court of the Equitable, in April 1800, page 164, the same Author states, that "the premiums required at its "first establishment, and which were continued for "nineteen years, were in most cases twice as high "as they are at present;" and in page 165 adds, "another, and by no means an inconsiderable, "source of profit, is the great number of assurances, which, from neglect or ignorance, are "annually forfeited to the society. Nay, so numerous have these forfeitures been in the *early* "*periods of the society*, that I do not believe *one* "*half* of the assurances which were made during "the first twenty-five years for the whole life,

" have been either continued till they became claims " or even surrendered for a valuable consideration."

These advantages, with the low price of Government securities during the late war and the consequent high rate of Interest at which its funds were uniformly invested for a long period, and the enormous increase in the money value of its capital within the last few years, could not fail to make this Society an object of national importance.

These sources, from which the immense surplus of the Equitable has been chiefly derived, have for some time pretty nearly ceased to exist, as the public are now aware that when the objects of their Assurances are realized, or their circumstances render it difficult for them to continue their payments, their Policies may be surrendered for a fair consideration, or sold in the market, sometimes for much more than what they are really worth. It is therefore well known to those connected with Assurance Offices, that only a small proportion of the Assurances now effected are made for limited periods, and still smaller proportion forfeited by non-payment of the premiums.

Laying aside the adventitious sources of profits already detailed, and comparing the premiums originally charged by the Equitable for the whole period of Life with those deduced from more recent observations on the duration of human Life, and regarding at the same time the superior talents of the late Thomas Simpson, James Dodson, and Dr. Price,

who were instrumental in forming that Society, and in fixing and modifying its rates as circumstances appeared to warrant, it surely must be absurd to attribute the want of information as the sole cause of their fixing the premiums at so high a standard; is it not as likely that their extensive researches into remote contingencies induced them to guard against those events which modern projectors of Assurance Offices have probably overlooked? Nor can it be supposed that want of information alone induced the Law Officers of the Crown at the close of the last century to reject the application of the Equitable for a Charter, on the ground that its premiums were insufficient; for the unexpected and almost incredible success of that Institution has certainly proved, that calculations relative to matters depending on such numerous and remote contingencies can at best be viewed but as loose approximations.

Further we observe that, however successful the Equitable and some other Institutions may have hitherto been, it would be no less than madness, under present circumstances, to expect that the like advantages can be realized in future; for, setting aside the exclusive advantages spoken of by Mr. Morgan, it is manifest that if the increased capital of the country should either cause a continuation of the present rate of Interest, or produce a further diminution in the same, the power of accumulation must be proportionably crippled: for instance, at

5 per cent. £100 in 50 years would amount to no less than £1146 15s; but at 3 per cent. in the same period it would only amount to £438 8s.; and should another protracted war happen, so as to increase the Rate of Interest on the premiums then received, it could little enhance the accumulation of the Funds previously invested, but on the contrary produce a most material diminution in the money value of that part laid out on Government securities. Nor is it less a delusion to expect at present, that the power of accumulation would be materially increased by investing the Funds on Mortgages and Annuities, or in the purchase of Reversionary property; for competition, produced by a superabundance of unemployed capital, has already reduced the profits on these speculations pretty nearly to a par with that derived from other modes of investments, taking into consideration the additional risk thereby incurred. And it is perhaps doubtful, whether the purchase of Reversions ever has been as profitable as the public were induced to suppose, from the mode by which they have been most commonly valued. For if the Northampton table has been found to afford to the grantors of Life Assurances, who are in fact the sellers of Reversions, a greater profit than that contemplated in the calculation, it necessarily follows that it must have had a contrary effect on the contemplated profit of those who may have invested their money in purchasing Reversions. Thus, by the Northampton table, the present value of £100 to be

received at the end of the year in which a life of 30 may fail, is at 5 per cent. £32 19s: whereas, according to the valuable table formed by Mr. Milne from the mortality at Carlisle, the present worth of the like sum to be received on the same contingency is but £31 6s 9d, allowing only 4 per cent. interest to the purchaser; if, therefore, an Insurance Office invested its premiums on a number* of similar Reversions according to the former valuation, with a view of making 5 per cent. of the purchase money, it is manifest that if the average duration of human life should approximate to the latter, the actual profit (supposing the securities &c. all good) would barely amount to 4 per cent. And even supposing an Insurance Office occasionally did find a favourable investment for small proportions of its funds, the bulk must be laid out in Government securities, or some other species of property, of a comeatible nature, bearing not much greater interest; so that the accumulation of the aggregate cannot by these means be materially accelerated. Nor is there any good reason to suppose that advancing money on Post-obit Bonds, and Deferred or Survivorship Annuities, will upon the long run be found more advantageous; but on the contrary, too much reason to fear that (notwithstand-

* From the uncertain duration of human life, the purchase of a single reversion is no other than lottery, and any thing like an approximation to a certain profit can only be derived from an average of a considerable number.

ing the avidity with which some of the Insurance Offices entertain these speculations) such modes of investments are pregnant with endless litigations, and the most ruinous consequences. And as to the practice of granting Life Annuities, though upon adequate terms beneficial to Insurance Offices, in case of seasons of more than common mortality, it can but little aid the improvement of their funds. Nay, at the rates held out by some of the Insurance Companies, such practice must be attended with inevitable loss. For instance, suppose a person on the eve of completing his 35th year, were to insure his life for £102 12s 4d with one of the new Assurance Companies, at an annual premium of £2 11s per cent., or £2 12s 4d for the whole, and that a few days after, when he shall have completed his 35th year, he were to sink £100 on annuity at the same Office, where he would be allowed £6 13s 5d* per annum for life; such annuity would enable him to pay his future premiums so to secure his capital (£102 12s 4d) at death, and also yield him an annual interest of £4 1s 1d for the £102 12s 4d paid by him in the first instance, which abstracting from the expences of deeds, that do not benefit the Company, is a clear interest of £3 19s per cent., and considerably more than the Institution can make of its money,

* Until very recently the Company from whose prospectus these rates were taken, granted an annuity of no less than £7 3s 8d on a life of 35.

when £95 are required to purchase £100 consols, bearing an interest of only £3 per annum. It therefore follows that at these rates, either Life Annuities or Life Assurances must be unprofitable, or else that whatever profit accrues from the assurance effected is more than swallowed up by the annuity granted; and if this be the case, from what source are the expences of management to be defrayed, and the inordinate expectations of the Shareholder to be realized, taking also into consideration that "a "liberal allowance is granted to Solicitors and others "recommending business to the Office," and that a considerable proportion of the receipts must lie for some time unproductive in the Bankers' and Agents' hands?

The diminution of mortality has also been urged to favour the reduction of the rates for Life Assurances; but granting that with the improvement of Medical science, and the successful extermination of several diseases by which our predecessors have suffered, connected with increased knowledge and more comfortable means of subsistence, as well as improved habits, greater degree of cleanliness, ventilation, &c. the Rate of Mortality to have been considerably reduced, it follows that the very fact of its being variable must strengthen the idea of the *possibility* of its again increasing with other changes of circumstances, which *may* take place before a considerable proportion of the risk now undertaken by Insurance Companies shall have been discharged.

It would therefore be nothing less than a delusion to suppose that even with the best means of information any set of Rates could be formed which would afford that security to the Assured which the nature of the subject demands, without allowing considerable margin for possibilities. Nor can it be fairly argued, that the Assured are injured by such a surplus premium as would afford a reasonable provision against these possibilities; although with favourable circumstances it might happen, that the overplus should tend to benefit the proprietors for the security afforded by their capital in the event of contrary circumstances happening, or to benefit the longest survivors of those that are mutually insured; for in case of premature death the representatives of the Assured must necessarily be benefited by the common fund, and surely, as an alternative of two evils, it must be more reasonable that those who may live the longest and contribute the most to that fund, should be benefited by an overplus of subscription in the way of Bonuses, than injured through their longevity by being the parties who must necessarily suffer in the event of inadequate contributions.

Another topic frequently alluded to as an argument for reducing the Rates of Life Assurances is *the selection of lives,* by which it is supposed that assured lives are materially better than a similar number taken indiscriminately from the community at large. That a very considerable advantage accrues to an Insurance Office by the rejection of bad lives, is too evident

to be denied; for without this caution there is reason to suspect that assured lives would be considerably worse than the average, as their acceptance could not fail to induce persons of weak constitutions to have recourse to Life Assurance for the benefit of their survivors; while those of robust health might think it more to their advantage to improve their own savings for the benefit of their families. But, whether the effect of selection from the class of persons composing the majority of those who have recourse to Life Assurances renders them better than the average, is not so evident, as it will appear by the Table herein deduced from the experience of the Equitable that the mortality among the members of that Institution approximated exceedingly near to that which obtained during the same period among the inhabitants of the town of Carlisle, shown by Mr. Milne's Table, which, there is reason to believe, with its author, affords a pretty fair index of the contemporaneous mortality among the community at large throughout England and Wales.* But even granting the mortality among

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* "Due weight..... it will then be found, that the "rate of mortality in the two parishes which include that city (Carlisle) "and its environs, has been nearly the same during the last 30 years, "as throughout England. So that although the Carlisle table has been "constructed from observations made upon two parishes only, the law "of mortality it exhibits probably differs very little from the general "law that obtains throughout the kingdom, taking towns and country "together; if we except the children under five years of age, or at most "those under ten."—*Milne on Annuities, Vol. II. pages* 450, 451.

the members of the Equitable to have been considerably less than the average among the community at large, the present competition and the possibility of fraud attending the practice of employing agents and medical examiners in distant parts of the Empire, not only increases the expence, but renders it doubtful whether, even with the advantage of a greater number of country lives, the persons assured by other Institutions are likely to average an equal longevity.

Another subject deserving serious attention is the comparative merit of Proprietary Companies and Societies for Mutual Assurances, and on which it may probably be suspected that the author, from the nature of his situation, cannot express an unbiassed opinion. Theoretically considered, it must be evident that, when favourable circumstances render a guarantee capital unnecessary, a greater advantage must accrue to the assured from Societies for Mutual Assurances established upon adequate rates, and conducted by wholesome regulations, than what can be realized from Proprietary Companies, where either a part or the whole of the profits reverts to the shareholders: and by reference to the Equitable and London Life Association, the two principal Institutions established in London for Mutual Assurances, it would seem that, practically considered, the advantages in their favour, as compared with other Institutions, not only supports that theory, but, owing to the favourable circumstances already alluded to, even surpassed the expectations of the most sanguine theorist. It is how-

ever possible that a little more attention to the subject may induce the public to entertain a less favourable opinion in future, as to the superior merits of the system of Mutual Assurance: for upon the long run, it will probably be found, that this system possesses one defect, in a greater degree than Proprietary Companies, and to which a remedy cannot easily be applied, which defect arises from the want of some distinct interest, to check the avaricious dispositions of the majority of those that are mutually assured.

The supereminent talents and amazing assiduity of the venerable character who has so long watched over the affairs of the Equitable, aided by a most cautious and firm line of conduct on the part of those on whom the management devolved, have hitherto successfully checked, or at least for a time smothered this disposition; but by a reference to his valuable addresses, it will be found that, notwithstanding all his efforts, that disposition, though kept down, he always entertained serious apprehensions that it was not obliterated: and notwithstanding the peculiar mode of division of profits in that Institution secured to the older members the full benefit of the surplus to which they were entitled,* and the regulation of 1816, limit-

* In his Address to the General Court, of April, 1800, page 167, Mr. Morgan, alluding to the advantages derived up to that period by those who had been for some time assured, says—"Were no further additions, "therefore, to be ever made to the Assurances, the *older Members*, at "least, would have the greatest reason to be satisfied with those benefits "which they already enjoy, and which must have exceeded their highest "hopes and expectations."

ing the number of participators to 5000, even gave them a monopoly, which at that time nothing but the alternative of two evils could warrant or excuse, recent occurrences afford indisputable proof that this passion, in proportion as it is fed, acquires increased strength, and seriously threatens to become, not only insatiable, but incapable of being controuled by any other means than the overwhelming power of the Court of Chancery.

CHAPTER II.

ON THE IMPROVEMENT OF MONEY.

Section I.

General Observations on Interest.

Interest is the compensation allowed for the loan or forbearance of a sum of money; the Rate of which is estimated according to the sum allowed for the use of £100 for one year. What is called interest may be distinguished into two parts:—First, that which is allowed for the mere use of the sum advanced; and, secondly, the premium charged for the risk of losing it.* Thus, the lender of a sum of money has an undoubted right to such compensation, for the advantage which he foregoes, in lending his capital to another, as would be afforded him by the ordinary rate of profit, if he retained it under his own controul, or laid it out in

* This distinction is of vital importance to Insurance Companies whose premiums are grounded on the improvement of money; as it is manifest that their calculations should be regulated by the compensation allowed for the *use* of money, and not by the casual rate which they may occasionally make by speculative investments.

such investments as are least subject to risk of loss or diminution; and, according to the nature of the security given, he has also an equal right to a reasonable compensation for the risk of losing, either wholly or in part, the capital he advances.

It is not, perhaps, an easy matter to determine, in what proportions these component parts make up the rates charged in various modes of investment, as it is evident, that the first must necessarily vary, according to the relative proportions of supply and demand, the stability of Government, the wealth of the country, the progress of population, and the different circumstances of peace and war; while the latter, also, must be equally subject to all degrees of variation, from the most perfect security to the wildest speculation. If, however, we regard the sentiment of our forefathers, in the application of the term *Real Property* to Freehold Estates, and the more modern doctrine of Political Economists, who hold, that the rate of profit on all other commodities is generally regulated by the returns afforded by Land, in the shape of Rent,—it is possible that we shall not be far from the truth, in laying down this standard as the measure of compensation charged for the *use* of money; and, if this be once determined, setting aside personal securities, it is not impossible that the number of Joint-Stock Investment Companies now existing

will, in due time, ascertain the additional premium which should be charged, in cases where money is advanced on Annuities, Reversions, Mortgages of Houses, &c., so as to afford, on an average of several investments, the necessary compensation for the risk of losing the sum advanced, as well as for the expences and delay which may be incurred in rendering it productive.

The Interest of money, where no life contingency is involved, is usually limited by legislative enactments; but, like the hire of any other commodity, that of money will always be found to bear a relation to the ordinary rate of profit afforded by the employment of capital, notwithstanding any restrictions put upon it. In 1545, the legal Rate of Interest was fixed at 10 per cent.; in 1624, it was reduced to 8 per cent; in 1651, to 6 per cent; and, in 1714, to 5 per cent., at which it still remains. If, however, a contract, bearing interest, were made in a foreign country, our Courts would direct payment of interest according to the law of that country; and, by statute 14 Geo. III., c. 79, all mortgages and other securities upon estates in Ireland, or the Plantations, bearing interest not exceeding 6 per cent., were made legal, though executed in Great Britain, unless the money lent should be known at the time to exceed the value of the property pledged.

From 1731 to the close of the last year, 1824, the Rate of Interest, afforded by Government Three per Cent. Consolidated Annuities, will appear by the following Table:—

Peace or War.	Year.	Average Price of Stock.	Average Interest per Cent.	Peace or War.	Year.	Average Price of Stock.	Average Interest per Cent.	Peace or War.	Year.	Average Price of Stock.	Average Interest per Cent.
		£	£ s. d.			£	£ s. d.			£	£ s. d.
Peace	1731	96	3 2 6	Peace	1763	89	3 7 6	War	1795	66	4 11 0
—	1732	98	3 1 3	—	1764	84	3 11 6	—	1796	62	4 13 0
—	1733	100	3 0 0	—	1765	88	3 8 3	—	1797	51	5 17 9
—	1734	92	3 5 3	—	1766	89	3 7 6	—	1798	50	6 0 0
—	1735	95	3 3 3	—	1767	89	3 7 6	—	1799	58	5 3 6
—	1736	102	2 18 9	—	1768	91	3 6 0	—	1800	63	4 15 3
—	1737	106	2 16 6	—	1769	88	3 8 3	—	1801	61	4 18 6
—	1738	104	2 17 9	—	1770	83	3 12 3	Peace	1802	70	4 5 9
War	1739	101	2 19 6	—	1771	86	3 9 9	War	1803	59	5 1 9
—	1740	99	3 0 6	—	1772	88	3 8 3	—	1804	56	5 7 3
—	1741	95	3 3 3	—	1773	87	3 9 0	—	1805	58	5 3 6
—	1742	100	3 0 0	War	1774	88	3 8 3	—	1806	61	4 18 6
—	1743	101	2 19 6	—	1775	89	3 7 6	—	1807	62	4 13 0
—	1744	94	3 3 9	—	1776	84	3 11 6	—	1808	66	4 11 0
—	1745	89	3 7 6	—	1777	78	3 17 0	—	1809	68	4 8 3
—	1746	82	3 13 3	—	1778	64	4 13 9	—	1810	68	4 8 3
—	1747	83	3 12 3	—	1779	61	4 18 6	—	1811	64	4 13 9
Peace	1748	86	3 9 9	—	1780	61	4 18 6	—	1812	59	5 1 9
—	1749	98	3 1 3	—	1781	58	5 3 6	—	1813	58	5 3 6
—	1750	100	3 0 0	—	1782	58	5 3 6	—	1814	56	4 11 0
—	1751	100	3 0 0	—	1783	64	4 13 9	Peace	1815	59	5 1 9
—	1752	104	2 17 9	Peace	1784	57	5 5 3	—	1816	61	4 18 6
—	1753	104	2 17 9	—	1785	60	5 0 0	—	1817	75	4 0 0
—	1754	103	2 18 3	—	1786	74	4 1 0	—	1818	78	3 17 0
War	1755	96	3 2 6	—	1787	74	4 1 0	—	1819	71	4 4 6
—	1756	89	3 7 6	—	1788	75	4 0 0	—	1820	68	4 8 3
—	1757	90	3 6 9	—	1789	77	3 18 0	—	1821	74	4 1 0
—	1758	92	3 5 3	—	1790	77	3 18 0	—	1822	79	3 16 0
—	1759	83	3 12 3	—	1791	83	3 12 3	—	1823	79	3 16 0
—	1760	82	3 13 3	—	1792	90	3 6 9	—	1824	93	3 4 6
—	1761	79	3 16 0	War	1793	76	3 19 0				
—	1762	75	4 0 0	—	1794	70	4 5 9				

For the whole period of.....94 years } Average Price Stock £79 5*s.* Averag. Int. £3 15*s* 8*d.*

For the.....46 do. Peace .....£85 6*s.* .....£3 10*s* 4*d.*

For the....48 do. War .....£73 10*s.* ..... £4 1*s* 8*d.*

This Table cannot, however, afford a just criterion of the Interest actually realized during the period comprehended therein, for, where the money was not allowed to remain, the interest realized must have been determined not only by the price at which the investment was made, but by that at which the capital was withdrawn. Thus, a person who bought into the funds in 1750, and sold out in 1785, made considerably less interest than that shown by the Table, because his principal must have been reduced in the ratio of 100 to 60, or of 5 to 3. On the contrary, a person who invested when stock was at 50, and sold out at 93, must have realized far greater interest than that shown by the Table, as his principal must have been nearly doubled in the interval: but it serves to show that, notwithstanding the uniformity in the legal Rate of Interest, the rate at which money could be improved has varied from about $2\frac{3}{4}$ to 6 per cent.; nor is it improbable, that the average rate it exhibits is somewhat lower than what is commonly supposed by those who have formed their notions of the Interest of money from what is generally charged on personal securities, what the law allows, or what they have been accustomed to make upon Government Securities, during the period over which their memories can grasp, without sufficiently considering, that, during the greater part of that period, the country was involved in expensive

wars; and for some years after its termination, not only labouring under the excessive weight of the debt thereby incurred, but suffering severely by a transition from a state of things, which a number of years had rendered habitual.

At the present time, Landed Property barely returns 3 per cent.; Eligible Mortgages on Freeholds, 3½ per cent.; Government Three per Cent. Perpetual Annuities, with the chance of a reduction of capital, in the event of Stock falling, yield only £3 3s per cent.; Long Annuities, for 35 years, selling for 23 years' purchase, afford to the purchaser (over and above the repayment of his principal,) no greater interest than 2½ per cent.; while Exchequer Bills, bearing a premium of about 3 per cent. and an interest of only 1½d per day for each £100, yield to the purchaser no greater interest than 2⅛ per cent. On comparing these securities, it will also be found, that Exchequer Bills bear a less interest than any other, because a person, advancing his money on them, suffers the least risk of a diminution of principal, on which account they are generally considered among the most eligible securities for temporary investments. The Long Annuities also bear considerably less interest than some of the other securities, because they afford a greater annual dividend, and cannot be paid off or reduced till they expire in 1860.

By reference to the different kinds of Public

Securities, it will also be found, that Three per Cent. Annuities bear a less interest than the Three and Half or Four per Cents., because they are more permanent, and less liable to a diminution of interest, and can only be paid off at £100 money for £100 Stock, while no greater money-value will be allowed for the others, in the event of their being paid off. The relative prices of these securities also vary, according to the times when the dividends become due. Thus, about the beginning of March, the Three per Cent. Reduced will generally be found to bear a higher price than the Three per Cent. Consols, because the dividends on the former become due in April, while those on the latter are not payable till July. Independent of these considerations, Table first, at the end of this work, exhibits the relative prices of different Funds, according to the annual interest which they respectively produce.

Simple Interest.

2. INTEREST is also distinguished into *Simple* and *Compound*. *Simple Interest* is that which is computed only on the sum advanced or forborn, and is supposed to be payable at stated periods, such as yearly, half-yearly, or quarterly, as may be regulated by custom, or agreed upon between

the borrower and the lender.* In the calculation of Interest, the following particulars are to be regarded, viz.

The *Principal*, on which interest is to be computed.

The *Time*, for which it is lent or forborn.

The *Rate*, at which the calculation is made.

The *Interest*, which is proportional to the Principal, Rate, and Time; and

The *Amount*, or sum of the Principal and Interest.

Instead of employing the sum allowed for the use of £100 for one year, it will be found more convenient in calculation to designate the Rate by the interest of £1 for the same time.

Thus, at 2	per cent., the interest of £1 is	.02	Amount 1.02
..... $2\frac{1}{2}$	.....	.025	..... 1.025
..... 3	.....	.03	..... 1.03
..... $3\frac{1}{2}$	.....	.035	..... 1.035
..... 4	.....	.04	..... 1.04
&c.	&c.		&c.

Then, taking 4 per cent, since .04 is the interest of £1 for one year, proportionably twice .04 or .08 must be the interest of £2 for the same time; three times .04 or .120 the interest of £3; and so on. And since .08 is the interest of £2 for one year, twice .08 or .160 gives the interest

* On Government Securities, Exchequer Bills excepted, the Interest is usually paid half-yearly.

for the like sum for two years, three times for three years, &c.; from which we deduce the following

RULE.—*Multiply together the Principal, Rate, and Time, and the product will be the Interest. Add the Principal and Interest together, and the result will be the Amount.*

Thus, the Interest of £240 12s, at 4 per cent. for $2\frac{1}{2}$ years, is $240.6 \times .04 \times 2\frac{1}{2} = 9.624 \times 2\frac{1}{2} = 24.060 =$ £24 1s $2\frac{1}{2}$.

Amount = £240 12s + £24 1s $2\frac{1}{2}$ = £264 13s $2\frac{1}{2}$d.

Similarly, the Interest of £784 6s 6d for 7 months, at 5 per cent. is $784.325 \times .05 \times \frac{7}{12} = 39.21625 \times \frac{7}{12} = 22.876 =$ £22 17s $6\frac{1}{4}$.

Amount = £784 6s 6d + £22 17s $6\frac{1}{4}$d = £807 4s $0\frac{1}{4}$.*

In order to generalize what has been stated,

Put p for the Principal, in pounds;

— t for the Time, in years;

— i for the Rate per pound, and

— a for the Amount.

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* In these and all other calculations, relative to Interest, &c. the reader will find a very considerable advantage, by a thorough acquaintance with decimal fractions, the contracted method of multiplication and division, and particularly the mode of converting shillings, pence, and farthings, into decimals of a pound, and vice versâ, by a mental process only. For these purposes, he may consult works on Arithmetic.

Then we have $a=p+pit$, or $=p(1+it)$, and $pit=a-p$, from which may be deduced,

$$p=\frac{a}{1+it}$$

$$i=\frac{a-p}{pt}$$

$$\text{And } t=\frac{a-p}{pi}$$

By these formulæ, any three of the above quantities being given, the fourth may be found.

3. Interest is not only applicable to find the accumulated amount of a sum of money, lent or forborn for time past, but also to determine the present values of future payments. By the *present worth* of a sum of money, to be received at a future period, is understood, that which, being laid out and improved at an assigned rate of interest during that period, will just amount to the proposed sum by the time it becomes due. The difference between the present worth and the sum itself is termed the *Discount*.†

† In mercantile transactions, the *Interest* is usually taken for the *Discount;* but, whatever sanction custom may have given to the practice, it is nevertheless incorrect, and, in some cases, of doubtful legality. Thus, supposing A, holding a bill for £100, due one year hence, applies to B to discount the same, at 5 per cent.; if B takes £5 as a discount, and gives to A only £95, does he not subject himself to the laws of usury, by claiming the £100 in payment at the end of the year, when the sum he advanced, with 5 per cent. interest thereon, only amounts to £99 15s.?

Thus, at 5 per cent. £100 is the present worth of £105, due one year hence; because that sum laid out and improved for one year, at 5 per cent. just amounts to the £105.

Since it is therefore manifest, that to find the present worth of a sum of money is no other than to determine what Principal, improved at an assigned rate for a given time, will amount to a given sum, from the formula $p=\frac{a}{1+it}$ deduced in the preceding page, we may derive the following

RULE.—*Divide the proposed sum by the amount of £1, improved at the assigned rate for the given time, and the quotient will be the Present worth. Subtract the present worth from the sum proposed, and the remainder will be the Discount.*

Thus, let it be required to find the Present worth of £480, due $2\frac{1}{4}$ years hence, reckoning interest at 4 per cent.?

Here, by Section 2, the interest of £1 for $2\frac{1}{4}$ years, at 4 per cent, is $.04\times 2\frac{1}{4}=.09$, its amount $=1.09$, and $480\div 1.09=440.367=$£440 7s 4d, the required Present worth. This again deducted from the £480, leaves £39 12s 8d for the Discount.

The Interest on £480, in $2\frac{1}{4}$ years, at 4 per cent. would be no less than £43 4s.

Similarly, the Present worth of £600, due 9 months hence, at 5 per cent. is $600\div(1+.05\times\frac{3}{4})=600\div 1.0375=578.313=$£578 6s 3d. Discount = £600 − £578 6s 3d = £21 13s 9d.

All the formulæ, deduced in the last section, are equally applicable in this, by merely substituting a, for the future sum to be received, and p, for its Present worth.

By the principles explained in this section, the third Table, at the end of this work, has been calculated.

4. Another case of frequent occurrence, and closely connected with the subject of Simple Interest, is the method of determining the Rate of Interest or Annuity, when the Principal and annual produce are given. For this purpose,

Divide the annual produce by the principal, and the result will be the rate per pound, from which the rate per cent. is immediately determined.

Thus, suppose $95\frac{1}{8}$, or £95 2s 6d, to be invested in the purchase of £100, Three per Cent. Stock; required the rate of Interest afforded by the purchase?

Here, the principal invested is £95 2s 6d, and the annual produce £3, exclusive of the advantage gained by the half-yearly payment of the Dividends. Hence, $3 \div 95.125 = .03154$ for the rate per pound, which, on £100, by removing the decimal point two places to the right, becomes 3.154 = £3 3s 1d for the rate per cent. as required.

This mode is no other than a contraction of the proportion £95 2s 6d : £3 : : £100 : £3 3s 1d, and therefore requires no further explanation.

Again, let it be required to determine the annual dividend per cent. yielded by the Long Annuities, when they sell for $23\frac{3}{8}$ years' purchase?

In this case, £23 7s 6d produces an annual dividend of £1. Hence $1 \div 23.375 = .04281$ per pound, or $4.281 =$ £4 5s $7\frac{1}{2}$d per cent. the rate required.

Moreover, suppose it were required to determine the rate of Annuity granted by Government on a life of 70, when the price of Consols is $94\frac{1}{2}$?

By the Government Table we find, that the Annuity granted on a life of 70, when Consols are at 80 or upwards, is £11 11s for every £100 *Stock;* and, if this Stock cost £94 10s money, we have $11.55 \div 94.5 = .12222$, the rate per pound, or $12.222 =$ £12 4s $5\frac{1}{4}$, the rate per cent. as required.

But, to determine the Purchase-money for a *given Annuity*, we must invert the proportion, or

Divide the price of Stock by the rate granted on the assigned life, and the quotient will be the answer in years purchase; and this, multiplied by the proposed Annuity, gives the required Purchase-money.

Thus, on a life of 70, the rate granted by Government for £100 Stock is £11 11s, when Consols are at $94\frac{1}{2}$. Hence, $94.5 \div 11.55 = 8.1818$ for the price of a given Annuity in years purchase, or, in other words, the price of each pound Annuity on the proposed life; and, if the Annuity proposed be £50, the Purchase-money will be $8.1818 \times 50 = 409.090 =$ £409 1s. $9\frac{1}{2}$.

Similarly, if it be required to find the value of a Perpetual Annuity, or a Freehold Estate, when money yields but 3 per cent. we divide the principal by the annual produce, and obtain $100 \div 3 = 33\frac{1}{3}$ for the required value in years purchase; and, if the Annuity or net Rental of the Estate be £300, the purchase-money will be $33\frac{1}{3} \times 300 = £10{,}000$.

For the Rates of Life Annuities, granted by the Government Office in the Old Jewry, see Table ii: those columns that are not found in their printed Tables have been calculated in the manner explained in this Section.

Compound Interest.

5. COMPOUND INTEREST is that which arises when the Simple Interest, due at the end of any period, instead of being paid, is added to the principal by which it was produced, thereby forming an increased principal, upon which Simple Interest is allowed during the succeeding period.*

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* Compound Interest is not allowed by law on a sum lent or forborn, as the party entitled to receive the Interest is supposed to have his remedy in demanding it as it becomes due; but, in the calculations of the Present worths of Leases, Annuities, &c. and on the running Accounts of Merchants and Bankers, whose balances are periodically advised and acknowledged, Compound Interest is charged and allowed.

In this subject, it is important to regard the periods at which the Interest is convertible into Principal, because that circumstance makes a difference in the annual rate thereby produced. Thus, if the Interest be convertible into Principal half-yearly, at the *nominal* rate of 5 per cent. but, correctly speaking, at the rate of $2\frac{1}{2}$ *per cent. half-yearly*, the amount of £100, at the end of the first half-year is £102 10s; and $2\frac{1}{2}$ per cent. on this again, during the succeeding half-year, gives £2 11s 3d for the second half-year's interest; so that the amount of the £100, at the end of the year, by this means becomes £105 1s 3d. In like manner it may be shown, that at the nominal rate of 5 per cent. if the Interest be convertible into Principal quarterly,

	£.	s.	d.
The amount of £100, at the end of the 1st quarter, is	101	5	0
..... 2nd	102	10	$3\frac{3}{4}$
..... 3rd	103	15	$11\frac{1}{2}$
..... 4th	105	1	$10\frac{1}{2}$

And from this it is manifest, that the shorter the periods of conversion be taken at a given nominal rate, the greater will be the annual interest.

As Compound Interest is nothing more than a repetition of Simple Interest, the method of calculation is sufficiently evident by what has been stated in Section 2. This method, however, is tedious when the term consists of more than two or three years, and in order to abridge the labour it has long

been found convenient to have recourse to Tables, the construction and use of which we shall here explain.

Since, by Section 2, the amount at Simple Interest is universally expressed by the formula $a=p(1+it)$, by making $t=1$, the amount of p pounds, in one year, becomes $=p(1+i)$, and, by substituting this amount for the principal, upon which interest is to be computed during the second year, the amount at the end of two years becomes $p(1+i)\times(1+i) = p(1+i)^2$. Similarly it may be shown, that

The amount of p pounds, in 3 years, becomes $p(1+i)^3$
. . . . . 4 . . . . . $p(1+i)^4$
. . . . . 5 . . . . . $p(1+i)^5$

and so on. And since these amounts are all proportional to the principal originally invested, it is manifest that, if the amount of £1 were given for any period, that of any other principal might be found by a single operation. Hence, making $p=1$, and calculating at 4 per cent. we have

$1+i$	$= 1.000000\times1.04 = 1.040000$	= Amount of £1 in	1 year.
$(1+i)^2$	$= 1.040000\times1.04 = 1.081600$	=	2 years.
$(1+i)^3$	$= 1.081600\times1.04 = 1.124864$	=	3 ....
$(1+i)^4$	$= 1.124864\times1.04 = 1.169859$	=	4 ....
$(1+i)^5$	$= 1.169859\times1.04 = 1.216653$	=	5 ....
&c.	&c.	&c.	&c.

In this manner, the indefatigable John Smart, formerly of Guildhall, London, calculated an extensive Table, exhibiting the amount of £1 for any

number of years, not exceeding 100, to 8 places of decimals, at the several rates of 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, 4, $4\frac{1}{2}$, 5, 6, 7, 8, 9, and 10 per cent. beside many other Tables equally valuable.* These Tables variously modified and abridged, have been copied by most of the authors that have since written on the subject, and the greater part of the v. vi. vii. and viii. Tables in this work have been derived from the same source. The 1 and $1\frac{1}{2}$ per cent. columns, which, in many cases, will be found more useful than the higher rates of 9 and 10 per cent. have been calculated for this work, as the author is not aware that they have before been made public. The fifth Table may be applied to find the amount of £1 in any number of years exceeding 100. Thus, the amount of £1 in 160 years is nothing more than the product of the amounts given in the Table opposite 100 and 60 years, which, at 2 per cent. is $7.244646 \times 3.281031 = 23.769907$.

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* These Tables give the amounts, &c. for the odd half years, as well as for the unbroken numbers; but, on examination, it will be found, that they are all formed on the supposition of *half-yearly Compound Interest*, and are, therefore, inapplicable to annual conversions, as far as the odd half years are concerned.

Thus, in the column showing the amount of £1, at 4 per cent., the amount in the first half-year is taken. $\sqrt{1.04} = 1.0198$ &c.

The amount in 2 half-years	$= (1.0198 \text{ \&c.})^2$	$= 1.04$
... . 3	$= 1.04 \times 1.0198$ &c.	$= 1.060596$
..... 4	$= 1.060596 \times 1.0198$ &c.	$= 1.081600$
..... 5	$= 1.081600 \times 1.0198$ &c.	$= 1.103020$

Whereas, on the supposition of yearly Compound Interest, the amount of £1, in two years, at 4 per cent. is 1.0816, and for the odd half-year, 2 per cent. on this amount being taken, will make the amount in $2\frac{1}{2}$ years $= 1.103232$.

Example I.

Let it be required to find the amount of £120, in 20 years, at 4 per cent.?

By Table v, under 4 per cent. opposite 20 years, we find the amount of £1 = 2.191123, and this, multiplied by 120, gives £262.93476 = £262 18s 8d, the amount required, supposing the Interest convertible into Principal yearly.

But if the Interest be convertible into Principal half-yearly, so as to make 2 per cent. half-yearly Compound Interest, we look in the Table under 2 per cent. and opposite 40 years, and find the amount of £1 = 2.20804; and this, multiplied as before by the £120, gives £264.9648 = £264 19s $3\frac{1}{2}$d, the required amount: and, by taking from the Table the amount under 1 per cent. opposite 80 years, and multiplying as before by the £120, the amount, on the supposition that the Interest is convertible into Principal quarterly, at the nominal rate of 4 per cent. per annum, or, more correctly speaking, at the rate of 1 per cent. quarterly, becomes $2.21671 \times 120 = 266.0052$ = £266 0s $1\frac{1}{4}$d.

Again, if the time be $20\frac{1}{4}$ years, and the Interest convertible into Principal yearly, the amount in 20 years, found as before, is 262.93476; and 1 per cent. on this amount, for the remaining quarter, gives the amount in $20\frac{1}{4}$ years = £265 11s $3\frac{1}{4}$d.

Example II.

Required the Principal, which will amount to £750 in 25 years, at 5 per cent.?

By Table v. we find, as before directed, that the amount of £1 in the given time, at 5 per cent. is 3.386355. Hence, since it has been shown, that the amounts are proportional to the principals, we say, as $3.386355 : 1 : : 750 : 750 \div 3.386355 = 221.477$ = £221 9s $6\frac{1}{2}$d for the principal required.

But, at $2\frac{1}{2}$ per cent. half-yearly Compound Interest, the amount of £1 in the given time is 3.43711, and the required principal, on that supposition, becomes $750 \div 3.43711 = 218.207$ = £218 4s $1\frac{3}{4}$d.

Example III.

In what time will £100 amount to £564, at 4 per cent?

Here, £100 : £564 : : £1 : $564 \div 100 = 5.64$, the amount of £1 at the given rate in the time required. Hence, seeking in Table v, under 4 per cent. we find the nearest number to 5.64 opposite 44 years, the approximated time; but, if the time be required to a greater degree of exactness,

From the amount of £1, as above - -	5.640000
Deduct that for 44 years - - - - -	5.616515
And the difference is - - - - - -	.023485
Also, from the amount of £1 for 45 years	5.841176
Deduct that for - - - - - 44 - -	5.616515
Difference - - - - - - - - -	.224661

then say, as .224661 : .023485 : : 1 year : .10454 : the required time is, therefore, 44.10454 years = 44 years, 38 days : But if the Interest be reckoned at 2 *per cent. half-yearly Compound Interest*, under 2 per cent. the nearest number to 5.64 is found in the Table opposite 87 years.

Then, from the given amount of £1 - -	5.64000
Deduct that opposite 87 years - - - -	5.60035
And their difference is - - - - - - -	.03965
Also, from the amount of £1 for 88 years	5.71235
Deduct that for - - - - - - 87 - -	5.60035
Difference - - - - - - - - - - -	.11200

And .03965 ÷ .112 = .354, from which it appears that, upon this supposition, the required time becomes 87.354 *half-years* = 43.677 years = 43 years, 247 days.

Example IV.

At what rate of Interest will £120 amount to £756 in 30 years ?

Proceeding, as in the last example, we find that 756 ÷ 120 = 6.3 = the amount of £1 in the same time; hence, turning to the Table opposite 30 years, the amount of £1, at 6 per cent. is 5.743, at 7 per cent. = 7.612, and the 6.3 falling between them, but as near again to the former as to the latter, indicates that the required rate is between 6 and 7 per cent. but nearer to the 6 than the 7 per cent. and therefore about $6\frac{1}{3}$. The true rate cannot, however, be ascertained by this means, though

sufficiently near for most practical purposes. For the method of determining the exact rate, the reader is referred to the algebraic formulæ at the end of this Chapter.

Present Worths of Future Payments.

6. In this case it is manifest, from what was said in Section 3, that finding the Present worth of a future payment is nothing more than to determine what Principal laid out and improved during the interval, will just amount to the proposed sum by the time it becomes due. Hence, by what has been shown in Example II. of the last Section, it is evident that, at 4 per cent. the Present worth of £1 to be received,

1 year hence	= 1 ÷ 1.04	=	.961538
2 years hence	= 1 ÷ 1.0816	=	.924556
3 - - - - -	= 1 ÷ 1.124864	=	.888996
4 - - - - -	= 1 ÷ 1.169859	=	.854804
5 - - - - -	= 1 ÷ 1.216653	=	.821927
&c.	&c.		&c.

In this manner, the construction of the Table VI. is nothing more than finding the reciprocals of the numbers given in Table V.

EXAMPLE.

What is the present value of £500, to be received 24 years hence, allowing 4 per cent. Compound Interest?

Supposing the Interest convertible into Principal yearly, we have the amount of £1 in 24 years, at 4 per cent. =2.563304, then 500÷2.563304= 195.060=£195 1s 2½d for answer.

Or, by Table VI. the present value of £1 to be received 24 years hence, is .390121, that of £500 is therefore 500 × .390121=195.0605=£195 1s 2½, the same as before. But if, instead of 4 per cent. per annum, we consider the rate as 2 *per cent. half-yearly Compound Interest*, the required Present worth found by either of the two Tables, taking 2 per cent. for 48 years, is £193 5s 4½, or, at 1 *per cent. quarterly Compound Interest*, £192 7s 2¾.

If, however, the time given be 24½ years, and the Interest convertible into Principal yearly, we have the

Amount of £1 in 25 years, at 4 per cent. 2.665836
- - - - - - 24 - - - - - - - 2.563304
Difference - - - - - - - - - - .102532

Half this difference, added to the amount in 24 years, gives 2.614570 for the amount of £1 in 24½ years; and dividing the 500 by this amount, we obtain 191.236=£191 4s 8½d, for the required Present worth.

The same mode of taking a part of the difference proportional to the fraction of a year will not, however, apply to the Table VI.; for the Present worth of any sum due 24½ years hence is not an arithmetical mean between the present value of the

like sum due 24 and 25 years hence.† But if we suppose 24 years of the term as expired, and take the then worth of £1 due in six months from Table III. at 4 per cent. it becomes =.980392; and taking the Present worth of this sum again for 24 years, we have the Present worth of £1 due 24½ years hence =.390121 × .980392, or =.390121 ÷ 102=.382472, which, multiplied by 500, gives £191 4s 8½d, the required Present worth as before: and if this sum be deducted from the £500, the remainder £308 15s 3½d shows the Discount of £500 due 24½ years hence, at 4 per cent. Compound Interest.

Periodical Payments,

OTHERWISE TERMED ANNUITIES CERTAIN.

7. These are principally distinguished into two kinds; first, those that are Forborn or in Arrears, and, secondly, those that hereafter become payable; in the former we regard the *Amount,* and in the latter the *Present worth;* the one exceeding the sum of the several payments of the Annuity by the Interest accumulated thereon, the other falling short of that sum by the Discount to be deducted: and both depending not only on the intervals at

† Thus, $\frac{1}{a+\frac{1}{2}}$ is not equal to the mean between $\frac{1}{a}$ and $\frac{1}{a+1}$, that mean being $=\frac{1}{2}\left\{\frac{a+1}{a(a+1)}+\frac{a}{a(a+1)}\right\}=\frac{2a+1}{2a(a+1)}$

which the payments successively become due, but also on the periods at which the Interest is supposed to be convertible in Principal. These calculations also are greatly facilitated by the assistance of Tables, showing the Amount or Present worth of an Annuity of £1, from which that of any other Annuity is easily determined.

Annuities in Arrears.

8. Suppose it were required to find the amount of an Annuity of £1 forborn for six years, the first payment of which became due five years ago, the second four years ago, the third three, the fourth two, the fifth one, and the sixth now just due?

By allowing 4 per cent. Interest, and taking the amount of the several payments from Table v. it is manifest that the amount of the payment just due, upon which no interest has accrued, is 1.000000

The payment due 1 year ago - - - - -	1.040000
Their sum - - - - - - - - - - -	2.040000
The payment due 2 years ago - - - -	1.081600
Sum of last three payments - - -	3.121600
The payment due 3 years ago - - -	1.124864
Sum of last four payments - - - -	4.246464
The payment due 4 years ago - - -	1.169859
Sum of last five payments - - - - :	5.416323
The payment due 5 years ago - - - -	1.216653
Sum of all the six payments - - - -	6.632976

The total, showing the amount of the whole six payments is, therefore, 6.632976 *years' purchase*, or £6 12s 8d.

By this mode of proceeding backwards, and successively adding the several payments to the sum of those that have subsequently become due, it is manifest that the

First total 2.040000 shows the amount of £1 per ann. for } 2 years.
Second - 3.121600 - - - - - - - 3 —
Third - 4.246464 - - - - - - 4 —
Fourth - 5.416323 - - - - - - - 5 —

and so on, from which the construction of Table VII. is sufficiently obvious.

If the Annuity be made payable at the *beginning* of the year, each payment, and consequently their aggregate, must be increased by one year's Interest, so that the amount of the last payment would, in that case, be - - - - - - - - - - - = 1.040000
Of that preceding it - - - - - - - 1.081600
Of the one preceding this - - - - - 1.124864
- - - - - - - - - - - - - - - 1.169859
- - - - - - - - - - - - - - - 1.216653
Total - - - - - - - - - - - - 5.632976

Showing the Amount of an Annuity, payable at the beginning of the year, for 5 years, which is less by unity than the amount of an Annuity for 6 years, payable at the end of the year, as taken from the Table. The same principle of deducting unity from the tabular number opposite one year more than the

number given, is equally applicable to any other period, and any other rate of interest.

Example.

Let it be required to determine the amount of an Annuity of £40 forborn for 20 years, at 4 per cent. Compound Interest?

By Table VII. the amount of £1 per annum for 20 years, at 4 per cent. is 29.77808, and this multiplied by 40, the amount of Annuity, gives 1191.1232 = £1191 2s $5\frac{1}{2}$ for the amount required.

But if the Annuity be payable half-yearly, and the Rate be considered as 2 per cent. half-yearly Compound Interest; under 2 per cent. opposite 40 years, we find the amount of £1 half-yearly for 40 half-years = 60.40198, and this multiplied by £20, the half-yearly payment of the Annuity, gives 1208.0396 = £1208 0s $9\frac{1}{2}$d, the amount required.

And if the Annuity be payable quarterly, and the rate considered as 1 per cent. quarterly Compound Interest; under 1 per cent. and opposite 80 years, the amount of £1 per quarter for 80 quarters will be found = 121.67153; and this multiplied by £10, the amount of quarterly payment, produces 1216.7153 = £1216 14s $3\frac{1}{2}$d for the amount required.

Again, if the Annuity be payable yearly, and the Interest convertible into Principal half-yearly, the amount may be found as follows:—Suppose £1 now invested in a fund producing 2 per cent. half-yearly Compound Interest, and divided between two

persons, A and B, in such a manner, that A or his heirs may receive the Interest for the next 20 years, and B or his heirs may come into possession of the Principal at the expiration of that period. Upon this supposition it is manifest, that if A received the dividends as they became due, he would be entitled to £.02 at the end of the first half-year; and if he added this dividend to the principal, there would arise £1.02, upon which an interest of 2 per cent. must be computed during the next half-year, and therefore amounting, by the end of the year, to $1.02 \times 1.02 = (1.02)^2 = 1.0404$; and if A were now to take the whole of the dividend produced during the year, he might receive £.0404, leaving the original principal to produce a like dividend during the succeeding year, and so on, for the 20 years; but if, instead of receiving his Annuity of $(1.02)^2 - 1 = £.0404$ as it became due, he were to suffer the whole to accumulate at the rate of 2 per cent. half-yearly Compound Interest until the end of the 20th year, the £1 invested would then amount to $(1.02)^{40}$ agreeably to what was stated in Section 5; and if from this we deduct the original principal then due to B, we obtain the amount of A's share $= (1.02)^{40} - 1$: and since it is manifest that this sum is the amount of A's Annuity of £.0404 for 20 years; for the proportional amount of £40 Annuity we have $.0404 : (1.02)^{40} - 1 :: 40 : \frac{40 \times \{(1.02)^{40} - 1\}}{.0404}$ which, by taking the value of $(1.02)^{40}$ from Table v. under

under 2 per cent. and opposite 40 years, becomes $\frac{40\times(2.208040-1)}{.0404}=\frac{40\times1.20804}{.0404}=1196.079=£1196$ 1s 7d for the amount required.

Similarly it may be shown that, if the Annuity be payable yearly, and the Interest convertible into Principal quarterly, the amount of the proposed Annuity is $\frac{40\times\left\{(1.01)^{80}-1\right\}}{(1.01)^4-1}$ which, by taking the values of $(1.01)^{80}$ and $(1.04)^4$ from Table v. becomes $\frac{40\times(2.216715-1)}{1.040604-1}=\frac{40\times1.216715}{.040604}=1198.616=£1198$ 12s $3\frac{3}{4}$d† And if the Annuity be payable half-yearly, and the Interest convertible into Principal quarterly, the required amount is $\frac{20\times\left\{(1.01)^{80}-1\right\}}{(1.01)^2-1}=\frac{20\times1.216715}{.0201}=$ 1210.661 = £1210 13s $2\frac{3}{4}$.†

Moreover, suppose the Interest convertible into Principal yearly, and the Annuity payable half-yearly, it is manifest, the party entitled to receive such Annuity half-yearly ought to be credited with half a year's interest on the first moiety, if he suffer it to be unpaid until the second moiety becomes due, but half a year's interest on a moiety is only a quarter's interest on the whole Annuity; and the

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† Since $(1.01)^4 = 1.040604$ is immediately found in Table IV. and $(1.01)^{80} = (1.040604)^{20}$, by taking the logarithm of 1.040604 as given in that Table, and multiplying it by 20, the logarithm of $(1.040604)^{20}$ is found to be 0.3457099, the natural number of which is $2.216715 = (1.01)^{80}$ or $= (1.040604)^{20}$, with which we proceed as above. This method enables us to proceed where the half-yearly or quarterly rate is not contained in Table v.

annual payment, equivalent to the two half-yearly instalments, therefore exceeds the sum of those instalments by a quarter's interest on their aggregate. Thus, at 4 per cent. the annual payment equivalent to each £1 Annuity is £1.01, and this sum multiplied into 29.77808, the amount of £1 per annum for 20 years, taken from Table VII. gives 30.07586 years' purchase for the corresponding amount of a similar Annuity payable half-yearly; and this again multiplied by £40 gives 1203.0344 = £1203 0s $8\frac{1}{4}$ for the required amount of the Annuity proposed.

But if the Interest be convertible into Principal yearly, and the Annuity payable by quarterly instalments, reasoning as before, it follows that if the annuitant suffer his instalments to remain unpaid until the end of the year, he ought to be credited with 3 quarters' interest on the first instalment, 2 on the second, and 1 on the third, making together 6 quarter's interest on *one* instalment, or $\frac{3}{8}$ of a year's interest on the sum of all the four payments, so that, upon this supposition, the equivalent yearly payment will be at 4 per cent. on each £1 Annuity $1 + \frac{3}{8} \times .04 = 1.015$, as stated in Table IV; and this multiplied by 29.77808, the amount of £1 per annum for 20 years, gives the value of the proposed Annuity = 30.22475 years' purchase, which, on an Annuity of £40, produces £1208 19s $9\frac{1}{2}$ for the required amount.

Finally, suppose the Annuity payable quarterly, and the Interest convertible into Principal half-yearly. In this case, referring the annuitant's claim to the end of the first half-year, when the first conversion takes place, it appears manifest that he will then be entitled to two quarterly instalments of the Annuity, with a quarter's interest on the first, making together half a year's Annuity, and $\frac{1}{16}$ of a year's interest on the whole four instalments, which, at 4 per cent. is, on each £1, $\frac{1}{2}+\frac{1}{16}\times .04 = .5 + .0025 = .5025$; and the amount of this periodical payment for 40 half-years, at 2 per cent. half-yearly Compound Interest, found as before directed, is $.5025 \times 60.401983 = 30.351997$ years' purchase, which, on £40, produces 1214.07988 = £1214 1s $7\frac{1}{4}$ for the required amount.

Collecting the various results, it appears that, at the nominal rate of 4 per cent. the amount of an Annuity of £40 for 20 years,

When the Annuity is payable	And the Interest convertible into Principal	is		
		£.	s.	d.
Yearly,	Yearly,	1191	2	$5\frac{1}{2}$
	Half-yearly,	1196	1	7
	Quarterly,	1198	12	$3\frac{3}{4}$
Half-yearly,	Yearly,	1203	0	$8\frac{1}{4}$
	Half-yearly,	1208	0	$9\frac{1}{2}$
	Quarterly,	1210	13	$2\frac{3}{4}$
Quarterly,	Yearly,	1208	19	$9\frac{1}{2}$
	Half-yearly,	1214	1	$7\frac{1}{4}$
	Quarterly,	1216	14	$3\frac{1}{2}$

Again, if we suppose the Annuity payable annually at the *beginning* of the year, and the Interest also convertible into Principal annually, its amount, found by deducting unity from the tabular amount opposite 21 years, agreeably to what was stated in page 49, is 30.969202 years' purchase, which, on £40, is 1238.768 = £1238 15s $4\frac{1}{4}$d.

Present Worth of Annuities.

9. In this branch, beside the particulars stated in the last section, we have to regard the period at which the Annuity is to commence,* and the term for which it is to continue. Thus an Annuity may commence immediately, and be continued either for a limited term, as Government long Annuities, or for an unlimited number of years as the Dividends on Stock, the Rents of Freehold Estates &c.; or the Annuity may commence at the expiration of a given period, and then be continued either for a given term, as the Reversions of Leases after the expiration of under-Leases, or run on in perpetuity as the Reversions to Freeholds after the expiration of terminable Leases, &c.

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* Note—An Annuity payable yearly is said to commence or be entered upon one year before the first payment becomes due; and an Annuity payable by half-yearly instalments, is said to commence half a year before the first instalment becomes due, and so on.

This subject may therefore be distinguished into *Immediate Annuities*, *Perpetual Annuities*, *Deferred Annuities*, and *Deferred Perpetuities*, all of which are valued by a Table, the construction of which is as follows: Thus at 4 per cent. by Table VI. it appears that

The present value of £1 due 1 year hence is 961538
To which adding that of £1 due 2 years hence.....924556
Their sum is the present value of £1 Annuity for 2 years 1.886094
To this again adding the value of £1 due 3 years hence...888996
Present value of 1 Annuity for three years.....2.775090
And adding to this the value of £1 due 4 years hence.....854804
Total, showing the value of £1 Annuity for 4 years....3.629894

The like operations repeated point out the construction of Table VIII.

If the Annuity were payable at the *beginning* of the year, the present value of

The 1st payment would evidently be =1.000000
That of the 2nd, discounted for 1 year, = .961538
That of the 3rd, - - - - 2 years, = .924556
That of the 4th, - - - - 3 years, = .888996
Total - - - - - - - - - - - - - 3.775090

which shews the present worth of an Annuity of £1 payable at the *beginning* of the year for 4 years, and is nothing more than unity added to the present value of a like Annuity payable at the *end* of the year for 3 years. The same principle, of adding unity to the Tabular number opposite 1 year less than the given term, is equally applicable to any other period, and to any other rate of Interest.

Example i.

Suppose it were required to find the present value of an Immediate Annuity of £50 for 24 years at 4 per cent?

By Table viii. the required value in years' purchase, found under 4 per cent. and opposite 24 years, is 15.246963, and this multiplied by 50, the amount of Annuity produces 762.34815 = £762 6s 11½d for the present value of the Annuity proposed.

In like manner it may be found, that if the Improved rent of a Tenement held under a Lease for 21 years be £60, reckoning interest at 5 per cent., the present value of the Lease is 12.821153 × 60 = 769 26918 = £769 5s 4½d.

Example ii.

What is the present value of a Perpetual Annuity of £20, reckoning interest at 4½ per cent.?

By Table viii. last line† in the 4½ per cent. column, the required value in years purchase is 22.222222, and this multiplied by 20, the annual payment, gives 444.444 &c. = £444 8s 10½d, the present value required. In like manner it may be shown that an Estate in Fee-simple producing a net annual rent of £600 is at 4 per cent. worth 25 × 600 = £15000.

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† For the mode of deducing this line, the reader is referred to page 38.

Example III.

Suppose an Annuity certain of £50 for the next 30 years divided between two persons A and B, so that A or his heirs may enjoy it during the next 10 years, and B or his heirs for the remaining 20 years; required the present value of B's Deferred Annuity, reckoning interest at $3\frac{1}{2}$ per cent. ?

By Table VIII. under $3\frac{1}{2}$ per cent., and opposite 30 years, we find the value of both shares collectively - } = 18.392045

From which deduct A's share found in the same Table, opposite 10 years } 8.316605

And there remains for B's share - - 10.075440

years' purchase, which multiplied by £50, the annual payment, produces 503.772 = £503 15s $5\frac{1}{4}$d for the present value required.

Otherwise, the present value of B's Interest on coming into possession, then an Immediate Annuity for 20 years, found as in Example I. is 14.212403 years' purchase, and the present value of this sum discounted for 10 years by section 6, is $14.212403 \times .708919 = 10.075442$ the same as before. ‡

In like manner it may be shown that the premium to be paid for adding 21 years to a Lease of £70 per annum, of which 12 years are unexpired, is at 4 per cent. $(18.147646 - 9.385074) \times 70 = 8.762572 \times 70 = 613.380 =$ £613 7s $7\frac{1}{4}$d.

‡ This mode of referring the value of a property first to one period, and then to another, will often be found of considerable service in facilitating calculation.

EXAMPLE IV.

Required the present value of the Reversion to a Freehold of £120 per annum to be entered upon 40 years hence, reckoning Interest at 5 per cent.?

Reasoning as in the last example, it is evident that if from the Perpetuity by Table VIII. - = 20.000000
we deduct the value of £1 annuity for 40 years as shown by ditto - } 17.159086
there remains for the Reversion in years' purchase - - - - - - } = 2.840914

and this multiplied by £120, the rent of the Estate, produces 340.90968 = £340 18s 2¼d, the present value required.

This sum, small as it appears, if laid out and improved at 5 per cent. for the next 40 years, would according to what was stated in section 5, amount to 7.039989 × 340.90968 = £2400, the interest on which at the like rate would evidently produce an annual income of £120 thenceforth, without diminution of principal.

EXAMPLE V.

Required the Present value of an Annuity of £100, payable at the *beginning* of each year for the next 20 years, reckoning interest at 4 per cent?

Here, by adding unity to the number found in Table VIII. under 4 per cent. and opposite 19 years, agreeably to what was stated in page 56, we obtain 14.133939 years' purchase, which, on an

Annuity of £100, is 1413 3939 = £1413. 7s 10½d, the present value required.

In comparing the *Amounts* of Annuities forborn, and the *Present Worths* of those that hereafter become payable, it will be found that in the former the higher the rate of interest the greater is the Amount of an Annuity, but in the latter the higher rate of interest the less is the present worth of the future payments, as a smaller sum now invested will provide for these payments as they successively become due; it will also be found that, in Annuities forborn, the Amount is increased both on account of the number of times in the year that the Annuity is made payable, and of the number of times that the Interest is converted into Principal; the one having the effect of increasing the Annuity, and the other of enhancing the rate of interest: whereas, in the Present worth of future Annuities, the oftener the Annuity is made payable in the year, the greater is its present value, but the oftener the Interest is converted into Principal, at a given nominal rate, the less is the Present worth of the Annuity: in Annuities forborn both causes combine to increase their values, but in those that hereafter become payable they produce contrary effects, the one tending to increase the Present worth of the Annuity, and the other to diminish it; and in Perpetuities, when the annuity and interest are payable at like intervals, these contrary tendencies balance each other so as to leave the value of the Annuity unaltered.

Example vi.

Required the Present value of an Annuity of £100 for 20 years, at 4 per cent.?

Supposing the Annuity payable yearly, and the Interest convertible into Principal yearly, the Present worth, found as in Example i. is 13.590326 × 100 = 1359.0326 = £1359 0s 8d.

And, if the Annuity be payable by half-yearly moieties, and the rate be considered as 2 per cent. half-yearly Compound Interest, by Table viii. under 2 per cent. and opposite 40 years, the present value of the forty half-yearly payments of £1 each is found to be 27.355479, and this multiplied by £50, the amount of half-yearly payment, produces 1367.77395 = £1367 15s $5\frac{3}{4}$d for the Present value required.

Similarly, if the Annuity be made payable quarterly, and the rate be considered as 1 per cent. quarterly compound Interest, by Table viii. under 1 per cent. and opposite 80 years, we find the amount of the 80 quarterly payments of £1 each = 54.888201, and this multiplied by £25, the amount of the quarterly payment, produces 1372.205 = £1372 4s $1\frac{1}{4}$d, for the Present worth required.

But if the Annuity be payable yearly, and the Interest convertible into Principal half-yearly, by supposing £1 now invested at 2 per cent. half-yearly Compound Interest, and divided between two persons, A and B, as in section 8, it is manifest, by

what was there stated, that the sum due to A, at the end of each year, is $(1.02)^2-1$; and the £1 due to B at the end of 20 years, at 2 per cent. half-yearly Compound Interest, is, by section 6, in present value, only $\frac{1}{(1.02)^{40}}$, and this deducted from the £1, now divided among the two parties, leaves $1-\frac{1}{(1.02)^{40}}$ for A's share, or the present value of his Annuity of $(1.02)^2-1$ for the next 20 years. Hence, by proportion we have $(1.02)^2-1 : 1-\frac{1}{(1.02)^{40}} :: 100 : \frac{100\times\left\{1-\frac{1}{(1.02)^{40}}\right\}}{(102)^2-1}$ which, by taking the value of $\frac{1}{(1.02)^{40}}$ from Table VI, opposite 40 years, and under 2 per cent. and that of $(1.02)^2$ from Table V. opposite 2 years, and under the like rate of interest, produces $\frac{100\times(1-.452890)}{1.0404-1}=\frac{100\times.547110}{.0404}=100\times 13.54232=$ 1354.232 = £1354 4s $7\frac{3}{4}$ for the Present value required.

Similarly it may be shown that, if the Annuity be payable yearly, and the Interest (considered as 1 per cent.) be convertible into Principal quarterly, the present value of the Annuity proposed is $\frac{100\times\left\{1-\frac{1}{(1.01)^{80}}\right\}}{(1.01)^4-1}=\frac{100\times(1-.451118)}{1.040604-1}=\frac{100\times.548882}{.040604}=100\times$ 13.5179 = 1351.79 = £1351 15s 9d.*

And, if the Annuity be payable by half-yearly moieties, and the Interest be convertible into Principal quarterly, the required present value is $\frac{50\times\left\{1-\frac{1}{(1.01)^{80}}\right\}}{(1.01)^2-1}=\frac{50\times(1-.451118)}{1.0201-1}=\frac{50\times.548882}{.0201}=50\times$ 27.30756 = 1365.378 = £1365 7s 7d.

Moreover, supposing the Annuity to be payable half-yearly or quarterly, and the Interest to be convertible into Principal yearly, by reasoning as in the corresponding cases in the last section, page 53, it may be shown that the equivalent annual payment for each £1 Annuity is, in the former case, 1.01, and in the latter 1.015, as represented by Table IV. and that the required present value is accordingly $100 \times 1.01 \times 13.590326 = 1372.623 =$ £1372 12s $5\frac{1}{2}$d, or $100 \times 1.015 \times 13.590326 = 1379.418 =$ £1379 8 $4\frac{1}{4}$.

And, if the Annuity be payable quarterly, and the Interest convertible into Principal half-yearly, the equivalent half-yearly payment, as shown in the last section, is, for each £1 Annuity, .5025, and the present value of 40 periodical payments made half-yearly, at 2 per cent. half-yearly Compound Interest, as shown by Table VIII. is 27.355479, from which we have $100 \times .5025 \times 27.355479 = 1374.6128 =$ £1374 12s 3d for the present worth of the Annuity proposed.

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* Since $(1.01)^4 = 1.040604$ is immediately found in Table IV. and $(1.01)^{80} = (1.040604)^{20}$ by taking the logarithm of 1.040604 as given in that Table, and multiplying it by 20, the logarithm of $(1.040604)^{20}$ is found to be .3457099, the complement of which is $\bar{1}.6542901 =$ log. of $\frac{1}{(1.01)^{80}}$; and from this logarithm, by taking out the number corresponding, the value of $\frac{1}{(1.01)^{80}}$ is found to be .451118, with which we proceed as above. This method has the advantage of enabling us to proceed where the half-yearly or quarterly rate is not contained in Table VI.

By collecting the various results obtained in this Example, it appears that, at the nominal rate of 4 per cent. the present value of an Annuity of £100 for 20 years,

When the Annuity is payable	And the Interest convertible into Principal	is
		£. s. d.
Yearly,	Yearly,	1359 0 8
	Half-yearly,	1354 4 $7\frac{3}{4}$
	Quarterly,	1351 15 9
Half-yearly,	Yearly,	1372 12 $5\frac{1}{2}$
	Half-yearly,	1367 15 $5\frac{3}{4}$
	Quarterly,	1365 7 7
Quarterly,	Yearly,	1379 8 $4\frac{1}{4}$
	Half-yearly,	1374 12 3
	Quarterly,	1372 4 $1\frac{1}{4}$

Example VII.

Required the Present value of a Perpetual Annuity of £120 at 4 per cent.?

Supposing the Annuity payable yearly, and the Interest convertible into Principal yearly, the required Present worth found as in Example II, is $25 \times 120 = £3000$.

And if the Annuity be payable half-yearly or quarterly, and the Interest be convertible into Principal at the like intervals, the required Present value, still found by Table VIII, is $50 \times £60$, or $100 \times £30$, each $= £3000$ the same as before, agreeably to what was stated in page 60.

But, if the Annuity be payable yearly, and the Interest be convertible into Principal half-yearly or quarterly, the rate of Interest being thereby increased, the Present value of the Annuity must be proportionably diminished. In the former case, the amount of £1 in one year at the nominal rate of 4 per cent. is $(1.02)^2$, which, by Table IV. is found to be 1.0404; its Annual Interest is therefore .0404; hence, proceeding as directed in section 4, page 38, we have the value of a Perpetual Annuity $1 \div .0404 =$ 24.752475 years' purchase, which on £120 produces $2970.297 =$ £2970 5s $11\frac{1}{2}$d; and in the latter, $1 \div \{(1.01)^4 - 1\} = 1 \div .040604 = 24.628115$, represents the required Present value in years' purchase, which on £120 produces $2955.3738 =$ £2955 7s $5\frac{3}{4}$d.

In like manner it may be shown, that if the Annuity be payable half-yearly, and the Interest be convertible into Principal quarterly, the required Present value $= 60 \times \frac{1}{(1.01)^2 - 1} = 60 \times \frac{1}{.0201} = 60 \times 49.751244 =$ 2985.07464 = £2985 1s 6d.

If, however, the Annuity be payable at less intervals than those at which the Interest is convertible into Principal, the required Present worth will be found to exceed that deduced, on the supposition of both annuity and interest being payable yearly: for, reasoning as in the corresponding cases of the last Example, it may be shown that, if the Annuity be payable half-yearly or quarterly, and the Interest convertible into Principal yearly, the Present worth of the proposed Annuity is, in the former case,

$120 \times 1.01 \times 25 = £3030$, or, in the latter case, $120 \times 1.015 \times 25 = £3045$.

And, if the Annuity be payable quarterly, and the Interest convertible into Principal half-yearly, the required Present worth, found as in the last Example, is $120 \times .5025 \times 50 = £3015$.

Collecting the several results, as in the last Example, it appears that the Present value of a Perpetual Annuity of £120 at 4 per cent.

When the Annuity is payable	And the Interest convertible into Principal	is		
		£	*s.*	*d.*
Yearly,	Yearly,	3000	0	0
	Half-yearly,	2970	5	$11\frac{1}{2}$
	Quarterly,	2955	7	$5\frac{3}{4}$
Half-yearly,	Yearly,	3030	0	0
	Half-yearly,	3000	0	0
	Quarterly,	2985	1	6
Quarterly,	Yearly,	3045	0	0
	Half-yearly,	3015	0	0
	Quarterly,	3000	0	0

From the last two Examples we may easily determine the Present value of either a Deferred Annuity or Deferred Perpetuity, when either the annuity or the interest, or both, are payable more than once a year, by first finding the Present value of the proposed Annuity, on the given supposition for the whole period, from the present time until its termination, and deducting from it the Present value

(found, on the same supposition, by Example VI,) of a like annuity to continue from the present time until the proposed Annuity is to be entered upon.

EXAMPLE VIII.

A, being entitled to the Reversion of certain property in Fee-simple producing £600 per annum, to be entered upon 12 years hence, is desirous of equalizing his income by disposing to B such part of that Reversion as would leave him and his heirs the same income in perpetuity as that granted him by B for the part so disposed, until his Deferred Annuity becomes payable; it is required to find what part of the said Reversion A must make over to B, and the permanent income which he can secure to himself?

This question being in effect a proposition to barter a Deferred for an Immediate Perpetuity, admits of different answers, according to the rate of interest employed in the calculation: but reckoning interest at $3\frac{1}{2}$ per cent., by Example IV, we find the present value of A's Deferred Annuity $= 600 \times (28.571429 - 9.663334) = 600 \times 18.908095 = 11344.857 =$ £11,344 17s 2d; and by Table VIII. we also find that at the like rate of interest the present value of £1 perpetuity is 28.571429; and in order to determine what perpetual Annuity A can purchase for £11344.857, the present value of his Reversion, we say,

As £28.571429 : £1 : : £11344.857 : $\frac{11344.857}{28.571429} =$ 397.070 = £397 1s 5d, for the equivalent perpetuity;

which is therefore the permanent income that A ought to secure for his Reversion; and deducting this Annuity from the £600 proposed, there remains £202 18s 7d to be made over to B. It therefore appears that, if interest be computed at $3\frac{1}{2}$ per cent. A must give to B a Deferred Perpetuity of £202 18s 7d, to be entered upon 12 years hence, as an equivalent for £397 1s 5d per annum, to be paid by B to A during that interval.

But, if interest be computed at 4 per cent., it may be found in like manner, that

B ought to give A an Immediate Annuity of - - - - -	£374 15s 2d
And A ought to give B a Deferred Perpetuity of - - - -	£225 4s 10d

And, reckoning interest at 5 per cent., it may be shown by a like process, that

B ought to give to A an Immediate Annuity of - - - - -	£334 2s 0d
And A ought to give to B a Deferred Perpetuity of - - - -	£265 18s 0d*

* In giving the *real* value in these calculations, we should always employ the rate of interest which money currently bears at the time; but in practice it will generally be found that one of the parties is induced to enter into the contract as a matter of convenience, while the other has no other motive than the beneficial improvement of his capital, upon which he therefore expects to obtain a higher rate of interest than that produced by other securities, where that capital would be more immediately under his own controul. In such cases, the rate of interest should evidently be determined by special agreement among the parties themselves.

14. Examples of this kind might be indefinitely multiplied, but the prescribed limits of the work renders it necessary to draw this Chapter to a close; the reader is therefore requested to observe, that in all cases where different kinds of property are bartered, the calculations must be conducted upon the self-evident principle of equalizing the present values of the commodities exchanged, and that he may always effect this purpose by the following

RULE.—*Divide the present value per pound of the property which has its amount given, by the present value of each £1 of that for which it is proposed to be exchanged, and the result multiplied by the given amount will be the answer.*

For let v denote the present value of each £1 of the property which has its amount given, and V that of the property which has its amount required; and let a and A denote their respective amounts: then will the present value of the former $=av$; that of the latter $=AV$; and these present values being made equal, we obtain $AV=av$, or $A=\frac{av}{V}=\frac{v}{V}\times a$, which shows the derivation of the rule from the principle upon which it is grounded.

Thus, in Example VIII of last section, the property given is a Deferred perpetuity of £600, and that for which it is proposed to be exchanged an Immediate perpetuity: the present value of each £1 of the former, found as in Example IV. page 59, is at $3\frac{1}{2}$ per cent. $28.571429-9.663334=18.908095$; and the present value of each £1 of the latter,

found by Table VIII. under the like rate of interest, is 28.571429. Hence, dividing the first of these sums by the second, the quotient is .661784; and this multiplied by £600, the given amount, produces 397.0704 = £397 1s 5d, as otherwise determined in Example VIII. page 67.

Similarly, if it be required to determine what Annuity for the next 20 years is an equivalent exchange for £5000 to be received at the end of 8 years, computing interest at 4 per cent., by Table VI. we find the present value per pound of the sum given = .730690; and by Table VIII. the present value of each £1 of the Annuity required = 13.590326. Hence, .730690 ÷ 13.590326 = .0537655, which, multiplied by the £5000, produces 268.8275 = £268 16s $6\frac{1}{2}$d for the Annuity required.

General Formulæ.

15. In order to generalize what has been advanced on Compound Interest and Periodical payments,

Let t and i represent as in page 33;

$r = 1 + i$ = Amount of £1 in one year;

and $v = \frac{1}{1+i} = \frac{1}{r}$ = Present value of £1, due 1 year hence. (Page 35.)

Also, let h, q, and m, represent the Amount of £1 in one year, as given in Table IV. according as the

Interest is convertible into Principal half-yearly, quarterly, or momently, while their reciprocals are respectively denoted by H, Q, and M.

And put λ for the word logarithm; then, in the

Amounts of Sums Forborn.

16. Putting p for the Principal and a for the Amount, and reasoning as in section 5, page 40, it may be shown that $a=p(1+i)^t=pr^t$.

Or, by the nature of logarithms, observing that $\lambda(pr)^t$ becomes $=\lambda p+t\lambda r$, we have

$$\lambda a=\lambda p+t\lambda r, \text{ or } =\lambda p-t\lambda v^*$$

$$\lambda p=\lambda a-t\lambda r, \text{ or } =\lambda a+t\lambda v$$

$$t=\frac{\lambda a-\lambda p}{\lambda r}$$

$$\text{and } \lambda r=\frac{\lambda a-\lambda p}{t}$$

But if the time be t years and a fraction f, the equation becomes $a=pr^t(1+if)$.

Again, if the Interest be convertible into Principal at x equal intervals in each year, at the nominal rate represented by i, it is manifest that the amount of £1, at the end of the first interval, is $1+\frac{i}{x}$; at the end of the second, $(1+\frac{i}{x})^2$; third, $(1+\frac{i}{x})^3$; at the

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* Since $v=\frac{1}{r}$, it follows that $r=\frac{1}{v}$ or $r^t=\frac{1}{v^t}$, and consequently $\lambda r=-\lambda v$ or $t\lambda r=-t\lambda v$; from which it appears, that $t\lambda r$ may be obtained by taking the complement of $\lambda(v^t)$ or of $t\lambda v$ given in Table IX.

end of the xth interval, or of 1 year, $=(1+\frac{i}{x})^x$; and consequently, at the end of t years, $=(1+\frac{i}{x})^{xt}$; from which we derive $p(1+\frac{i}{x})^{xt}=a$,

$$\text{or, } \lambda a=\lambda p+xt\lambda(1+\tfrac{i}{x})$$

$$\lambda p=\lambda a-xt\lambda(1+\tfrac{i}{x})$$

$$t=\frac{\lambda a-\lambda p}{x\lambda\,(1+\frac{i}{x})}$$

$$\text{and } \lambda(1+\tfrac{i}{x})=\frac{\lambda a-\lambda p}{xt}$$

and if in the expression $(1+\frac{i}{x})^x$ we make x successively $=1$, 2, 4 or infinite, we find the

Amount of £1 in 1 year, when the Interest is convertible into Principal,

Yearly $=(1+\frac{i}{1})^1=r$
Half-yearly $=(1+\frac{i}{2})^2=h$
Quarterly $=(1+\frac{i}{4})^4=q$
Momently $=(1+\frac{i}{x})^x=m$

} as given in Table IV.

To show the mode of finding the quantity m, let the expression $(1+\frac{i}{x})^x$ be expanded by the Binomial Theorem, and it becomes $=1+i+\frac{x-1}{2}\cdot\frac{i^2}{x}+\frac{x-1}{2}\cdot\frac{x-2}{3}\cdot\frac{i^3}{x^2}$ &c. which, by making x infinite, and considering $x-1$, $x-2$, $x-3$, &c. each $=x$, is transformed into $1+i+\frac{i^2}{2}+\frac{i^3}{2.3}+\frac{i^4}{2.3.4}$ &c. a series shown by writers on Logarithms to be equal to the natural number whose hyp. log. $=i$, or common log. $=i\times$ .43429448 &c. From this, it becomes an easy matter to determine the Amount to which £1, im-

proved at a given nominal rate, will be increased in one year at Compound Interest, by increasing the periods of conversions without limit.

Thus, at 4 per cent. $i = .04$, and $i \times .43429448$ &c. $= .0173717792$ for the logarithm, to which the corresponding natural number is $1.040811 = m$, as given in Table IV.

From what has been stated, it appears that when the Interest is convertible into Principal,

Yearly,	$a = pr^t$,	or $\lambda a = \lambda p + t\lambda r$.
Half-yearly,	$a = ph^t$,	or $\lambda a = \lambda p + t\lambda h$.
Quarterly,	$a = pq^t$,	or $\lambda a = \lambda p + t\lambda q$.
Momently,	$a = pm^t$,	or $\lambda a = \lambda p + t\lambda m$.

From these equations, which are all applicable to determine the value of a, others may be derived, as in page 71, to find the remaining quantities. Thus, from the second equation, $\lambda a = \lambda p + t\lambda h$,

$$\lambda p = \lambda a - t\lambda h$$

$$t = \frac{\lambda a - \lambda p}{\lambda h}$$

$$\lambda h = \frac{\lambda a - \lambda p}{t}$$

And when h is thus found, from the equation $\left(1 + \frac{i}{2}\right)^2 = h$, we have, $1 + \frac{i}{2} = h^{\frac{1}{2}}$ or $i = 2(h^{\frac{1}{2}} - 1)$.

Similarly it may be shown that, in the

Third equation $i = 4(q^{\frac{1}{4}} - 1)$,

Fourth —— $i = \lambda m \div .43429448$.

Present Worths of future Payments.

17. Let s represent the future Sum to be received,
w — its Present worth;
and the remaining symbols as in sect. 15.

Then since the present worth of a future payment is that which, being improved during the proposed term, will just amount to the sum to be received by the time it becomes due, it follows, from the last section, that if the Interest be convertible into Principal x times in each year for t years at the nominal rate denoted by i,

$$w\left(1+\tfrac{i}{x}\right)^{xt}=s, \text{ from which}$$

$$\lambda s=\lambda w+xt\lambda\left(1+\tfrac{i}{x}\right)^{xt}$$

$$\lambda w=\lambda s-xt\lambda\left(1+\tfrac{i}{x}\right)$$

$$t=\frac{\lambda s-\lambda w}{x\lambda\left(1+\frac{i}{x}\right)}, \text{ and}$$

$$\lambda\left(1+\tfrac{i}{x}\right)=\frac{\lambda s-\lambda w}{tx}$$

and if in the expression $s=w\left(1+\frac{i}{x}\right)^{xt}$ we make x successively $=1$, 2, 4 & infinite, we find that if the Interest be convertible into Principal,

Yearly,	$s=wr^t$,	and $w=\frac{s}{r^t}=sv^t$
Half-yearly,	$s=wh^t$,	$w=\frac{s}{h^t}=sH^t$
Quarterly,	$s=wq^t$,	$w=\frac{s}{q^t}=sQ_t$
Momently,	$s=wm^t$,	$w=\frac{s}{m^t}=sM^t$

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Annuities Forborn.

18. Let p here denote the Yearly payment,
a ——— the Amount of the Annuity,
and the remaining symbols as before.

Then, supposing the Interest of £1 to be assigned to A for the next t years, and the Principal to revert to B at the expiration of that period, it is obvious that if A suffer his annuity to accumulate until the end of the given term, the collective shares of A and B will be the amount of £1 improved at Compound Interest for t years, which, on the supposition of yearly conversions, $=r^t$; and if from this be deducted the £1 then due to B, there remains r^t-1 for the amount of A's annuity of $£i$.

Hence, proportionally, for the amount of an annuity of $£p$, we have

$i : r^t-1 \; : : \; p : a$, from which are deduced,

$$a=\tfrac{p}{i}(r^t-1)$$

$$p=\tfrac{ai}{r^t-1} \text{ or } =\tfrac{a(r-1)}{r^t-1}$$

$$t=\tfrac{\lambda(ai+p)-\lambda p}{\lambda r}, \quad \text{and}$$

$r^t-nr+n-1=0$, n being put $=\frac{a}{p}$;

from which the value of r may be found by approximation; for the method of performing which, the reader is referred to the 20th section.

But if the annuity be made payable by z equal instalments in each year, at z equal intervals therein, and the Interest be convertible into Prin-

cipal y times in each interval, or zy times annually; by putting $zy=x$, or $y=\frac{x}{z}$, and supposing £1 divided between A and B as before, it follows from what has been stated, that the Dividend due to A, at the end of the first interval, after the first y periods of conversions, is $(1+\frac{i}{x})^{y}-1=(1+\frac{i}{x})^{\frac{x}{z}}-1$; or if he suffer the whole to accumulate for t years, the amount then due to him as an equivalent to z periodical payments of $(1+\frac{i}{x})^{\frac{x}{z}}-1$ in each year, is $(1+\frac{i}{x})^{n}-1$. Hence proportionally, for the amount of an annuity of £p for the same term, payable in like manner by z equal instalments of $\frac{p}{z}$ in each year, we have

$$(1+\tfrac{i}{x})^{\frac{x}{z}}-1 : (1+\tfrac{i}{x})^{n}-1 : : \tfrac{p}{z} : a, \text{ from which}$$

$$a=\frac{p}{z}\cdot\frac{(1+\frac{i}{x})^{n}-1}{(1+\frac{i}{x})^{\frac{x}{z}}-1} \quad - - - - - \quad \mathbf{A}$$

And if y be taken $=1$, so that the Annuity and the Interest may both be payable at the same time, x becomes $=z$, and the above formula is transformed into

$$a=p\left\{\frac{(1+\frac{i}{x})^{n}-1}{i}\right\} \quad - - - - - - \quad \mathbf{B}$$

Moreover, if the Interest be convertible into Principal x times in each year, and the Annuity made payable y times in each period of conversion, or xy times in the year, by instalments of $\frac{p}{xy}$ each, at xy equal intervals therein; by putting $xy=z$ or $y=\frac{z}{x}$,

and referring to the end of the first xth part of a year, when the first conversion of interest into principal takes place, and the first y instalments of the Annuity shall have become due, it is evident that, on the last of those instalments, no interest will have accrued; but, on the preceding instalment, an interest ought to be computed for one interval, which interest, therefore, produces $\frac{p}{xy} \cdot \frac{i}{xy} = \frac{pi}{x^2y^2}$; on the third instalment, reckoning backwards an interest for two intervals $\left(=\frac{2pi}{x^2y^2}\right)$ on the fourth three, and so on to the yth instalment, on which an interest must be computed for the $y-1$ intervals elapsed after it shall have become due; the interest then accrued on the various instalments is, therefore, $= \frac{pi}{x^2y^2}\left\{1+2+3+4 \text{ - - - - } y-1\right\} = \frac{pi}{x^2y^2}\left\{\frac{y(y-1)}{2}\right\} = \frac{pi}{x^2y}\left(\frac{y-1}{2}\right)$; to which adding the sum, $y\left\{\frac{p}{xy}\right\} = \frac{p}{x}$, of the y instalments of the Annuity, we have the aggregate due at the end of the first xth part of a year $= \frac{p}{x} + \frac{pi}{x^2y}\left\{\frac{y-1}{2}\right\} = \frac{p}{x}\left\{1+\frac{(y-1)i}{2xy}\right\} = \frac{p}{x}\left\{1+\frac{(z-x)i}{2xz}\right\}$. Hence, for the amount of this periodical payment, receivable x times in the year, with the Interest convertible into Principal at the like intervals, by formula B we have,

$$\frac{p}{x} : p\left\{\frac{\left(1+\frac{i}{x}\right)^x - 1}{i}\right\} : : \frac{p}{x}\left\{1+\frac{(z-x)i}{2xz}\right\} : a, \text{ from which}$$

$$a = p\left\{1+\frac{(z-x)i}{2xz}\right\} \times \left\{\frac{\left(1+\frac{i}{x}\right)^x - 1}{i}\right\} \text{ - - - } \mathbf{C}$$

From the three formulæ A, B, and C, herein deduced, by observing, that if x is made $=1$, 2, 4, or infinite $(1+\frac{i}{x})^{x}$ becomes $=r$, h, q, or m, and that in formula C, the expression $\frac{(z-x)i}{2xz}$ becomes $\frac{zi}{2xz}=\frac{i}{2x}$ when z is infinite, and vanishes when x also is infinite, it is evident that

The Amount of £p per annum, for t years, at the nominal rate denoted by i is, when the

Annuity is payable	And the Interest convertible into Principal			
	YEARLY,	HALF YEARLY,	QUARTERLY,	MOMENTLY,
YEARLY,	$p\left(\frac{r^t-1}{i}\right)$	$p\left(\frac{h^t-1}{h-1}\right)$	$p\left(\frac{q^t-1}{q-1}\right)$	$p\left(\frac{m^t-1}{m-1}\right)$
HALF YEARLY,	$p\left(1+\frac{i}{4}\right)\left(\frac{r^t-1}{i}\right)$	$p\left(\frac{h^t-1}{i}\right)$	$\frac{p}{2}\left(\frac{q^t-1}{\sqrt{q}-1}\right)$	$\frac{p}{2}\left(\frac{m^t-1}{\sqrt{m}-1}\right)$
QUARTERLY,	$p\left(1+\frac{3i}{8}\right)\left(\frac{r^t-1}{i}\right)$	$p\left(1+\frac{i}{8}\right)\left(\frac{h^t-1}{i}\right)$	$p\left(\frac{q^t-1}{i}\right)$	$\frac{p}{4}\left(\frac{m^t-1}{\sqrt[4]{m}-1}\right)$
MOMENTLY,	$p\left(1+\frac{i}{2}\right)\left(\frac{r^t-1}{i}\right)$	$p\left(1+\frac{i}{4}\right)\left(\frac{h^t-1}{i}\right)$	$p\left(1+\frac{i}{8}\right)\left(\frac{q^t-1}{i}\right)$	$p\left(\frac{m^t-1}{i}\right)$

Present Worth of Annuities.

19. Let p here represent the Yearly payment,
w — — its Present worth,
t — — the time of its continuance,
d — — the number of years for which the Annuity may be deferred;
and the remaining symbols as before.

Then referring to the *termination* of the Annuity, by the last section, we have its value at that period considered as forborn $= a$; and this, being discounted for the t years during which the Annuity is made payable, gives its value at its *commencement*, on the supposition of yearly conversions, $= av^t$ or $= ar^{-t}$, and this again being discounted for the d years that the Annuity is deferred, gives its *present worth* $w = av^t \cdot v^d = av^{d+t} = \frac{ar \cdot t}{r^d}$, or by substituting the value of $a = p\left(\frac{r^t - 1}{i}\right)$ as found in the last section, we have, $w = \frac{p}{r^d}\left\{\frac{1 - r^{-t}}{i}\right\} = \frac{p}{r^d}\left\{\frac{1 - v^t}{i}\right\} = pv^d\left\{\frac{1 - v^t}{i}\right\}$, any of which is equally applicable to Immediate, Perpetual, or Deferred Annuities, or Deferred Perpetuities, by merely observing, that when $d = 0$ $r^d =$ or $v^d = 1$, and when t is infinite v^t vanishes ; for the present value of £1 to be received an infinite number of years hence, is that of £1 never to be received, and therefore $= 0$. Hence it appears that,

When the Annuity is Immediate and Terminable,

$$w = p\left\{\frac{1 - v^t}{i}\right\}$$

$$p = \frac{wi}{1 - v^t}$$

$$t = \frac{\lambda p - \lambda(p - wi)}{\lambda r}$$

$r^{t+1} - (n+1)\, r^t + n = 0$; n being $= \frac{p}{w}$

When the Annuity is Immediate and Perpetual,

$$w = \frac{p}{i}$$

$$p = wi$$

$$i = \frac{p}{w}$$

When the Annuity is Deferred and Terminable,

$$w = \frac{p}{r^d}\left\{\frac{1-v^t}{i}\right\}$$

$$p = \frac{wir^d}{1-v^t}$$

$$d = \frac{\lambda p + \lambda(1-v^t) - \lambda w - \lambda i}{\lambda r}$$

$$t = \frac{\lambda p - \lambda(p - wir^d)}{\lambda r}$$

$$r^{d+t+1} - r^{d+t} - nr^t + n = 0^*, \; n \text{ being} = \frac{p}{w}$$

When the Annuity is Deferred and Perpetual,

$$w = \frac{p}{ir^d} \text{ or } = \frac{pv^d}{i}$$

$$p = wir^d$$

$$d = \frac{\lambda p - \lambda w - \lambda i}{\lambda r}$$

$$r^{d+1} - r^d - n = 0, \quad n \text{ being } \frac{p}{w}$$

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* In all cases of Annuities, Immediate Perpetuity excepted, the value of r can only be determined by approximation, for which the reader is referred to the next section.

Again supposing the Annuity payable by z equal instalments in the year, and the Interest controvertible into principal x times in the year; substituting $(1+\frac{i}{x})^{x}$ for r in the Equation $w=\frac{ar^{-t}}{r^{dt}}$ and taking the value of a as given in the Formulæ A, B and C, pages 76 and 77, it follows that

When z is an aliquot part of x,

$$w=\frac{p}{z(1+\frac{i}{x})^{dt}}\left\{\frac{1-(1+\frac{i}{x})^{-xt}}{(1+\frac{i}{x})^{\frac{x}{z}}-1}\right\} \quad \text{D}$$

When z is equal to x,

$$w=\frac{p}{i}\left\{\frac{1-(1+\frac{i}{x})^{-xt}}{(1+\frac{i}{x})^{dx}}\right\} \quad \text{E}$$

When z is a multiple of x,

$$w=\frac{p}{i}\left\{1+\frac{(z-x)i}{2xz}\right\}\times\left\{\frac{1-(1+\frac{i}{x})^{-xt}}{(1+\frac{i}{x})^{dx}}\right\} \quad \text{F}$$

Moreover, by making x in these expressions $=1$, 2, 4, or infinite, and observing as in the last section, that, $(1+\frac{i}{x})^{x}$ accordingly becomes $=r$, h, q, or m, and $(1+\frac{i}{x})^{-x}=v$, H, Q, or M, and making z also $=1$, 2, 4, or infinite, we find that

The Present value of an Annuity of £p, at the Nominal Rate denoted by i is, when the

Annuity is			And the Interest is convertible into Principal			
Payable.	Deferred for	To continue for	YEARLY,	HALF-YEARLY,	QUARTERLY,	MOMENTLY,
	Yrs.	Yrs.				
YEARLY,	o	t	$p\frac{1-v^t}{i}$	$p\left(\frac{1-\mathrm{H}^t}{h-1}\right)$	$p\left(\frac{1-\mathrm{Q}^t}{q-1}\right)$	$p\left(\frac{1-\mathrm{M}^t}{m-1}\right)$
	o	*inf.*	$p\left(\frac{1}{i}\right)$	$p\left(\frac{1}{h-1}\right)$	$p\left(\frac{1}{q-1}\right)$	$p\left(\frac{1}{m-1}\right)$
	d	t	$\frac{p}{r^d}\left(\frac{1-v^t}{i}\right)$	$\frac{p}{h^d}\left(\frac{1-\mathrm{H}^t}{h-1}\right)$	$\frac{p}{q^d}\left(\frac{1-\mathrm{Q}^t}{q-1}\right)$	$\frac{p}{m^d}\left(\frac{1-\mathrm{M}^t}{m-1}\right)$
	d	*inf.*	$\frac{p}{r^d}\left(\frac{1}{i}\right)$	$\frac{p}{h^d}\left(\frac{1}{h-1}\right)$	$\frac{p}{q^d}\left(\frac{1}{q-1}\right)$	$\frac{p}{m^d}\left(\frac{1}{m-1}\right)$
HALF-YEARLY,	o	t	$p\left(1+\frac{i}{4}\right)\left(\frac{1-v^t}{i}\right)$	$p\left(\frac{1-\mathrm{H}^t}{i}\right)$	$\frac{p}{2}\left(\frac{1-\mathrm{Q}^t}{\sqrt{q}-1}\right)$	$\frac{p}{2}\left(\frac{1-\mathrm{M}^t}{\sqrt{m}-1}\right)$
	o	*inf.*	$p\left(1+\frac{i}{4}\right)\left(\frac{1}{i}\right)$	$p\left(\frac{1}{i}\right)$	$\frac{p}{2}\left(\frac{1}{\sqrt{q}-1}\right)$	$\frac{p}{2}\left(\frac{1}{\sqrt{m}-1}\right)$
	d	t	$\frac{p}{r^d}\left(1+\frac{i}{4}\right)\left(\frac{1-v^t}{i}\right)$	$\frac{p}{h^d}\left(\frac{1-\mathrm{H}^t}{i}\right)$	$\frac{p}{2q^d}\left(\frac{1-\mathrm{Q}^t}{\sqrt{q}-1}\right)$	$\frac{p}{2m^d}\left(\frac{1-\mathrm{M}^t}{\sqrt{m}-1}\right)$
	d	*inf.*	$\frac{p}{r^d}\left(1+\frac{i}{4}\right)\left(\frac{1}{i}\right)$	$\frac{p}{h^d}\left(\frac{1}{i}\right)$	$\frac{p}{2q^d}\left(\frac{1}{\sqrt{q}-1}\right)$	$\frac{p}{2m^d}\left(\frac{1}{\sqrt{m}-1}\right)$
QUARTERLY,	o	t	$p\left(1+\frac{3i}{8}\right)\left(\frac{1-v^t}{i}\right)$	$p\left(1+\frac{i}{8}\right)\left(\frac{1-\mathrm{H}^t}{i}\right)$	$p\left(\frac{1-\mathrm{Q}^t}{i}\right)$	$\frac{p}{4}\left(\frac{1-\mathrm{M}^t}{\sqrt[4]{m}-1}\right)$
	o	*inf.*	$p\left(1+\frac{3i}{8}\right)\left(\frac{1}{i}\right)$	$p\left(1+\frac{i}{8}\right)\left(\frac{1}{i}\right)$	$p\left(\frac{1}{i}\right)$	$\frac{p}{4}\left(\frac{1}{\sqrt[4]{m}-1}\right)$
	d	t	$\frac{p}{r^d}\left(1+\frac{3i}{8}\right)\left(\frac{1-v^t}{i}\right)$	$\frac{p}{h^d}\left(1+\frac{i}{8}\right)\left(\frac{1-\mathrm{H}^t}{i}\right)$	$\frac{p}{q^d}\left(\frac{1-\mathrm{Q}^t}{i}\right)$	$\frac{p}{4m^d}\left(\frac{1-\mathrm{M}^t}{\sqrt[4]{m}-1}\right)$
	d	*inf.*	$\frac{p}{r^d}\left(1+\frac{3i}{8}\right)\left(\frac{1}{i}\right)$	$\frac{p}{h^d}\left(1+\frac{i}{8}\right)\left(\frac{1}{i}\right)$	$\frac{p}{q^d}\left(\frac{1}{i}\right)$	$\frac{p}{4m^d}\left(\frac{1}{\sqrt[4]{m}-1}\right)$
MOMENTLY,	o	t	$p\left(1+\frac{i}{2}\right)\left(\frac{1-v^t}{i}\right)$	$p\left(1+\frac{i}{4}\right)\left(\frac{1-\mathrm{H}^t}{i}\right)$	$p\left(1+\frac{i}{8}\right)\left(\frac{1-\mathrm{Q}^t}{i}\right)$	$p\left(\frac{1-\mathrm{M}^t}{i}\right)$
	o	*inf.*	$p\left(1+\frac{i}{2}\right)\left(\frac{1}{i}\right)$	$p\left(1+\frac{i}{4}\right)\left(\frac{1}{i}\right)$	$p\left(1+\frac{i}{8}\right)\left(\frac{1}{i}\right)$	$p\left(\frac{1}{i}\right)$
	d	t	$\frac{p}{r^d}\left(1+\frac{i}{2}\right)\left(\frac{1-v^t}{i}\right)$	$\frac{p}{h^d}\left(1+\frac{i}{4}\right)\left(\frac{1-\mathrm{H}^t}{i}\right)$	$\frac{p}{q^d}\left(1+\frac{i}{8}\right)\left(\frac{1-\mathrm{Q}^t}{i}\right)$	$\frac{p}{m^d}\left(\frac{1-\mathrm{M}^t}{i}\right)$
	d	*inf.*	$\frac{p}{r^d}\left(1+\frac{i}{2}\right)\left(\frac{1}{i}\right)$	$\frac{p}{h^d}\left(1+\frac{i}{4}\right)\left(\frac{1}{i}\right)$	$\frac{p}{q^d}\left(1+\frac{i}{8}\right)\left(\frac{1}{i}\right)$	$\frac{p}{m^d}\left(\frac{1}{i}\right)$

Finally, supposing p to represent the periodical fine on a Copyhold, payable every yth year for t payments, w its present worth, and the Interest to be convertible into Principal x times in each year; by conceiving £1 divided between A. and B. as before, it may be shown that if the first fine becomes due y years hence, the present value of the £1 due to B. at the end of ty years $(1+\frac{i}{x})^{-xty}$, and the present value of A.'s Annuity of $(1+\frac{i}{x})^{xy}-1$, receivable every yth year, is therefore $=1-(1+\frac{i}{x})^{-xty}$

Hence, $(1+\frac{i}{x})^{xy}-1 : 1-(1+\frac{i}{x})^{-xty} : : p : w$

from which $w=p.\dfrac{1-(1+\frac{i}{x})^{-xty}}{(1+\frac{i}{x})^{xy}-1}$

Or, in case of yearly conversions, $w=p.\dfrac{1-v^{ty}}{r^y-1}$

20. For the purpose of determining the Rate of Interest in Annuities certain, it should be remarked that (r) the amount of £1 seldom exceeds unity by more than a few hundredth parts, and that it may be easily approximated by the assistance of the Tables, or even assumed in most cases sufficiently near to form the ground-work of further approximation. Hence, let an assumed value of r be put $=e$, and the error between that and its true value $=y$. Then $r=e+y$, and

In Annuities Forborn,

By substituting $e+y$ for r in the general equation $r^t-nr+n-1=0$, obtained in page 75,

we have $(e+y)^t-n(e+y)+n-1=0$;

or by expanding the first term of this expression, and rejecting the square and higher powers of y,

$$e^t + te^{t-1}y - ne - ny + n - 1 = 0;$$

$$\text{Or, } \left(\tfrac{t}{e}e^t - n\right) y = n(e-1) - e^t + 1,$$

$$\text{from which } y = \frac{n(e-1) - e^t + 1}{\frac{t}{e}e^t - 1} \text{ nearly.}$$

In the Present Worth of Annuities,

Taking the general Equation deduced in page 80, we obtain $r^{d+t+1} - r^{d+t} - nr^t + n = 0$,

and by dividing each term of it by r^t there arises

$$r^{d+1} - r^d - n + nr^{-t} = 0;$$

then substituting $e+y$ for r, expanding and rejecting the higher powers of y as before, we derive

$$e^{d+1} + (d+1)\,e^d y - e^d - de^{d-1}y - n + ne^{-t} - nte^{-(t+1)}y = 0;$$

$$\text{Or, } \left\{e^d\left(d+1-\tfrac{d}{e}\right) - \tfrac{n}{e}te^{-t}\right\} y = n(1-e^{-t}) - e^d(e-1)$$

$$\text{from which } y = \frac{n(1-e^{-t}) - e^d(e-1)}{e^d\left(d+1-\frac{d}{e}\right) - \frac{n}{e}\cdot te^{-t}} \text{ nearly.}$$

Or if in this expression $d=0$ $e^d=1$, and if t be infinite e^{-t} vanishes, agreeably to what was stated in page 79. Hence it follows, that in

Immediate Annuities $y = \dfrac{n(1-e^{-t}) - (e-1)}{1 - \frac{n}{e}\cdot te^{-t}}$ nearly;

Perpetual Annuities $\begin{cases} y = n - e + 1, \text{ and} \\ r = e + y = n + 1 \text{ correct.} \end{cases}$

Deferred Annuities $y = \dfrac{n(1-e^{-t}) - e^d(e-1)}{e^d\left(d+1-\frac{d}{e}\right) - \frac{n}{e}te^{-t}}$ nearly;

Deferred Perpetuities $y = \dfrac{n - e^d(e-1)}{e^d\left(d+1-\frac{d}{e}\right)}$ nearly.

In each of these cases when an approximated value of y is determined by the proper Formula, the corresponding value of r becomes known by adding e and y together; and by substituting the result as a new value of e and repeating the operations, the value of r, and therefore that of $i\,(=r-1)$ may be determined to any degree of exactness.

In most cases however the value of i may be found at once sufficiently exact by the assistance of the following Formulæ taken from Mr. Baily's ingenious work on Interest and Annuities, but modified to the foregoing notation.

Annuities Forborn

$$i = \frac{[12+(t+1)c]c}{12+2(t+1)c}; \qquad c \text{ being} = \left(\frac{a}{pt}\right)^{\frac{2}{t-1}} - 1$$

Immediate Annuities

$$i = \frac{[12-(t-1)c]c}{12-2(t-1)c}; \qquad c \text{ being} = \left(\frac{pt}{w}\right)^{\frac{2}{t+1}} - 1$$

Deferred Annuities

$$i = \frac{[12g-(t^2-1)c]c}{12g-2(t^2-1)c}; \begin{cases} g \text{ being} = 2d+t+1 \\ c \text{ - - } = \left(\frac{pt}{w}\right)^{\frac{2}{s}} - 1 \end{cases}$$

Deferred Perpetuities.

$$i = \frac{[6+(5d+1)c]c}{6+4(2d+1)c}; \; c \text{ being} = \left(\frac{w+pd}{w}\right)^{\frac{1}{d}} - 1$$

Or if 1 be added to the value of i as thus determined, and e be substituted for the result, the approximation may be continued by the preceding method.

CHAPTER III.

ON THE RATE OF MORTALITY.

Section I.

General Observations.

In our enquiry concerning the Rate or Law of human mortality, it is implied that out of a great number of persons living at any given age, some determinate proportion will die off during the year: were this the case, and that proportion at each age once assertained, and always to continue invariably the same; upon the principle of association mentioned in page 2, it would not be more difficult to deduce the premiums to be required for life Assurances &c, than to determine the amount or present value of a sum of money improved or discounted for a given number of years. It does not however appear that any such law exists among the human race at large, or even obtains for any length of time in a particular country. On the contrary it may be shown, from indubitable evidence, that human life is much more precarious in one climate than in another;

and that in the same country the Rate of Mortality is subject to the most striking variations at different periods and under different circumstances.

For instance, in England and Wales it will be found by the Population Abstracts, published by order of the House of Commons in 1801, 1811, and 1821, that,

From the Year	The average number of annual Baptisms was	The average number of annual Burials was	Number of Burials for every 1000 Baptisms.
1720 to 1730	163210	174267	1068
1730 — 1740	170375	177655	1043
1740 — 1750	180026	166375	924
1750 — 1760	189365	160512	848
1760 — 1770	203170	170700	840
1770 — 1780	220894	189372	857
1780 — 1790	239714	188576	787
1790 — 1800	261776	195469	747
1800 — 1805	278353	194094	697
1805 — 1810	297428	195944	659
1810 — 1815	316996	193847	612
1815 — 1820	334017	208153	623

It will likewise be found by the same documents, that,

In the year	The estimated Population was	The returned Baptisms in the preceding year were	Making one Baptism to every —— persons.
1801	9,168,000	254870	35.97
1811	10,502,500	298853	35.14
1821	12,218,500	343660	35.55

From which it appears, that the returned Baptisms at these periods were nearly in proportion to the estimated Population. Hence, supposing the average annual Baptisms in each of the above periods

to be proportional to the contemporaneous population, and the returned Burials the same constant proportion of the annual deaths which actually took place, it follows that

Out of the same Population that			
1068	died annually	between	1720 and 1730
1043	- - -	-	1730 - 1740
924	- - -	-	1740 - 1750
848	- - -	-	1750 - 1760
840	- - -	-	1760 - 1770
857	- - -	-	1770 - 1780
787	- - -	-	1780 - 1790
747	- - -	-	1790 - 1800
697	- - -	-	1800 - 1805
659	- - -	-	1805 - 1810
612	- - -	-	1810 - 1815
623	- - -	-	1815 - 1820

From this abstract it appears, that the Rate of Mortality in this country has gradually decreased for the last 100 years, and the average duration of human life has therefore reciprocally increased during the same period, but not in so great a ratio as that of 623 to 1068; for it is manifest, that in a community where the number of Births annually exceed the number of Deaths, (and new settlers supply the places of those that emigrate into foreign countries) not only the population, but the proportion of young persons to those of a more advanced age must necessarily increase; the average age of the whole community must be proportionally diminished, and

independent of any improvement in the average duration of human life, the proportion of annual deaths to the contemporaneous population must for some time necessarily diminish.

We cannot however attribute to this cause so great an effect in the diminution of the Rate of Mortality, as that shown by the foregoing abstract, without admitting the most striking improvement in the average duration of human life; but if, in connection with what has been stated, we regard the improvements made in the preservation of juvenile life by the introduction of Vaccination &c. it will not be found a difficult matter to account for the diminution of mortality, already pointed out, in a mixed community consisting of a very considerable proportion of Children,* without admitting so great an improvement as has been commonly supposed, in the duration of *Assured* lives whose average age may be reckoned from 35 to 40.

Comparing what has been here stated with the observations made on the improvement of money, (Chap. ii. 1.) we are led to conclude, that during the greater part of the last 100 years the Rate of Mortality in this Country has been less subject to variation than that of Interest: and if we regard the important purposes to which calculations grounded

* In the summary of the population for 1821 it appears, that out of the 10,557,671 persons whose ages were returned in England and Wales, 2,942,583 were children under 10 years of age.

on the improvement of money and the average duration of human life may be applied, it must be granted that every effort made to determine that average must deserve the encouragement, not only of those connected with life Insurance Offices, but of a considerable proportion of the inhabitants of the British Empire.

The first Table of Mortality upon record is that deduced by Dr. Halley, from the mortuary registers of Breslaw for five years, ending with 1691; the next that of Mr. Kerrsboom, constructed from the registers of Life annuitants in Holland and West Friesland, published in 1738. Soon after Mr. Thomas Simpson published a Table from the London Bills of Mortality; and in 1746 M. Deparcieux constructed a valuable Table from the lists of French Tontines. Since this period, the celebrated Dr. Price, fully aware of the importance of the subject, extended its boundaries by a number of additional Tables deduced from observations on the Mortality in London, Norwich, Northampton, Vaud in Switzerland, Brandenburg, Holy Cross near Shrewsbury, Vienna, Berlin, Chester, Sweden and Finland, and the Town of Stockholm. In 1806 Mr. Duvillard published a Table of Mortality for the Kingdom of France; and in 1815 Mr. Joshua Milne added to the list three valuable Tables; the first (and doubtless the best index of British Lives hitherto published) constructed from very accurate observations made at Carlisle by Dr. Hysham; the second from observations made in Sweden and Finland, for 20 years ending

with 1795; and the third from the registers of mortality at Montpellier for 21 years ending with 1792.†

Mortality among Assured Lives.

2. Dr. PRICE, in his Observations on Reversionary Payments, speaking of the Equitable society in the year 1782, states, "that the rate of mortality "among the persons assured has continued much "below that in the Tables by which it has hitherto "made its calculations; and that upon the whole, "it appears at present to possess such a *Surplus* of "income and stock as places it (if no mismanagement takes place) above danger, except from events "the most extraordinary. In these circumstances, "the society, not willing to raise an exorbitant "capital, or to take unreasonable profits, came to "a resolution at the beginning of the last year, or "1781, to make such abatements in its demands as "its present circumstances render safe, and to settle such new arrangements in its business as may "contribute to make is as great a benefit as possible "to the public." And his relative Mr. Morgan, (to whom we are not less indebted,) as the Editor of that Work adds, in a note to the seventh Edition, pages

† For an account of several other Tables, and some valuable observations on those enumerated above, the reader is referred to Mr. Milne's Treatise on Life Annuities &c. and to the Articles Mortality, and Bills of Mortality, written by the same Author, in the Supplement to the Encyclopædia Britannica.

182—3, " Until the time nearly in which this ac-
" count was written, the Society had computed all
" their premiums from the London Table of Obser-
" vations, and it appeared from the experience of
" 12 years, (or from 1768 to 1780) that the decre-
" ments of life among the members had been from

	To those in London.	To those in Breslaw.
" the age of		
" 10 to 20 in the ratio of	1 to $1\frac{1}{4}$	1 to 1
" 20 to 30	3 to 7	2 to 3
" 30 to 40	4 to 9	2 to 3
" 40 to 50	3 to 5	8 to 9
" 50 to 60	2 to 3	6 to 7
" 60 to 70	9 to 10	9 to 8

" In consequence, they determined to compute
" the premiums in future from a table which should
" give the probability of life *higher* than that which
" they had hitherto used; and for this purpose they
" adopted one which had been just formed by *Dr.*
" *Price*, from very accurate observations made in
" the town of *Northampton*. Compared with the
" decrements of life in that Table from the year
" 1768 to the year 1810, the decrements of life
" in the Society appear to have been

" from the age of 10 to 20 in the ratio of	1 to 2
" 20 to 30	1 to 2
" 30 to 40	3 to 5
" 40 to 50	3 to 5
" 50 to 60	5 to 7
" 60 to 80	4 to 5

" or at all ages together in the ratio of 2 to 3."

These proportions were first made public in Mr. Morgan's Address to a General Court of the Society in the year 1800 (page 163,) as the "result of an experience of thirty years," and in the note referred to they are repeated as expressing the rate of mortality among the members of the Society up to 1810; and that this latter number is not put in a mistake for 1800, we are informed by the same author in another note, contained in page 192 of edition before quoted, published in 1812, where he gives a further account of the Equitable Society, that, "The many thousands of which it is now "composed render the decrements of life almost "uniform among its members, and from the ex- "perience of *more than forty years* those decre- "ments are found to be even *one-third* lower than "they are in the Table from which the premiums "are computed." Nor does it appear that up to the present time any material alteration has taken place in the mortality among the members of that Society; for we are told by Mr. Morgan in his Address delivered in December, 1816, (page 203,) "that the probabilities of life in the Society bear "the same ratio to those in the Table of observa- "tions from which its premiums are computed, "that they have done for the last forty years;" and in that delivered on the 3rd of March, 1825, that— "The events of the four preceding years have by "no means been so particularly favourable as to "justify a departure from the established rules of

" the Society. The decrements of life have been " much the same as they have always been; in the " last two years, indeed, they have rather exceeded " their usual number."

It cannot, however, be denied that a considerable improvement has taken place in the average duration of Life among the members of the Equitable, in common with other classes of the community; for it is manifest that had not this been the case the mortality among them must have gradually increased, as the effect of selection necessarily became less perceptible.

But taking all these circumstances into consideration, it must be allowed that, however doubtful the limited experience of a new Institution might be regarded, the proportions stated by Mr. Morgan, repeated and confirmed as they have been for a period exceeding half a century, afford more satisfactory data for determining the *Rate of Mortality among Assured Lives* than any registers hitherto made public.

By the Table referred to in these proportions, we find that at the Town of Northampton, from 1735 to 1780, out of 11650 children all born alive,

8650	attained	the	age	of	1 year,	and	3000	died	during the 1st year
7283	-	-	-		2 years	-	1367	- -	2nd ——
6781	-	-	-		3 ——	-	502	- -	3rd ——
6446	-	-	-		4 ——	-	335	- -	4th ——

and so on, as in the Register known by the name of the

Age.	Number Living.	Deaths.	Sum of Nos. Living	Sum of Deaths.
0	11650	3000		
1	8650	1367		
2	7283	502		
3	6781	335		
4	6446	197		
5	6249	184	70599	5975
6	6065	140		
7	5925	110		
8	5815	80		
9	5735	60		
10	5675	52		
11	5623	50		
12	5573	50		
13	5523	50		
14	5473	50		
15	5423	50	54444	543
16	5373	53		
17	5320	58		
18	5262	63		
19	5199	67		
20	5132	72		
21	5060	75		
22	4985	75		
23	4910	75		
24	4835	75		
25	4760	75	47972	747
26	4685	75		
27	4610	75		
28	4535	75		
29	4460	75		
30	4385	75		
31	4310	75		
32	4235	75		
33	4160	75		
34	4085	75		
35	4010	75	40475	750
36	3935	75		
37	3860	75		
38	3785	75		
39	3710	75		
40	3635	76		
41	3559	77		
42	3482	78		
43	3404	78		
44	3326	78	32866	778
45	3248	78		
46	3170	78		
47	3092	78		
48	3014	78		
49	2936	79		

Age.	Number Living.	Deaths.	Sum of Nos. Liviug.	Sum of Deaths.
50	2857	81		
51	2776	82		
52	2694	82		
53	2612	82		
54	2530	82		
55	2448	82	24889	819
56	2366	82		
57	2284	82		
58	2202	82		
59	2120	82		
60	2038	82		
61	1956	82		
62	1874	81		
63	1793	81		
64	1712	80		
65	1632	80		
66	1552	80		
67	1472	80		
68	1392	80		
69	1312	80	25488	1569
70	1232	80		
71	1152	80		
72	1072	80		
73	992	80		
74	912	80		
75	832	80		
76	752	77		
77	675	73		
78	602	68		
79	534	65		
80	469	63		
81	406	60		
82	346	57		
83	289	55		
84	234	48		
85	186	41		
86	145	34		
87	111	28		
88	83	21	2465	469
89	62	16		
90	46	12		
91	34	10		
92	24	8		
93	16	7		
94	9	5		
95	4	3		
96	1	1		
Total--	299198	11650		

543

Instead of fixing upon a given number of children just born, and ascertaining how many survive each successive year of their age until they all become extinct, let us suppose a community, subject to the law of mortality exhibited by the foregoing Table, formed by the birth of 11650 children annually for any period not less than 96 years; then conceiving all the children born simultaneously at the beginning of the year,* and none to be admitted into the community except by birth, or removed from it otherwise than by death, it is manifest that at the beginning of the 97 or any subsequent year, such community must necessarily consist of

11650 children just born,
8650 of the age of 1 year, survivors of those born 1 year before
7283 - - 2 years - - - - 2 years —
6781 - - 3 —— - - - - 3 —— —
6446 - - 4 —— - - - - 4 —— —

and so on, as expressed in the foregoing Table.

Or by the classification we have made, it appears that such community would consist of

70599	children under 10,	of whom	5975	would die	during the year.			
54444	persons between 10 and 20	-	543	—	—	—		
47972	- - - 20 - 30	-	747	—	—	—		
40475	- - - 30 - 40	-	750	—	—	—		
32866	- - - 40 - 50	-	778	—	—	—		
24889	- - - 50 - 60	-	819	—	—	—		
25488	- - - 60 - 80	-	1569	—	—	—		
2465	- - - 80 & upwards		469	—	—	—		
299198	Total Population		11650	Total Deaths.				

* This supposition is, however, inconsistent with matters of fact, but is not on that account less applicable to the illustration of the subject here considered.

Hence, comparing the number living in each class with the annual deaths in the same, it will be found, according to the rate of mortality exhibited by the Northampton Table, that

under	10, one person dies annually out of	$\frac{70599}{5975}$ =	11.816
between	10 and 20 - - - - -	$\frac{54444}{543}$ =	100.265
——	20 - 30 - - - - -	$\frac{47972}{747}$ =	64.219
——	30 - 40 - - - - -	$\frac{40475}{750}$ =	53.967
——	40 - 50 - - - - -	$\frac{32866}{778}$ =	42.244
——	50 - 60 - - - - -	$\frac{24889}{819}$ =	30.389
——	60 - 80 - - - - -	$\frac{25488}{1569}$ =	16.245
——	80 and upwards - - - -	$\frac{2465}{469}$ =	5.256

And by applying to these results the proportions stated in page 92, we find that among the members of the Equitable from 1768 to the present year,

between	10 and 20, one died annually out of	2(100.265) =	200.530
——	20 - 30 - - - - -	2 (64.219) =	128.438
——	30 - 40 - - - - -	$\frac{5}{3}$(53.964) =	89.940
——	40 - 50 - - - - -	$\frac{5}{3}$(42.244) =	70.407
——	50 - 60 - - - - -	$\frac{7}{5}$(30.389) =	42.545
——	60 - 80 - - - - -	$\frac{5}{4}$(16.245) =	20.306

Again, by observing the gradual approximation of the deaths at the Equitable, to those exhibited by the Northampton Table, towards the more ad-

vanced periods of life, we find their relative proportions

between 40 and 50 as 3 to 5, or as 6 to 10,
——— 50 - 60 as 5 to 7, or as 7 to 10,
——— 60 - 80 as 4 to 5, or as 8 to 10,

from the gradual progression of which we may fairly assume that had there been a sufficient number of old lives at the Equitable to produce an average number of deaths from 80 upwards, the proportion taken as above would have been about as 9 to 10; or in other words the annual deaths among persons above 80 years of age, may from these proportions fairly be taken as one out of $\frac{10}{9}(5.256)$ or one out of 5.840. From these data, and as is always done in the construction of Tables of Mortality,* the fixing of the limiting age somewhat arbitrarily, Table X. in this work, has been deduced by the method explained at the end of this chapter.

In order however to show the coincidence between the rate of Mortality thus deduced and that which hitherto obtained among the Members of the Equitable, according to Mr. Morgan's statements, let us suppose as before a community subject to this law of mortality, formed by the simultaneous admission of 2844 Children of the age of 10, at the beginning of each year for a period not less than 87 years, and no person to be admitted into the

* Milne on Annuities, Vol. ii. pages 410—11.

community of any other age, or removed from it otherwise than by death; then, by classifying the numbers given in Table X. as we have done with those in the Northampton Table, it will appear that at the beginning of the 88th or any subsequent year, such community must, if the law of mortality should continue invariable during the interval, necessarily consist of

27882	persons between	10	and	20,	of whom	137	would die during the year		
26187	- - -	20	-	30	-	204	—	—	—
23853	- - -	30	-	40	-	265	—	—	—
21060	- - -	40	-	50	-	299	—	—	—
17605	- - -	50	-	60	-	413	—	—	—
20892	- - -	60	-	80	-	1044	—	—	—
2820	- - -	80 & upwards				480	—	—	—
140299	Total Population.					2844	Total Deaths.		

And comparing as before the number living in each of these classes with the annual deaths in the same, it will appear that, according to this rate of mortality,

between	10	and	20,	one would die annually out of			200.59
——	20	-	30	- - - - -			128.37
——	30	-	40	- - - - -			90.01
——	40	-	50	- - - - -			70.43
——	50	-	60	- - - - -			42.63
——	60	-	80	- - - - -			20.01
——	80 and upwards.			- - - -			5.87

all agreeing as nearly as possible with the numbers deduced in page 97, from which our Table has been constructed, and therefore proving that the rate of

mortality exhibited thereby is identical with that which obtained among the members of the Equitable Society.

Nor is it incompatible with the other proportion stated by Mr. Morgan, viz. that, at all ages together, the decrements of life in the Northampton Table have been to those which took place in the Equitable Society, in the ratio of three to two: for in order to compare the decrements in this view, we ought to know the relative proportions of the number of lives assured at different ages. By the *Nosological* Table annexed to the last edition of Mr. Morgan's work on Assurances, it will be found that, *including Renewals*, the Assurances effected at the Equitable during the first twenty years of the present century were as under, viz.

1494	on lives between	10 and 20
8996		20 - 30
33850		30 - 40
45429		40 - 50
36489		50 - 60
19042		60 - 70
5880		70 - 80
574*		80 and upwards.

* Mr. Morgan in the Table here referred to states, that the number of *Assurances* effected from 70 upwards was 6454, and that the number of deaths from 70 to 80 was 345, from 80 upwards 82. Also by Table X. it appears that, from 70 to 80, one person died annually out of 14.244; and, from 80 upwards, one out of 5.87: hence $345 \times 14.244 + 82 \times 5.87 = 4921 + 481 = 5402$; and 5402 : 6454 : : 4921 : 5880, for the number assured from 70 to 80, and 5402 : 6454 : : 481 : 574, for the number assured from 80 upwards.

Hence, supposing these Insurances to have been effected on as many different lives, and the whole made simultaneously at the beginning of one and the same year, it is manifest that among the first class $\frac{1494}{100.265} = 15$ deaths might have been expected to happen in the year, according to the Northampton Table; or $\frac{1494}{200.59} = 7$ deaths, according to the Table we have formed from the experience of the Equitable; and by proceeding in like manner through all the classes, and collecting the results into a Tabular form, it appears that,

Out of the Insurances effected from 1800 to 1820	On Lives between	The number of Deaths would be	
		By the Northampton Table	By the experience of the Equitable
1494	10 and 20	15	7
8996	20 — 30	140	70
33850	30 — 40	627	376
45429	40 — 50	1075	645
36489	50 — 60	1201	856
19042	60 — 70	917	722
5880	70 — 80	513	413
574	80 and upwards	109	98
151,754	Totals	4597	3187

And since the totals 4597 and 3187 are to each other very nearly as three to two, the additional proportion stated by Mr. Morgan, relative to the comparison between the decrements of life in the Northampton Table and the deaths which took place at the Equitable, instead of militating against

the accuracy of the Table herein deduced, evidently tends to confirm its coincidence with the rate of mortality among the members of that Institution.

Nor is it difficult to reconcile the apparent incongruity between the number of deaths exhibited in the last column of this abstract, and the number of deaths (1930) which actually took place at the Equitable from 1800 to 1820; for the 151,754 in the first column represents the number of *Insurances*, and not the number of *Lives* on which those Insurances were effected; and it is well known to those connected with Life Offices, that it is no uncommon thing for *several Insurances* to be effected on the *same life;* the number of lives out of which the 1930 deaths happened must therefore have been materially less than 151,754, otherwise Mr. Morgan's statements in 1812, 1816 and 1825, must have been altogether erroneous. Nor must it be forgotten, that in order to deduce the rate of mortality from the experience of an Office (an object evidently not contemplated by Mr. Morgan in the *Nosological* Table above referred to,) allowance must be made for policies effected for periods less than a year, as well as for those that are cancelled, by expiration, forfeiture, or redemption, before the time that the renewal premiums would have become due.

Considering the importance of obtaining as accurate an index as possible of the duration of human life, it may not be thought tedious to compare the

Table herein deduced with that formed by Mr. Milne from observations at Carlisle.

For this purpose, by classifying the numbers in Mr. Milne's Table, it appears, that at Carlisle, out of

Persons living	between the age of	there died annually	Making one out of every	At the Equitable one died out of
63083	10 and 20	370	170	200
58980	20 — 30	448	132	128
53894	30 — 40	567	95	90
47642	40 — 50	678	70	70
40953	50 — 60	754	54	43
30826	60 — 70	1242	25	26
17502	70 — 80	1448	12	14
5725	80 & upwds.	953	6	6
318,605	10 upwards	6460	49½	49½

From which it appears that the rates of mortality, indicated by the two Tables, at all ages from 10 upwards agree to a fraction, although at particular ages they materially differ.

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3. *Application of the Rate of Mortality.*

Example 1.

Let it be required to determine, out of 400 persons living at the age of 30, how many may be expected to survive to attain the age of 50, supposing them subject to the same law of mortality as the members of the Equitable?

By Table X. we find that out of 2501 persons living at the age of 30, 1937 will survive to attain

the age of 50. Hence 2501 : 1937 : : 400 : 310 the Answer.

Similarly it may be shown, that out of 764 persons living at the age of 55, 450 may be expected to survive to the age of 70.

Example II.

Required the number of deaths which may be expected to happen within one year among 500 persons living at the age of 40, supposing them subject to the same law of mortality as the members of the Equitable?

By Table X. it appears that out of 2236 persons living at the age of 40, 28 may be expected to die off during the year.

Hence $2236 : 28 : : 500 : \frac{28}{2236} \times 500 = 6\frac{261}{1000}$ Answer. The fraction $\frac{261}{1000}$ though absurd when referred to an individual, serves to show that out of 1000 times as many persons as the number proposed, (i. e. 500,000 persons,) 6261 deaths may be expected to happen within the year.

We may here further remark from the expression $\frac{28}{2236} \times 500$, that if the *decrement* opposite each age be divided by the number living at the same age, the result points out the *proportion* which may be expected to die off during the year. Thus $28 \div 2236 = .012522 =$ about $1\frac{1}{4}$ per cent. represents the proportion of the number living at 40, which may be expected to die off within one year: and deducting

the .012522 from unity, there remains .987478, or about 98$\frac{3}{4}$ per cent. as the proportion which may be expected to survive one year to attain the age of 41. In this manner Table XII. has been constructed.

By the assistance of this Table the question proposed may be solved by a single operation. Thus .012522 × 500=6.261 the Answer as before.

Example III.

Suppose a society consisting of 300 persons of the age of 20, 250 of the age of 30, 400 of the age of 50, and 200 of the age of 60; it is required to find the number of deaths which may be expected to happen within one year, admitting the different classes to be all subject to the same law of mortality as the members of the Equitable?

Here, proceeding as in the last example, we find the deaths to be expected as under,

Thus	.006654 × 300 =	1.9962	deaths in 1st class
—	.009596 × 250 =	2.3990	- - - 2nd
—	.018069 × 400 =	7.2276	- - - 3rd
—	.030184 × 200 =	6.0368	- - - 4th
		17.6596 = 17$\frac{2}{3}$	Answer.

A repetition of the same process with the number insured at each age would evidently enable an Insurance Office to determine the number of deaths, which might be expected to happen during the year

according to the experience of the Equitable, or any other rate of mortality. And comparing the result with matters of fact, such Office might ascertain at the end of each year whether the deaths had been more or less in number than what might have been reasonably expected.* In order however to determine, whether the *Losses* had proved more or less favourable than what might have been expected, it would be necessary to regard not only the number of deaths, but the average amount of the sums insured by the different Policies. Thus it is manifest that an Insurance Office having the average amount of its Policies = £1500, would have reason to consider its losses as unfavourable by 18 deaths, averaging £2000 each, although 20 deaths might have been expected, according to the most correct calculation.

Example iv.

Let it be required to determine the number of deaths which may be expected, within the next seven years, out of 1000 persons living at the age of 50, supposing them subject to the same law of mortality as the members of the Equitable?

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* With respect to the fresh Insurances effected during the year, and the Policies cancelled by forfeiture or redemption during the same period, as some are insured or cancelled at the beginning of the year, some at the middle and some at the end, it is evident that on an average each of these policies should only be considered in force for *half* a year, or half their number for the whole year.

By Table x. it appears, that out of 1937 persons living at the age of 50, 1659 may be expected to attain the age of 57; the difference, $1937 - 1659 = 278$, may therefore be expected to die off within the next seven years.

Hence 1937 : 278 : : 1000 : 143.52 the Answer.

But if the number of deaths which may be expected in each year were required, we should take out the successive decrements, and proceed thus :—

As 1937 : 35 : : 1000 : 18.07 =	deaths in	first	year.
1937 : 37 : : 1000 : 19.10 =	—	2nd	—
1937 : 39 : : 1000 : 20.13 =	—	3rd	—
1937 : 41 : : 1000 : 21.17 =	—	4th	—
1937 : 41 : : 1000 : 21.17 =	—	5th	—
1937 : 42 : : 1000 : 21.68 =	—	6th	—
1937 : 43 : : 1000 : 22.20 =	—	7th	—
Total deaths = 143.52	in the whole	7	years.

Example v.

Supposing a given number of marriages contracted between males of the age of 30, and females of the age of 25; it is required to find the proportion remaining undissolved by death at the end of 10 years, admitting the husbands and the wives † to be

† This supposition is not strictly correct, for the most accurate observations that have been made on this subject tend to show that female life is better than male, and married life better than single.—See Milne on Annuities, Vol. II.

all subject to the same law of mortality as the members of the Equitable?

Here, were the number of marriages contracted to be 2501, (the number living in Table x. at the age of 30,) it is manifest by inspection that the number of husbands which may be expected to attain the age of 40 is 2236.

And out of the same number of wives living at the age of 25, the number which may be expected to survive 10 years to attain the age of 35, is found as in example 1. thus

$$2611 : 2374 : : 2501 : \tfrac{2374\times 2501}{2611} = 2274.$$

It therefore appears that, out of the 2501 marriages contracted, 2236 husbands and 2274 wives would survive the proposed interval. But as it is manifest that some of these husbands would be widowers and some of the wives widows, (unless married again, which is excluded by the supposition,) we must further enquire how many *pairs* may be expected to remain undissolved by death. For this purpose conceive the 2236 pairs, of which the husbands will survive, to be separated from the rest in the first instance, then to find how many of their wives will survive, we have

$$2611 : 2374 : : 2236 : \tfrac{2374\times 2236}{2611} = 2033$$

for the number of marriages undissolved at the end of 10 years.

The same conclusion might be derived by conceiving the 2274 pairs, whereof the wives may be

expected to survive, to be as before separated from the 2501 married couples in the first instance, and then enquiring how many of their husbands may be expected to survive the proposed term; for in that case we have $2501 : 2236 : : 2274 : \frac{2236 \times 2274}{2501}$ which, by substituting for the 2274 its value $\frac{2374 \times 2501}{2611}$ as already found, becomes $\frac{2236}{2501} \times \frac{2374 \times 2501}{2611} = \frac{2236 \times 2374}{2611} = 2033$, the same as before.

Again, suppose the number of marriages contracted to be double of 2501, then the number remaining undissolved at the end of 10 years may be expected to be double of 2033, or of $\frac{2236 \times 2374}{2611}$.

Similarly it may be shown, that out of

3 times 2501 marriages,	3 times $\frac{2236 \times 2374}{2611}$	may be expected to survive.
4 times 2501 - - -	4 times ditto	ditto.
2611 times 2501 - - -	2611 times $\frac{2236 \times 2374}{2611} = 2236 \times 2374$	

may be expected to survive the given interval: the required proportion is therefore

$$\frac{2236 \times 2374}{2501 \times 2611} = \frac{5308264}{6530111} = .8129 = \text{about } 81\tfrac{1}{4} \text{ per Cent.}$$

By applying the same mode of reasoning to any other ages, and observing that out of a number of marriages represented by the product of the numbers living in a Single Life Table, opposite the respective ages of the parties, the number of surviving pairs, at the end of any given term, is always represented by the product of the numbers living in the same table opposite the then advanced ages of the married couples, we deduce an easy method of

forming a table showing the rate of extinction of two Joint Lives by multiplying the numbers living in the Single Life Table opposite the proposed ages, and also those opposite ages respectively 1, 2, 3, 4, &c. years older than the ages given. Thus taking the ages at 30 and 25 as before, and multiplying the numbers living at these ages, and also those at

31 and 26
32 and 27
33 and 28
34 and 39 and so on; it appears that out of

2501 × 2611 pairs living at the ages of 30 and 25,
2477 × 2591 = 6417,907 will survive 1 year,
2452 × 2570 = 6301,640 - - - 2 years,
2426 × 2548 = 6181,448 - - - 3 -
2400 × 2525 = 6060,000 - - - 4 -
2374 × 2501 = 5937,374 - - - 5 -
2347 × 2477 = 5813,519 - - - 6 -
2320 × 2452 = 5688,640 - - - 7 -
2292 × 2426 = 5560,392 - - - 8 -
2264 × 2400 = 5433,600 - - - 9 -
2236 × 2374 = 5308,264 - - - 10 -
&c. &c. &c.

But as it is unnecessary, in Tables formed for these purposes, to retain so many figures as would be thus produced, the labour might be materially reduced by dividing the results by any number, (say 1000, which is easily done by simply cutting off the last three figures,) and the quotients retaining the

same proportion will equally answer the end in view. In this manner Table XIV. has been formed.

Example VI.

Supposing, as in the last example, that a certain number of marriages were contracted between males of 30, and females of 25; it is required to determine the proportions of widowers and widows which may be expected to survive 10 years, admitting the males and females to be subject to the same law of mortality as the members of the Equitable, and no second marriages to be entered into?

By Table x. it appears that, of 2501 husbands living at 30, 2236 may be expected to survive 10 years; and by the last example it has been shown, that out of a like number of marriages at the ages proposed, 2274 of the wives and 2033 complete pairs may be expected to survive the given interval; hence, by deducting the number of complete pairs from the number of surviving husbands, there remains 203 for the number of widowers, and proceeding in like manner with the wives, we have 241 for the number of widows. Or by dividing each of these results by 2501, the number of marriages from which they remain, we have

$\frac{203}{2501} = .0812 = 8$ per Cent, = proportion of Widowers.

$\frac{241}{2501} = .0964 = 9\frac{2}{3}$ per Cent. = ditto - - Widows.

Otherwise, since it appears by Table x, that out of

2501 living at 30, in 10 years 2236 will survive, and 265 will have died,
2611 — — 25, — — 2374 — — — 237 — —

by reasoning as in the foregoing examples it may be shown, that out of $2501 \times 2611 = 6530111$ marriages contracted, at the end of 10 years we may expect them to be distinguished as follows:—

Husbands living & wives dead	2236×237	$= 529932$
Wives living & husbands dead	2374×265	$= 629110$
Husbands and wives living	2236×2374	$= 5308264$
Husbands and wives dead	265×237	$= 62805$
Making together the number proposed.....		6530111

Hence, dividing each result by the number of marriages, we have

$529932 \div 6530111 = .08115 =$ Proportion of Widowers,
$629110 \div 6530111 = .09634 =$ — — Widows,
$5308264 \div 6530111 = .81289 =$ — — Pairs remaining
$62805 \div 6530111 = .00962 =$ — — ditto extinct.

Example vii.

Suppose a number of families proposed, each consisting of husband, wife, and child, of the respective ages of 40, 35, and 10 years; it is required to determine what proportion of them may be expected to escape the ravages of death for the next 20 years, admitting them all to be subject to the same law of mortality as the members of the Equitable?

By Table x. it will be found, that out of 2236 husbands living at 40, 1524 will survive 20 years
2374 wives - - - 35, 1744 — — —
2844 children - - - 10, 2501 — — —

And, by what has been shown in example v, page 107, it follows, that out of 2236×2374 pairs of husbands and wives, existing at the beginning of the proposed interval, 1524×1744 may be expected to remain entire at the end of that interval. Hence, conceiving the number of families, in the first instance, to be expressed by 2236×2374, and of these the 1524×1744, whereof the husband and wife may be expected to survive the given term, to be separated from the rest as supposed in the foregoing examples; to find how many of them will have their children also alive at the end of the proposed term, we have

$$2844 : 2501 : : 1524 \times 1744 : \frac{1524 \times 1744 \times 2501}{2844.}$$

for the number of families which may be expected to remain entire at the end of the proposed interval; and proportionally out of twice 2236×2374 families of the proposed ages, twice $\frac{1524 \times 1744 \times 2501}{2844}$ may be expected to survive 20 years: out of three times as many families, three times as many survivors, and so on; consequently out of 2844 times the proposed number of families, which is $2236 \times 2374 \times 2844$, 2844 times $\frac{1524 \times 1744 \times 2501}{2844}$ which is $1524 \times 1744 \times 2501$ may be expected to survive the given interval. Then dividing

the number of survivors, by the number of families out of which they remain, we have $\frac{1524\times1744\times2501}{2236\times2374\times2844} = .44031$, or rather better than 44 per cent. for the proportion required.

By observing the result herein deduced, it appears that out of a number of families represented by con-continual product of the numbers living in a Single Life Table, opposite the respective ages of the individuals of which each is composed, the number of surviving families, at the end of any given term, is denoted by the continual product of the numbers living in the same table opposite the then advanced ages of the parties, we deduce an easy and general method of forming a table to represent the gradual extinction of joint lives, taken three and three: but the labour in forming such table may be materially reduced by cutting off a given number of figures from the results, as we have done in page 110, when forming a similar table for two joint lives.

Thus, taking the ages to be 30, 25, and 10 years, and multiplying together the numbers living in the table opposite the ages,

30, 25, and 10,
31, 26, — 11,
32, 27, — 12,
33, 28, — 13,

} and cutting off a given number of figures from the result,

or, taking the product of the first two numbers in each case from the Two Joint Life Table already

formed, as directed in page 110, it will be found that out of

Combinations of Three Joint Lives.							
6531×2.844	= 18574	—	existing at the ages of	30,	25,	and	10,
6417×2.833	= 18179	—	will jointly survive to	31,	26,	—	11,
6302×2.822	= 17785	—	— — —	32,	27,	—	12,
6181×2.810	= 17369	—	— — —	33,	28,	—	13,
&c	&c.			&c.			&c.

In this manner Table xv. has been constructed.

Example viii.

A number of families, each consisting of husband, wife, and child, of the respective ages of 40, 35, and 10, being proposed, and all subject to the same law of mortality as the members of the Equitable; it is required to determine what proportions of them will have surviving at the end of 20 years;

1st. Husband, wife, and child;
2nd. Husband and wife only;
3rd. Husband and child only;
4th. Wife and child only;
5th. Husband only;
6th. Wife only;
7th. Child only;
8th. None?

By Table x. it will be found, that out of

			Will survive 20 years.		Will die in the interval.
2236	husbands living at	40,	1524,	and	712
2374	wives - -	35, -	1744,	- -	630
2844	children -	10, -	2501,	- -	343

Hence, representing the husbands, wives, and children by h, w, and c; and reasoning as in the foregoing examples, it may be shown that out of a number of families represented by $2236 \times 2374 \times 2844$, there may be expected to survive the proposed term,

h. w. & c.	$1524 \times 1744 \times 2501$,	which divided by $2236 \times 2374 \times 2844$	= .44031
h. & w. only,	$1524 \times 1744 \times 343$	- - - - - -	= .06039
h. & c. only,	$1524 \times 2501 \times 630$	- - - - - -	= .15906
w. & c. only,	$1744 \times 2501 \times 712$	- - - - - -	= .20571
h. only,	$1524 \times 630 \times 343$	- - - - - -	= .02181
w. only,	$1744 \times 712 \times 343$	- - - - - -	= .02822
c. only,	$2501 \times 712 \times 630$	- - - - - -	= .07431
extinct,	$712 \times 630 \times 343$	- - - - - -	= .01019
Total	- - - - - - - - - - - -		1.00000

And if we suppose the number of families to be 1000, it follows, from these results, that at the end of 20 years

440 of them would survive entire;
61 pairs of husbands and wives without children,
159 widowers with children,
206 widows with children,
22 widowers without children,
28 widows without children,
74 orphans, without fathers or mothers; and
10 families would altogether become extinct.

1000 making together the number proposed.

The same mode of proceeding will equally apply to any other ages or number of lives.

Example ix.

A number of lives of the age of 85 being proposed, all subject to the same law of mortality as the members of the Equitable; it is required to determine how long, on an average, each of them may be expected to live?

Referring to Table x. it will be found, that out of 224 persons living at 85,

43 may be expected to die off in the 1st year,
38 - - - - - - 2nd —
32 - - - - - - 3rd —
26 - - - - - - 4th —
20 - - - - - - 5th —
and so on.

It is also probable, that of those who die off in the first year, some will become extinct at the beginning of the year, some in the middle, and others towards the end; we may therefore assume, that on an average, they will live half a year each; and those who die off in the second year must evidently live the whole of the first year, and (as before assumed) on an average half the second year, making together an average of $1\frac{1}{2}$, or $\frac{3}{2}$ years each: in like manner it may be shown, that those who die off in the third year, must on an average live $2\frac{1}{2}$, or $\frac{5}{2}$ years; those dying in the fourth, $3\frac{1}{2}$, or $\frac{7}{2}$ years,

and so on. Hence, considering the duration of each life a separate portion of time, as though they lived in succession, and not coexistently, it follows that the

43	who	die	in	the	1st	year,	must	among	them	live $43\times\frac{1}{2}=21.5$ years
38	-	-	2nd	—	-	-	-	-		$38\times1\frac{1}{2}=57.0$ —
32	-	-	3rd	—	-	-	-	-		$32\times2\frac{1}{2}=80.0$ —
26	-	-	4th	—	-	-	-	-		$26\times3\frac{1}{2}=91.0$ —
20	-	-	5th	—	-	-	-	-		$20\times4\frac{1}{2}=90.0$ —
16	-	-	6th	—	-	-	-	-		$16\times5\frac{1}{2}=88.0$ —
13	-	-	7th	—	-	-	-	-		$13\times6\frac{1}{2}=84.5$ —
11	-	-	8th	—	-	-	-	-		$11\times7\frac{1}{2}=82.5$ —
9	-	-	9th	—	-	-	-	-		$9\times8\frac{1}{2}=76.5$ —
7	-	-	10th	—	-	-	-	-		$7\times9\frac{1}{2}=66.5$ —
5	-	-	11th	—	-	-	-	-		$5\times10\frac{1}{2}=52.5$ —
3	-	-	12th	—	-	-	-	-		$3\times11\frac{1}{2}=34.5$ —
1	-	-	13th	—	-	-	-	-		$1\times12\frac{1}{2}=12.5$ —
The 224 persons existing at 85 must therefore live in all -										837. years,

and this total divided by the number of persons gives $\frac{837}{224}=3.73$, or nearly $3\frac{3}{4}$ years, for the average duration of each life as required.

Otherwise, by reference to the same Table as before, it will be found that out of 224 persons living at 85,

181	may	be	expected	to	survive	1 year,
143	-	-	-	-	-	2 years,
111	-	-	-	-	-	3 —
&c.						&c.

It therefore follows, that in the view here taken, the

			Half Years.
181 persons who survive 1 year must together live	181 years —	to which adding for those that die	43
143 - - - -	2 years - 143 years more	- - -	38
111 - - - -	3 — - 111 — —	- - -	32
85 - - - -	4 — - 85 — —	- - -	26
65 - - - -	5 — - 65 — —	- - -	20
49 - - - -	6 — - 49 — —	- - -	16
36 - - - -	7 — - 36 — —	- - -	13
25 - - - -	8 — - 25 — —	- - -	11
16 - - - -	9 — - 16 — —	- - -	9
9 - - - -	10 — - 9 — —	- - -	7
4 - - - -	11 — - 4 — —	- - -	5
1 - - - -	12 — - 1 — —	- - -	3
0 - - - -	13 — - 0 — —	- - -	1
	Totals - -	725 years, and - - -	224
But 224 half years - -		=112 years, which being added to the	
first total, gives - - -		837, with which we proceed as before.	

In like manner the *Expectation* of Life at any other age may be determined by either of the two methods here suggested. From the latter of these two methods it is manifest that the Expectation of Life may be deduced by the following

RULE.—*To the sum of the numbers living (in* Table x.) *opposite all ages above the age proposed, add half the number living at that age, and the result divided by the number living opposite the given age will be the Expectation of Life. Or, divide the sum of the numbers living at all higher ages by that at the given age, and the quotient increased by* $\frac{1}{2}$ *will be the Expectation of Life.*

But if it be required to form a Table showing the Expectations of Life at all ages, the labour may be materially reduced by beginning at the oldest age, and taking the successive sums of the numbers living, as in Table XVI.

Thus, the numbers in column **D** of that Table being the same as those living at the different ages in Table X., it is manifest that the

sum of the living above 97 = 0 as in col. **N** at 97

the sum of the living above

96 = 0 + 1 = 1 - - - - - - 96
95 = 1 + 4 = 5 - - - - - - 95
94 = 5 + 9 = 14 - - - - - - 94
93 = 14 + 16 = 30 - - - - - - 93
92 = 30 + 25 = 55 - - - - - - 92

and so on to the youngest age.

This preparatory Table being formed, we have only to divide the number given in column **N** opposite any given age, by that in column **D**, and the result, increased by $\frac{1}{2}$, will be the Expectation of life at that age.

Thus at the age of 50; $\frac{39380}{1937} = 20.33$; and this quotient increased by $\frac{1}{2}$, $= 20.33 + .5 = 20.83$ for the Expectation of Life at that age. In this manner column third of Table XVII. has been deduced. Taking the quotients thus obtained, without adding the $\frac{1}{2}$, and registering the results, we derive the numbers given in column second of that Table, which, after the example of Mr. Milne, we have termed the *Curtate* Expectation of Life.

Example x.

Suppose a number of marriages contracted between males of 85 and females of 65; it is required to determine the average duration of each marriage, admitting both the males and females to be subject to the same law of mortality as the members of the Equitable?

Reasoning as in the last example, it will be found by Table xiv, difference of age 20 years, that out of 289 pairs existing at the proposed ages,

224 will remain undissolved at the end of 1 year,
170 - - - - - - 2 years,
126 - - - - - - 3 —

and so on. Hence, conceiving the duration of the different marriages as so many successive portions of time, it follows, that those which remain undissolved at the end of the

1st year, must among them exist 224 years,

2nd year	- - - -	170 years more,
3rd —	- - - -	126 — —
4th —	- - - -	92 — —
5th —	- - - -	67 — —
6th —	- - - -	48 — —
7th —	- - - -	33 — —
8th —	- - - -	22 — —
9th —	- - - -	13 — —
10th —	- - - -	7 — —
11th —	- - - -	3 — —
12th —	- - - -	1 — —
Total	- - - - -	806 years;

hence, this total divided by the 289 marriages, and the result increased by $\frac{1}{2}$, as directed in the last example, produces $2.79 + .5 = 3.29$, or rather better than $3\frac{1}{4}$ years for the average duration of each marriage, as required to be found.

From this mode of adding the numbers of combinations given by Table XIV. at all ages higher than those proposed, we deduce an easy method of forming a Preparatory Table for determining the Expectation of Two Joint Lives. Table XVIII. is a specimen of this kind, the numbers in column $\mathbb{D}$. being found from the Single Life Table as directed in page 110, and column $\mathbb{N}$. nothing more than the successive sums of the numbers in column $\mathbb{D}$. By this Table, looking for younger age 65, and difference between the two ages 20 years, then dividing the number in column $\mathbb{N}$. at the angle of intersection by the adjoining number in column $\mathbb{D}$. we have $\frac{804.0}{288.5} = 2.79$ for the *Curtate Expectation*, to which adding $\frac{1}{2}$, we deduce the same result as before.† In this manner Table XIX. showing the *Curtate Expectation* of Two Joint Lives has been constructed.

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† The result obtained by adding $\frac{1}{2}$ to the *Curtate Expectation* of joint lives is however somewhat greater than their correct Expectation, though sufficiently near in most practical purposes to which such calculations may be applied. To show that $\frac{1}{2}$ is too much to be added on account of the fraction which two persons may live together, during the year wherein their joint existence fails, let us adhere to the supposition of marriages; then it is manifest that of the number of marriages be-

Example xi.

Let it be required by the assistance of Table xix. to find the Expectation of Two Joint Lives whose ages are respectively 20 and 30 years?

Opposite younger age 20, and under difference of age 10 years, the Curtate Expectation is found to be 26.48 years; to which adding $\frac{1}{2}$ or .5, as before directed, the result is 26.98, or very nearly 27 years, the Expectation required.

If the proposed ages are not to be found in the Table, the Expectation may be approximated by taking out the Expectations given opposite the younger age, under the differences nearest to that obtaining between the proposed ages as follows:

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coming extinct during the year, some are dissolved by the death of the husbands, some by the death of the wives, and others altogether become extinct by the death of both parties in the same year. Each of those that are dissolved by the death of *one* party may evidently be considered as remaining in force one half of the year in which it fails; but each of those whereof the two parties die off during the same year, the first death must be considered on an average as happening at the end of *one third* of the year, and the second death at the end of *two thirds* of the same period; and since the marriage becomes dissolved by the *first* death, it can only continue in force $\frac{1}{3}$ of the year in which it fails. The exact fraction to be added must therefore be $\frac{1}{2}$ for those marriages which become extinct by the death of either party, and $\frac{1}{3}$ for those in which both parties die in the same year; to add $\frac{1}{2}$ for each is therefore too much by $\frac{1}{2}-\frac{1}{3}$ or $\frac{1}{6}$ of those in which both parties die in the same year. For the mode of determining the correct Expectation of joint lives, the reader is referred to the next section.

Thus, suppose the ages to be 37 and 49, the difference between which is 12 years.

Opposite the younger age 37, and under difference 10 years	the Expectation of 37 and 47	=17.42
Also, opposite younger age 37 and under difference 15 years	that of - - - 37 and 52	=15.54
The 5 years difference in the older age is therefore found to make a difference in the Expectation of	- - -	1.88
The fifth of which may be considered as the difference for each year of difference in the age	- - - -	.37

and since 49 is two years worse than 47, double of .37 taken from 17.42, the Expectation of 37 and 47 leaves 16.68, which may be taken as the approximated *Curtate* Expectation of 37 and 49; and adding $\frac{1}{2}$ to this result, the required Expectation becomes 17.18, or rather better than 17 years.

The same mode of approximation may be applied to find the Expectation of Two Joint Lives of any other ages that cannot be immediately found in the Table.

Example xii.

Suppose a number of families proposed, each consisting of three persons, whose ages are respectively 70, 80, and 90 years; it is required to determine the number of years which each family, on an average, may be expected jointly to survive before the first death shall happen, admitting all the individuals to be subject to the same law of mortality as the members of the Equitable?

Proceeding as directed in Example VII. page 114, it may be shown that out of

$1028 \times .480 \times 65$ families of the proposed ages,

$974 \times .426 \times 49 =$	20331	will survive 1 year.
$919 \times .373 \times 36 =$	12340	- - - - 2 years.
$864 \times .321 \times 25 =$	6934	- - - - 3 - -
$808 \times .271 \times 16 =$	3503	- - - - 4 - -
$752 \times .224 \times 9 =$	1516	- - - - 5 - -
$697 \times .181 \times 4 =$	505	- - - - 6 - -
$642 \times .143 \times 1 =$	92	- - - - 7 - -
	45221	Total.

Hence, reasoning as in page 119, and dividing this total by $1028 \times .480 \times 65 = 32074$, the number of families at the proposed ages, the quotient $\frac{45221}{32074} = 1.41$ shows the *Curtate Expectation :* to which adding $\frac{1}{2}$ we obtain $1.41 + .5 = 1.91$, or $1\ \frac{91}{100}$ years for an approximation to the required Expectation.

The Expectation of Three Joint Lives may also be approximated by the assistance of a Table exhibiting the Expectation of Two Joint Lives, by the following

RULE.—*Find the Curtate Expectation of the joint existence of the two oldest lives, as directed in p.* 124, *and by Table* XVII. *find the age of a Single Life having its Curtate Expectation equal to the result: then find the Curtate Expectation of the joint existence of this life combined with the youngest of the proposed ages, and the result may be considered as the Curtate Expectation of the lives proposed.*

This again increased by $\frac{1}{2}$ will be an approximation to the Expectation required.

Thus, in the Example proposed, the Curtate Expectation of 80 and 90, found by Table XIX, is 1.58, answering to the Curtate Expectation of a Single Life of $91\frac{6}{7}$, then combining this age with the 70, we have the Curtate Expectation of

70 and 90 by Table XIX.	1.88
70 and 95 by ditto	.52
Difference for 5 years	1.36
Ditto for 1 year	.27
For 6-7ths of a year	.23
For $1\frac{6}{7}$ of a year	.50

This taken from the Expectation of 70 and 90, leaves 1.38, which being increased by $\frac{1}{2}$ produces 1.88 for the approximated Expectation of the lives proposed; which does not materially differ from that deduced by a more laborious process in p. 125.

Example XIII.

Required the Expectation of the last Survivor of two Lives, aged 30 and 40, according to the experience of the Equitable?

Here it is manifest, that the joint existence of two lives terminates with the first death, and the last survivor with the second. The Expectation of the joint lives and that of the last survivor are, therefore, together equal to the sum of the Expectations of the two single lives; and, as a consequence,

The Expectation of the last survivor of two lives, is equal to the sum of the Expectations of the two single lives, diminished by that of their joint existence.

Hence, the Curtate Expectation of - -	30 = 33.48
that of - - - - - -	40 = 26.90
their sum - - - - -	= 60.38
the Expectation of 30 and 40 jointly -	= 21.07
Curtate Expectation of last Survivor -	= 39.31
And this, increased by $\frac{1}{2}$, gives -	39.81

or $39\frac{4}{5}$ years for the Expectation required.

Example xiv.

Suppose a number of marriages to be contracted between males of 30, and females of 20; it is required to determine the number of years which each of the females, on an average, may expect to outlive her husband, admitting the males and females to be subject to the same law of mortality as the members of the Equitable?

By Table xvii. it will be found that the females of 20 may, on an average, expect to live 41.06, or rather better than 41 years; and in Example xi. it has been shown, that two persons of 20 and 30 may expect to live together about 27 years. Hence it follows that if out of the 41 years which each of the females may expect to live, either married or single,

we deduct the 27 years she may expect to continue in the married state, the remainder 14 years shows the average duration of widowhood as required.

By the same kind of reasoning as that employed in the foregoing examples, though perhaps too complex to be followed without the aid of symbols, it may be shown that

The approximated Expectation of the last survivor of three lives A, B, and C, is found by adding together the Curtate Expectations of A, B, and C, taken singly, and that of A, B, and C, taken jointly; then taking from the result the sum of the Curtate Expectations of A & B, A & C. and B & C, taken two and two, and increasing the remainder by $\frac{1}{2}$.

Thus, supposing it were required to find the Expectation of the last survivor of three lives of 20, 30, and 40, according to the rate of mortality among the members of the Equitable?

Curt. Expn. of 20 = 40.56		Ditto of 20 & 30 = 26.48	
that of - - 30 = 33.48		- - 20 & 40 = 22.59	
- - 40 = 26.90		- - 30 & 40 = 21.07	
Do. of 20, 30, & 40 or of 20 & 49 } = 18.39		second sum	70.14
First sum -	= 119.33		
Second - -	= 70.14		
Curt. Expn.	= 49.19		
the $\frac{1}{2}$ to be added =	.5		
	= 49.69 = $49\frac{2}{3}$ years. Answer.		

4. *General Problems.*

For the purpose of generalizing what has been advanced relative to the application of the Rate of Mortality to the determination of the various contingencies of Life, Death, or Survivorship, among several persons designated by A, B, C, &c.*; in any table of mortality

Let the number living opposite the age of - $A=a$
that opposite the age of - - - $B=b$
- - - - - - - - $C=c$
that opposite an age t years older than $A='a$
- - - - - - - - $B='b$
- - - - - - - - $C='c$
that opposite an age t years younger than $A=a_t$
- - - - - - - - $B=b_t$
- - - - - - - - $C=c_t$

Put $'a.'b.'c\&c.={}'(abc\&c.)$, $a_t.b_t.c_t$ &c. $={}_t(abc\&c.)$,

$\frac{'a}{a}={}'a$, $\frac{'b}{b}={}'b$, &c. and $\frac{'a.'b.'c\&c.}{abc\&c.}={}'(abc\&c)$.

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* All calculations of this kind are grounded on the supposition that, out of the same number of persons of any age as are expressed to be living opposite that age in the Table of Mortality employed, a corresponding number of survivors will attain any higher age, and that out of any other number of persons, greater or less than that given in the Table, the number of survivors, at the end of any given interval, will be greater or less in proportion.

Then it is manifest, from the nature of a Table of Mortality, that out of

Persons living	of the Age of	may be expected to	
		survive t years.	die in the interval
a	A	${}^{t}a$	$a-{}^{t}a$
b	B	${}^{t}b$	$b-{}^{t}b$
c	C	${}^{t}c$	$c-{}^{t}c$
&c.	&c.	&c.	&c.

Problem I.

A certain number (n) of families or combinations of lives being proposed, each consisting of a like number of individuals of given ages, and subject to a certain law of mortality; it is required to determine how many of them may be expected to have a given number of survivors left at the end of t years?

Case 1. Suppose each family to consist of only one person of the age of A; then we have

$$a : {}^{t}a :: n : \frac{{}^{t}an}{a} = {}_{t}an = \begin{cases} \text{No. expected to survive} \\ \text{the given interval.} \end{cases}$$

$$a : a-{}^{t}a :: n : \frac{(a-{}^{t}a)n}{a} = (1-{}_{t}a)n \begin{cases} \text{Ditto to die in} \\ \text{ditto.} \end{cases}$$

Case 2.—Suppose each family, or combination, to consist of two persons, designated by A & B;

Then, by the last case, it follows that at the end of the proposed interval we may expect the number of

A's surviving $= \frac{{}'an}{a}$, number of A's dead $= \frac{(a-{}'a)n}{a}$

Hence, conceiving the proposed number of combinations to be separated, in the first instance, into two classes, the one being composed of those combinations in which the A's may be expected to survive the given interval, and the other of those in which the A's may be expected to fail within that interval; by the last case the number in the former class will be $\frac{{}'an}{a}$, that in the latter $\frac{(a-{}'a)n}{a}$.

And since each of these combinations in the first instance contains a B as well as A, to subdivide each of these classes according as the B's may be alive or dead at the end of the proposed term, we have, in the

First Class,

$b : {}'b :: \frac{{}'an}{a} : \frac{{}'a.{}'bn}{ab} =$ No. having the B's surviving,

$b : b-{}'b :: \frac{{}'an}{a} : \frac{{}'a(b-{}'b)n}{ab} =$ No. having the B's dead;

Second Class,

$b : {}'b :: \frac{(a-{}'a)n}{a} : \frac{{}'b(a-{}'a)n}{ab} =$ { No. having the B's surviving.

$b : b-{}'b :: \frac{(a-{}'a)n}{a} : \frac{(a-{}'a)(b-{}'b)n}{ab} =$ { No. having the B's dead

Case 3.—Suppose each of the proposed combinations to consist of three persons designated respectively by A, B and C;

Then it is manifest from the last case that, without reference to the third life C, the proposed combinations may be divided into the four following classes, viz.

1st. the combins. in which the A's & B's both live - - $= \frac{{}'a.{}'bn}{ab}$

2nd. ditto - - A's live and B's die off $= \frac{{}'a(b-{}'b)n}{ab}$

3rd. ditto - - A's die and B's live $= \frac{{}'b(a-{}'a)n}{ab}$

4th. ditto - $\left\{\begin{matrix} A\text{'s \& } B\text{'s both} \\ \text{die off - -} \end{matrix}\right\} = \frac{(a-{}'a)(b-{}'b)n}{ab}$

And, by the introduction of a third life, C, each of these classes becomes subdivided into two others, according as the C's may survive the proposed term, or die off in the interval.

Thus, with regard to the first of the above four classes, we have

$c : {}'c :: \frac{{}'a.{}'bn}{ab} : \frac{{}'a.{}'b.{}'cn}{abc}$ for the number of the proposed combinations which may be expected to remain entire to the end of the given period; and

$c : c-{}'c :: \frac{{}'a.{}'bn}{ab} : \frac{{}'a.{}'b(c-{}'c)n}{abc}$ for the number in which the A's and B's may be expected to survive t years, and the C's to die off in the interval. Proceeding in like manner with the remaining classes,

and collecting the results into a tabular form, it will be found that, out of n combinations of Three Joint Lives, each consisting of one person of the age of A, another of the age of B, and a third of the age of C, at the end of t years we may expect them distributed into

COMBINATIONS	in which there will be	
	living	dead
$\frac{{}^{t}a.{}^{t}b.{}^{t}cn}{abc}$	A, B & C	0
$\frac{{}^{t}a.{}^{t}b(c-{}^{t}c)n}{abc}$	A & B	C
$\frac{{}^{t}a.{}^{t}c(b-{}^{t}b)n}{abc}$	A & C	B
$\frac{{}^{t}b.{}^{t}c(a-{}^{t}a)n}{abc}$	B & C	A
$\frac{{}^{t}a(b-{}^{t}b)(c-{}^{t}c)n}{abc}$	A	B & C
$\frac{{}^{t}b(a-{}^{t}a)(c-{}^{t}c)n}{abc}$	B	A & C
$\frac{{}^{t}c(a-{}^{t}a)(b-{}^{t}b)n}{abc}$	C	A & B
$\frac{(a-{}^{t}a)(b-{}^{t}b)(c-{}^{t}c)n}{abc}$	0	A, B & C

Similarly, by subdividing each of these classes in the ratio of ${}^{t}d$ to $d-{}^{t}d$, we may proportion n com-

binations of Four Joint Lives into the sixteen classes to which they may be expected to be distributed at the end of t years: and by subdividing each of these results again in the ratio of $'e$ to $e-'e$, we may further deduce the 32 classes into which n combinations of Five Joint Lives may be apportioned at the end of the proposed interval. The same mode of proceeding is evidently applicable to any other number of lives; the number of classes continually increasing in the same geometrical progression: but from the formation of the different classes, the law of which is evident from the foregoing cases, any one class may be determined without reference to the others.

Thus, out of n combinations of Five Joint Lives A, B, C, D & E; the number in which the A's, B's & C's may be expected to survive t years, and the remaining two to fail in the interval, is $\frac{'a.'b.'c(d-'d)(e-'e)n}{abcde}$; and out of n combinations of Four Joint Lives A, B, C & D, the number of combinations which may be expected not to have a single survivor left at the end of t years is $\frac{(a-'a)(b-'b)(c-'c)(d-'d)n}{abcd}$.

The expressions here deduced may, however, be materially simplified by means of the notation adopted in page 129, where $\frac{'a}{a}$ has been put $=,a$, $\frac{'a.'b}{ab}=,(ab)$, &c.

By this means it will be found that, out of n combinations of Two Joint Lives A & B,

$\frac{{}_{t}a \cdot {}_{t}bn}{ab}$ becomes $= {}_{t}(ab)n$ - { for the number of surviving pairs.

$\frac{{}_{t}a(b - {}_{t}b)n}{ab} = \{{}_{t}a - {}_{t}(ab)\}n$ { ditto having the A's only surviving.

$\frac{{}_{t}b(a - {}_{t}a)n}{ab} = \{{}_{t}b - {}_{t}(ab)\}n$ { ditto having the B's only surviving.

$\frac{(a - {}_{t}a)(b - {}_{t}b)n}{ab} = \{1 - {}_{t}a - {}_{t}b + {}_{t}(ab)\}n$ for the number of combinations in which both the A's and B's may be expected to die off within the next t years.

And by adding together the second and third of these classes we obtain

$$\{{}_{t}a + {}_{t}b - 2\,{}_{t}(ab)\}n$$

for the number of combinations which may be expected to have but one survivor in each at the end of t years; and adding the first to these we deduce

$$\{{}_{t}a + {}_{t}b - {}_{t}(ab)\}n$$

for the number of combinations in which some one or both of the persons proposed may be expected to survive the given interval.

Proceeding in like manner with the various expressions given in the Table page 133, it will be found that, out of n combinations of Three Joint Lives, the different classes in which three, two, or one of the proposed lives may be expected to survive t years, will be represented as follows, viz.

COMBINATIONS	in which there will be	
	living	dead
${}_{t}(abc)n$	A, B & C	none
${}_{t}(ab)n - {}_{t}(abc)n$	A & B	C
${}_{t}(ac)n - {}_{t}(abc)n$	A & C	B
${}_{t}(bc)n - {}_{t}(abc)n$	B & C	A
${}_{t}an - {}_{t}(ab)n - {}_{t}(ac)n + {}_{t}(abc)n$	A	B & C
${}_{t}bn - {}_{t}(ab)n - {}_{t}(bc)n + {}_{t}(abc)n$	B	A & C
${}_{t}cn - {}_{t}(ac)n - {}_{t}(bc)n + {}_{t}(abc)n$	C	B & C

The first of these expressions ${}_{t}(abc)n$ represents the number of the proposed combinations in each of which all the three lives may be expected to survive the given interval.

The sum of the second, third and fourth

$$= \{{}_{t}(ab) + {}_{t}(ac) + {}_{t}(bc) - 3{}_{t}(abc)\}n$$

denotes the number in each of which some two of the proposed lives may be expected to survive the given period.

The sum of the fifth, sixth, and seventh

$$= \{{}_{t}a + {}_{t}b + {}_{t}c - 2{}_{t}(ab) - 2{}_{t}(ac) - 2{}_{t}(bc) + 3{}_{t}(abc)\}n$$

represents the number of the proposed combinations

in each of which some one of the three lives may be expected to survive t years.

The sum of the first, second, third, and fourth

$$= \{{}_t(ab) + {}_t(ac) + {}_t(bc) - 2{}_t(abc)\}n$$

denotes the number of the proposed combinations, in each of which some two, or all three, of the given lives may be expected to survive the given interval.

And the sum of all the seven, which is

$$\{{}_ta + {}_tb + {}_tc - {}_t(ab) - {}_t(ac) - {}_t(bc) + {}_t(abc)\}n$$

represents the number of the proposed combinations, in each of which some one, some two, or all three, of the given lives may be expected to survive t years.

The same mode of proceeding is equally applicable to any other number of lives.

Scholium.

In the foregoing investigation we have uniformly supposed it required to determine the number of the proposed combinations, in each of which, the A may be expected to be living or dead at the end of any given interval, and the B or C to be also living or dead at the end of the *same* interval. But cases not unfrequently occur where it may be necessary to determine, out of n families, or combinations of lives, how many may be expected to have the A in each living or dead at the end of one interval, and the B &c. living or dead at the end of another.

Thus suppose it were required to determine, how

many of the n proposed combinations may be expected to have in each, the A living at the end of r years, the B at the end of s years, and the C at the end of t years. Substituting r for t in the foregoing results, it is manifest that the number of the proposed combinations, in each of which the A, B & C may be expected to be living at the end of r years is $\frac{(^ra.^rb.^rc)n.}{abc}$. Now, supposing s greater than r it is also evident that out of ^rb persons, living at an age r years older than B, ^sb may be expected to survive to attain an age s years older than B, and the remaining $^rb - {}^sb$ may be expected to drop off in the interval between the end of the rth and the end of the sth year. Hence proportionally we have

$$^rb : {}^sb : : \frac{(^ra.^rb.^rc.)n}{abc} \quad : \frac{(^ra.^sb.^rc.)n}{abc}$$

for the number of the proposed combinations, in each of which the A & C may be expected to survive r years, and the B to survive s years. And similarly

$$^rc : {}^tc : : \frac{(^ra.^rb.^rc)n}{abc} \quad : \frac{(^ra.^rb.^tc)n}{abc}$$

for the number of the proposed combinations, in each of which the A may be expected to survive r years, the B to survive s years, and the C to survive t years.

$$\text{Also } ^rb : {}^rb - {}^sb : : \frac{(^ra.^rb.^rc)n}{abc} \quad : \frac{\{^ra(^rb - {}^sb).^rc\}n}{abc}$$

for the number of the proposed combinations in each of which the A & C may be expected to survive r years, and the B to die off during the interval

between the end of r years and the end of s years. In like manner it may be shown that

$$\frac{(a-{}^{r}a)({}^{r}b-{}^{s}b)({}^{s}c-{}^{t}c)n}{abc}$$

represents the number of the proposed combinations, in each of which the A may be expected to die in the next r years, the B in the following $s-r$ years, and the C in the succeeding $t-s$ years.

It therefore appears that the principle deduced in the foregoing pages is equally applicable, whether the existence or nonexistence of the different individuals, of which each combination is composed, be referred to the same, or to different intervals of time.

Problem II.

A certain number (n) of families, or combinations of lives being proposed, each consisting of a like number of individuals, of the respective ages of A, B, C, &c. and subject to a certain law of mortality; it is required to determine the average duration of each entire combination;—or, in other words, the Expectation of the Joint Lives of the individuals whereof it is composed?

Case 1.—To find the Curtate Expectation, or the average duration of each combination, supposing those that fail within any year to exist no part of that year, but to fail simultaneously at its commencement.

In our enquiry concerning the Expectation of Life, we must consider the duration of each combination a separate portion of time, as though they existed in succession; so that two combinations, both existing together for one year, must be regarded as collectively enjoying two years existence; three combinations, all surviving one year, must be considered among them as existing three years, and so on. Upon this principle it is manifest, that the collective existences of the proposed combinations of lives is nothing more than the aggregate of the several quantities of existence which may, in this sense, be enjoyed by them in the first, second, third, &c. year till they all become extinct. But, upon the supposition made in this case, the quantity of existence to be enjoyed by the proposed combinations, within the first year, is identical with the number of combinations which may be expected to survive that year, and is, therefore, by the last Problem $= {}_1(abc\&c.)n$; and the quantity of existence which they may expect to enjoy during the second year is, in like manner, denoted by the number of combinations, ${}_2(abc\&c.)n$, which may be expected to survive that year, and so on to the extremity of life; the sum of which

$= {}_1(abc\&c)n + {}_2(abc\&c)n + {}_3(abc\&c)n + {}_4(abc\&c)n\&c.$

represents, on the supposition made, the aggregate existence of the combinations proposed; hence, dividing this aggregate by n, the number of combinations given, we have

$_1(abc\&c.) + {}_2(abc\&c.) + {}_3(abc\&c.) + {}_4(abc\&c.) + \&c.$ for the Curtate Expectation of each.

From this result it follows, that the Curtate Expectation of

A Single Life, A,

$= {}_1a + {}_2a + {}_3a + {}_4a + {}_5a + \&c.$ which put $= a$

Two Joint Lives, A, B,

$= {}_1(ab) + {}_2(ab) + {}_3(ab) + {}_4(ab) + \&c. \quad = ab$

Three Joint Lives, A, B, C,

$= {}_1(abc) + {}_2(abc) + {}_3(abc) + {}_4(abc)\&c. \quad = abc$

But by the notation employed in this section

$${}_1a = \frac{{}^1a}{a},\ {}_2a = \frac{{}^2a}{a},\ \&c.\ {}_1(ab) = \frac{{}^1(ab)}{ab},\ {}_2(ab) = \frac{{}^2(ab)}{ab},\ \&c.$$

hence, by substituting these values in the above expressions, we obtain

$$a = \frac{1}{a}({}^1a + {}^2a + {}^3a + {}^4a + \&c.) = \frac{N}{D} \text{ by Table XVI.}$$

$$ab = \frac{1}{ab}\left\{{}^1(ab) + {}^2(ab) + {}^3(ab)\&c.\right\} = \frac{N}{D} \text{ by Table XVIII.}$$

$$abc = \frac{1}{abc}\{{}^1(abc) + {}^2(abc) + {}^3(abc) + {}^4(abc) + \&c.\}$$

Case 2.—To find the average duration of each family, or combination, supposing the individuals, of which the several combinations failing during any year are composed, to die off in equal numbers, during equal portions of time, from the beginning of that year to the end thereof.

Upon this supposition, if the year be divided

into m equal intervals, and x be taken to denote any variable number of such intervals, when each family consists of only one person, A, and n is taken $=a$, it is evident that, out of the $a-{}^1a$ persons which may be expected to die within one year, $\frac{x}{m}(a-{}^1a)$ is the proportion which may be expected to drop off during the next $\frac{x}{m}$th part of a year, and this deducted from the a persons living at the proposed age gives

$$a-\frac{x}{m}(a-{}^1a)=\frac{1}{m}\left\{(m-x)a+x.{}^1a\right\}$$

for the number which may be expected to survive the next $\frac{x}{m}$ th part of a year.

And if, in this expression, we make m indefinitely great, so as to represent the number of *instants* in the year, and take x successively $= 1, 2, 3, \text{- - -} m$, of such instants, by reasoning as in the last case, it follows that those which survive

1 inst. must exist $\frac{1}{m}\left\{(m-1)a+{}^1a\right\}$ instants.

2 insts. - - - $\frac{1}{m}\left\{(m-2)a+2.{}^1a\right\}$ ditto more.

3 - - - - - - $\frac{1}{m}\left\{(m-3)a+3.{}^1a\right\}$ — —

4 - - - - - - $\frac{1}{m}\left\{(m-4)a+4.{}^1a\right\}$ — —

m - - - - - $\frac{1}{m}\left\{(m-m)a+m.{}^1a\right\}$ — —

And these being added by Arithmetic of Infinites *

$$=\frac{1}{m}\left\{(m^2-\frac{m^2}{2})a+\frac{m^2}{2}.{}^1a\right\}=\frac{m}{2}.a+\frac{m}{2}{}^1a=m(\frac{a+{}^1a}{2})$$

for the number of *instants*, $=\frac{a+{}^1a}{2}$ *years*, existence which a persons now living may, among them, expect to enjoy during the next year.

Similarly it may be shown that $\frac{{}^1a+{}^2a}{2}$ years represents the quantity of existence which may be enjoyed, by the a persons now proposed, during the second year, $\frac{{}^2a+{}^3a}{2}$, during the third year, and so on.

The aggregate quantity of existence which a persons now living may henceforth expect to enjoy is therefore

$$=\tfrac{1}{2}(a+{}^1a)+\tfrac{1}{2}({}^1a+{}^2a)+\tfrac{1}{2}({}^2a+{}^3a)+\tfrac{1}{2}({}^3a+{}^4a)+\&c.$$

$=\frac{1}{2}a+{}^1a+{}^2a+{}^3a+{}^4a+{}^5a+\&c$; and this divided by a, the number of persons proposed, gives

$$\tfrac{1}{2}+\tfrac{1}{a}({}^1a+{}^2a+{}^3a+{}^4a+\&c.)=\tfrac{1}{2}+a$$

for the required Expectation, or average duration of each life.

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* By the principles of Arithmetic of Infinites it may be shown that when m is indefinitely great with respect to unity,

$1+2+3 \text{ - - - } m=\frac{m^2}{2}$, the same as f^t. of $\dot{m}\dot{m}$

$1^2+2^2+3^2 \text{ - - - } m^2=\frac{m^3}{3}$ - - f^t. of $m^2\dot{m}$

$1^3+2^3+3^3 \text{ - - - } m^3=\frac{m^4}{4}$ - - f^t. of $m^3\dot{m}$

Again, supposing each combination to consist of two persons, of the respective ages of A & B, it follows that of a persons now living $\frac{1}{m}\{(m-x)a+x.^1a)\}$ may be expected to survive the $\frac{x}{m}$th part of a year, and of b persons now living $\frac{1}{m}\{(m-x)b+x.^1b\}$ may be expected to survive the like interval. Hence, reasoning as in the last problem, page 131, it may be shown that, out of n combinations, each consisting of one person of the age of A, and another of the age of B, the number of surviving pairs at the end of the next $\frac{x}{m}$th part of a year, is denoted by

$$\frac{1}{m}\left\{(m-x)a+x.^1a\right\}\times\frac{1}{m}\left\{(m-x)b+x.^1b\right\}\times\frac{n}{ab}$$

which, by making $n=ab$, and actually multiplying, becomes

$$\tfrac{1}{m^2}\{(m-x)^2.ab+(mx-x^2)a.^1b+(mx-x^2).^1ab+x^2.^1a.^1b\}$$

Hence, making x successively $=1, 2, 3, \text{- - -} m$, and adding the results as in page 142, the sum found agreeably to the last note is

$$\frac{1}{m^2}\left\{\frac{m^3}{3}.ab+\left(\frac{m^3}{2}-\frac{m^3}{3}\right)a.^1b+\left(\frac{m^3}{2}-\frac{m^3}{3}\right).^1ab+\frac{m^3}{3}.^1a.^1b\right\}$$

$$=\frac{m}{3}ab+\frac{m}{6}.a.^1b+\frac{m}{6}.^1ab+\frac{m}{3}.^1a.^1b \textit{ instants,}$$

$$=\tfrac{1}{3}ab+\tfrac{1}{6}a.^1b+\tfrac{1}{6}.^1ab+\tfrac{1}{3}.^1a.^1b \textit{ years.}$$

For the quantity of existence which the n $(=ab)$ combinations proposed may, among them, expect to enjoy during the next year.†

† Were we to suppose the combinations (instead of the individuals of which they are composed) to fail in equal portions during equal

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In like manner it may be shown that the quantity of existence which the proposed combinations may expect to enjoy during the second year is, $\frac{1}{3}.{}^{1}a.{}^{1}b + \frac{1}{6}.{}^{1}a.{}^{2}b + \frac{1}{6}.{}^{2}a.{}^{1}b + \frac{1}{3}.{}^{2}a.{}^{2}b$, in the third year $\frac{1}{3}.{}^{2}a.{}^{2}b + \frac{1}{6}.{}^{2}a.{}^{3}b + \frac{1}{6}.{}^{3}a.{}^{2}b + \frac{1}{3}.{}^{3}a.{}^{3}b$, and so on, to the greatest tabular duration of the oldest of the proposed lives.

The required Expectation is therefore equal to the sum of the four following series, divided by ab, the number of combinations proposed.

1st series $= \frac{1}{3}\{ab + {}^{1}(ab) + {}^{2}(ab) + {}^{3}(ab) + {}^{4}(ab)\&c.\}$

2nd — $= \frac{1}{6}\{a.{}^{1}b + {}^{1}a.{}^{2}b + {}^{2}a.{}^{3}b + {}^{3}a.{}^{4}b + {}^{4}a.{}^{5}b\&c.\}$

3rd — $= \frac{1}{6}\{{}^{1}ab + {}^{2}a.{}^{1}b + {}^{3}a.{}^{2}b + {}^{4}a.{}^{3}b + {}^{5}a.{}^{4}b\&c.\}$

4th — $= \frac{1}{3}\{{}^{1}(ab) + {}^{2}(ab) + {}^{3}(ab) + {}^{4}(ab) + {}^{5}(ab)\&c.\}$

But the sum of the first series is equal to $\frac{1}{3}$ of the number in column N. of Table XVIII. corresponding to two ages respectively 1 year younger than A & B, which number we shall denote by $_{,}\mathrm{N}_{,}$. That of the second series $= \frac{1}{6}$ of the number in the same Table, column N, corresponding to two ages, whereof one is a year younger than A, and the other of the same age as B, which we shall designate by $_{,}\mathrm{N}$. The sum of the third $= \frac{1}{6}$ of the number found in like manner opposite two ages,

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intervals of the same year, the quantity of existence, which would be enjoyed by ab combinations of joint lives during the next year, would evidently be ${}^{1}(ab) + \frac{1}{2}\{(ab) - {}^{1}(ab)\} = \frac{1}{2}\{(ab) + {}^{1}(ab)\}$ which will be found greater by $\frac{1}{6}(a - {}^{1}a)(b - {}^{1}b)$ than the quantity deduced above, agreeably to what was stated in the note pages 122–3.

whereof the former is the same age as A, and the latter a year younger than B, which we shall designate by $\mathrm{N}_{,}$. And the sum of the fourth series $=\frac{1}{3}$ of the number in the same Table opposite the proposed ages, which we put $=\mathrm{N}$. The ab being also found in column D. of the same Table opposite the given ages.

The Expectation of Two Joint Lives is therefore

$$=\frac{\frac{1}{3}\mathrm{N}_{,}+\frac{1}{6}\mathrm{N}+\frac{1}{6}\mathrm{N}_{,}+\frac{1}{3}\mathrm{N}}{\mathrm{D}}.*$$

Otherwise, since it has been shown in page 141,

that $\frac{1}{ab}\{{}^{1}(ab)+{}^{2}(ab)+{}^{3}(ab)+\&c.\}=\mathrm{ab}$,

we have, ${}^{1}(ab)+{}^{2}(ab)+{}^{3}(ab)+\&c.=ab.\mathrm{ab}$,

or, $ab+{}^{1}(ab)+{}^{2}(ab)+{}^{3}(ab)+\&c.=ab+ab.\mathrm{ab}$,

similarly $a.{}^{1}b+{}^{1}a.{}^{2}b+{}^{2}a.{}^{3}b+\&c.=a_{,}b.\mathrm{a_{,}b}$,

and ${}^{1}ab+{}^{2}a.{}^{1}b+{}^{3}a.{}^{2}b+\&c.=ab_{,}.\mathrm{ab_{,}}$.

The required Expectation may therefore be represented by

$$\{\tfrac{1}{3}(ab+ab.\mathrm{ab})+\tfrac{1}{6}.a_{,}b.\mathrm{a_{,}b}+\tfrac{1}{6}ab_{,}.\mathrm{ab_{,}}+\tfrac{1}{3}ab.\mathrm{ab}\}\div ab$$

$$=\tfrac{1}{3}+\tfrac{2}{3}\mathrm{ab}+\tfrac{1}{6}\left(\frac{1}{a_{,}}.\mathrm{a_{,}b}+\frac{1}{b_{,}}\mathrm{ab_{,}}\right)$$

where it may be observed that ab, $\mathrm{a_{,}b}$, and $\mathrm{ab_{,}}$ may be found in Table XIX, or approximated, when the ages are not given in that table, as in Example XI, page 123, and the quantities $\frac{1}{a_{,}}$ and $\frac{1}{b_{,}}$ are found

* This Formula cannot be conveniently applied unless the Tables were calculated for every difference of age, as it requires the numbers from column N. to be taken out from three successive differences.

already calculated in Table XII, column marked $\frac{1}{a'}$, opposite one year younger than the respective ages of A & B.

Moreover, suppose each of the proposed combinations to consist of three lives, one of the age of A, another of the age of B, and a third of the age of C.

Then reasoning as before it will be found that the number of combinations which may be expected to remain entire at the end of the $\frac{x}{m}$th part of a year is

$$\frac{1}{m}\Big\{(m-x)\,a+x.{}^1a\Big\}\times\frac{1}{m}\Big\{(m-x)b+x.{}^1b\Big\}$$
$$\times\frac{1}{m}\Big\{(m-x)\,c+x.{}^1c\Big\}\times\frac{n}{abc}$$

from which, by making $n=abc$, actually multiplying, taking x successively $=1,\ 2,\ 3\ ----\ m$, and adding the results as before, we deduce the aggregate existence which may be enjoyed by the proposed combinations during the next year $=$

$$\tfrac{1}{4}abc+\tfrac{1}{12}(ab.{}^1c+a.{}^1bc+a.{}^1b.{}^1c)$$
$$+\tfrac{1}{12}({}^1abc+{}^1ab.{}^1c+{}^1a.{}^1bc)+\tfrac{1}{4}.{}^1(abc)$$

Proceeding in like manner to determine the quantity of existence which may be enjoyed by the proposed combinations in the second, third, &c. year to the greatest tabular duration of the oldest life, and adding the results together, we may deduce eight different series the sum of which, divided by abc, the number of combinations proposed,

gives the average duration of each. These operations being performed, it will be found that

The Expectation of the joint existence of three lives, A, B & C is,

$$= \tfrac{1}{4} + \tfrac{1}{2}\,abc + \frac{1}{12,a,}\left(\frac{1}{,b,}a,b,c + \frac{1}{,c,}a,bc, + a,bc\right)$$

$$+ \frac{1}{12,b,}\left(\frac{1}{,c,}ab,c, + ab,c\right) + \frac{1}{12,c,}abc,$$

Finally, suppose each combination to consist of any number of lives, A, B, C, &c. and the complete Expectation of their joint existence to be denoted by (abc &c.)'

Then it is manifest, that out of a, b &c. persons now living at the respective ages of A, B &c. $'a,$, $'b,$ &c. may be expected to survive $t-1$ years, or to enter upon the tth year, and $'a$, $'b$ &c. to survive it; the differences, $'a, - 'a$, $'b, - 'b$ &c. must therefore represent the numbers which may be expected to die off during the tth year, and consequently $\frac{z}{m}('a, - 'a)$, $\frac{z}{m}('b, - 'b)$ &c. must be the proportional decrements for the $\frac{z}{m}$th part of that year; hence deducting these results from the $'a,$, $'b,$ &c. which enter upon that interval, there remains $'a, - \frac{z}{m}('a, - 'a)$ $'b, - \frac{z}{m}('b, - 'b)$ &c or $\frac{1}{m}\{(m-x).'a, + 'a\}$, $\frac{1}{m}\{(m-x).'b, + x.'b\}$ &c for the numbers which may be expected to survive to the end of the $\frac{z}{m}$th part of the tth year.

Then, reasoning as in Problem 1, it follows that of n combinations of Joint Lives A, B, C &c.

$$\left[\tfrac{1}{m}\{(m-x).'a, + x.'a\} \times \tfrac{1}{m}\{(m-x).'b, + x.'b\}.\text{&c.}\right]\frac{n}{abc\,\text{&c.}}.$$

represents the number remaining entire at the end of the interval stated. If therefore $f.$ be put for the sum of the results arising by making x, in this expression, successively $= 1, 2, 3 \text{ - - - } m$, and $\int.$ for the sum of these results again, by making $t = 1, 2, 3$ &c. we have the aggregate quantity of existence which the n proposed combinations may henceforth expect to enjoy in an entire state,

$$= \int. f. \left[\tfrac{1}{m}\{(m-x).'a_{,} + x.'a\} \times \tfrac{1}{m}\{(m-x).'b_{,} + x.'b\} \&c.\right]_{\frac{n}{ab\&c.}}$$

And this result divided by n, gives the required Expectation or average duration of each,

$$(ab_{\&c})' = \int f. \left[\tfrac{1}{m}\{(m-x).'a_{,} + x.'a\} \times \tfrac{1}{m}\{(m-x).'b_{,} + x.'b\}_{\&c}\right]. \tfrac{1}{ab\&c}$$

If in this expression the operations denoted by the smaller sign $f.$ be performed we have

$$a' = \int. \tfrac{1}{a}\{\tfrac{1}{2}.'a_{,} + \tfrac{1}{2}.'a\}$$

$$(ab)' = \int. \tfrac{1}{ab}\{\tfrac{1}{3}.'(ab)_{,} + \tfrac{1}{6}.'(a_{,}b) + \tfrac{1}{6}.'(ab_{,}) + \tfrac{1}{3}.'(ab)\}$$

$$(abc)' = \int. \tfrac{1}{abc}\left\{\begin{array}{l} \tfrac{1}{4}.'(abc)_{,} + \tfrac{1}{12}\{'(a_{,}b,c) + '(a,bc_{,}) + '(a,bc)\} \\ + \tfrac{1}{12}\{'(ab_{,}c_{,}) + '(ab_{,}c)\} + \tfrac{1}{12}.'(abc_{,}) + \tfrac{1}{4}.'(abc) \end{array}\right\}$$

The same mode of proceeding is equally applicable to any other number of lives; the terms being doubled for every additional life.

Problem III.

A certain number (n) of combinations of lives being proposed, each consisting of one person of the age of A, another of the age of B, a third of the age

of C, &c. all subject to some assigned law of mortality; it is required to determine the average duration of the longest liver in each combination,—or in other words, the Expectation of the last survivor of the individuals whereof it is composed?

Case 1.—To find the Curtate Expectation.

By Problem 1, page 130, it is manifest that the number of the proposed combinations in which not a single survivor may be expected to remain at the end of t years, is

$$\frac{n(a-{}_t a)(b-{}_t b)(c-{}_t c)\&c.}{abc\&c.} = n.\frac{a-{}_t a}{a}.\frac{b-{}_t b}{b}.\frac{c-{}_t c}{c}.\&c.$$

$$= n(1-{}_t a)(1-{}_t b)(1-{}_t c)\&c.,$$

and this deducted from n, the number of combinations proposed, leaves $n\{1-(1-{}_t a)(1-{}_t b)(1-{}_t c)\&c.\}$ for the number in which some one or more of the proposed lives may be expected to survive t years: hence, reasoning as in the last Problem, making t successively $=1, 2, 3$ &c., adding the results together, and dividing the sum by n, the required Expectation becomes

$$= f.\{1-(1-{}_t a)(1-{}_t b)(1-{}_t c)\&c.\}$$

$f.$ being put for the sum of the several results arising by making $t=1, 2, 3$ &c. to the greatest Tabular duration of the youngest of the proposed lives.

Suppose each combination to consist of only two lives A and B, then the required Expectation becomes $= f.\{1-(1-{}_t a)(1-{}_t b)\} = f.\{{}_t a+{}_t b-{}_t(ab)\}$ which by actually performing the operations indicated by the sign $f.$ becomes

$$= \begin{cases} {}_1a + {}_2a + {}_3a + {}_4a \text{ \&c., which by last Prob.} = a \\ {}_1b + {}_2b + {}_3b + {}_4b \text{ \&c.} \quad - \quad - \quad - \quad = b \\ -{}_1(ab) - {}_2(ab) - {}_3(ab) - {}_4(ab) \text{ \&c.} \quad - \quad = -ab \end{cases}$$

Hence it appears that the Curtate Expectation of the last survivor of two lives, A & B, is

$$= a + b - ab$$

Again, suppose each combination to consist of three lives, designated by A, B & C, then the required Expectation

$f. \{1-(1-{}_ia)(1-{}_ib)(1-{}_ic)\}$, becomes

$= f. \{{}_ia + {}_ib + {}_ic - {}_i(ab) - {}_i(ac) - {}_i(bc) + {}_i(abc)\}$, which, by actually performing the operations indicated by $f.$, becomes

$$= a + b + c - ab - ac - bc + abc.$$

In the same manner it may be shown that

The Curtate Expectation of the last survivor of Four or more Joint Lives is equal to the sum of the Curtate Expectations of the several Lives taken singly, minus the sum of the Curtate Expectations of the same Lives combined two and two, plus the Curtate Expectations of the same lives taken three and three, minus that of the several Lives combined by four and four, and so on.

Case 2.—To find the Complete Expectation, taking into consideration the fraction of a year's existence enjoyed by the last survivor in each combination, during the year in which it may become extinct.

Here it is manifest that out of a persons living, of the age of A, ${}^{t-1}a$ or ${}^{\prime}a$, may be expected to survive $t-1$ years, and therefore enter upon the tth year, and ${}^{\prime}a$ will outlive the tth year, consequently ${}^{\prime}a_{,}-{}^{\prime}a$ is the number which may be expected to fail within the tth year; hence, upon the supposition of equal decrements during equal portions of that year, $\frac{x}{m}({}^{\prime}a_{,}-{}^{\prime}a)$ must be the proportional number which may be expected to die off during the fraction $\frac{x}{m}$th of that year, and this deducted from the ${}^{\prime}a_{,}$ which enter upon it, leaves ${}^{\prime}a_{,}-\frac{x}{m}({}^{\prime}a_{,}-{}^{\prime}a) = \frac{1}{m}\{(m-x).{}^{\prime}a_{,}+x.{}^{\prime}a\}$ for the number, out of a persons now living of the age of A, which may be expected to be living at the end of the $\frac{x}{m}$th part of the tth year, and consequently $a-\frac{1}{m}\{(m-x).{}^{\prime}a_{,}+x.{}^{\prime}a\}$ is the number which may be expected to die off prior to that period.

In like manner it may be shown that, out of b,c,&c. persons now living at the respective ages of B, C &c. $b-\frac{1}{m}\{(m-x).{}^{\prime}b_{,}+x.{}^{\prime}b\}$, $c-\frac{1}{m}\{(m-x).{}^{\prime}c_{,}+x.{}^{\prime}c\}$&c. will respectively denote the numbers which may be expected to die in the like interval.

Hence reasoning as in Problem 1. it follows that, of the n proposed combinations,

$$\left\{[a-\frac{1}{m}\{(m-x).{}^{\prime}a_{,}+x.{}^{\prime}a\}]\times[b-\frac{1}{m}\{(m-x).{}^{\prime}b_{,}+x.{}^{\prime}b\}]\right.$$
$$\left.\times[c-\frac{1}{m}\{(m-x).{}^{\prime}c_{,}+x.{}^{\prime}c\}]\&c.\right\}\times\frac{n}{abc\,\&c.}$$

represents the number which may be expected not to have a single survivor left at the end of the in-

terval stated. This expression being expanded and the result deducted from n, the number of combinations proposed, leaves the number of combinations in each of which some one or more of the proposed persons may be expected survive the said interval: and from the remainder the required Expectation may be determined by the process employed in the second case of last Problem.

Thus, confining the investigation to three lives we deduce

$$\frac{n}{am}\left\{(m-x).{}^{t}a_{t}+x.{}^{t}a\right\}-\frac{n}{abm^{2}}\left\{(m-x).{}^{t}a_{t}+x.{}^{t}a\right\}\times\left\{(m-x).{}^{t}b_{t}+x.{}^{t}b\right\}$$
$$+\frac{n}{bm}\left\{(m-x).{}^{t}b_{t}+x.{}^{t}b\right\}-\frac{n}{acm^{2}}\left\{(m-x).{}^{t}a_{t}+x.{}^{t}a\right\}\times\left\{(m-x).{}^{t}c_{t}+x.{}^{t}c\right\}$$
$$+\frac{n}{cm}\left\{(m-x).{}^{t}c_{t}+x.{}^{t}c\right\}-\frac{n}{bcm^{2}}\left\{(m-x).{}^{t}b_{t}+x.{}^{t}b\right\}\times\left\{(m-x).{}^{t}c_{t}+x.{}^{t}c\right\}$$
$$+\frac{n}{abcm^{3}}\left\{(m-x).{}^{t}a_{t}+x.{}^{t}a\right\}\times\left\{(m-x).{}^{t}b_{t}+x.{}^{t}b\right\}\times\left\{(m-x).{}^{t}c_{t}+x.{}^{t}c\right\}$$

for the number of the proposed combinations, which may be expected to have one or more survivors in each, at the end of the $\frac{x}{m}$th part of the tth year.

And if, after further expanding the terms wherein the multiplications are only indicated, we proceed as in the last problem, by making m infinite, taking x successively $= 1, 2, 3 \text{ ----- } m$, adding the results, and dividing their sum by m, we obtain the quantity of existence to be enjoyed by one or more survivors of the proposed combinations during the tth year

$$=\frac{n}{a}(\tfrac{1}{2}{}'a_{,}+\tfrac{1}{2}{}'a)-\frac{n}{ab}\left\{\tfrac{1}{3}{}'(ab)_{,}+\tfrac{1}{6}{}'a_{,}{}'b+\tfrac{1}{6}{}'a{}'b_{,}+\tfrac{1}{3}{}'(ab)\right\}$$

$$+\frac{n}{b}(\tfrac{1}{2}{}'b_{,}+\tfrac{1}{2}{}'b)-\frac{n}{ac}\left\{\tfrac{1}{3}{}'(ac)_{,}+\tfrac{1}{6}{}'a_{,}{}'c+\tfrac{1}{6}{}'a{}'c_{,}+\tfrac{1}{3}{}'(ac)\right\}$$

$$+\frac{n}{c}(\tfrac{1}{2}{}'c_{,}+\tfrac{1}{2}{}'c)-\frac{n}{bc}\left\{\tfrac{1}{3}{}'(bc)_{,}+\tfrac{1}{6}{}'b_{,}{}'c+\tfrac{1}{6}{}'b{}'c_{,}+\tfrac{1}{3}{}'(bc)\right\}$$

$$+\frac{n}{abc}\left\{\begin{array}{l}\tfrac{1}{4}{}'(abc)_{,}+\tfrac{1}{12}\{{}'a_{,}{}'b_{,}{}'c+{}'a_{,}{}'b{}'c_{,}+{}'a_{,}{}'b{}'c\}\\+\tfrac{1}{12}\{{}'a{}'b_{,}{}'c_{,}+{}'a{}'b_{,}{}'c+{}'a{}'b{}'c_{,}\}+\tfrac{1}{4}{}'(abc)\end{array}\right\}$$

Then making t successively $=1, 2, 3$ &c. to the greatest Tabular duration of the youngest of the given lives, adding the results, and dividing the sum by n, the number of combinations proposed, we ultimately deduce the Expectation required.

But by observing the identity of the several results here derived, by making $t=1, 2, 3$ &c., with the expressions deduced in page 149, we find that, the three series resulting from the first terms of the first three lines in the above expressions are equal to the respective Expectations of A, B & C; the several series resulting from the second terms of the same lines are equal to the respective Expectations of the joint lives of A & B, A & C, B & C; and the series derived from the remaining expression is equal to the Expectation of the joint lives of A, B & C; whence it follows that the Expectation of the last survivor of three lives, A, B & C, is

$$a'+b'+c'-(ab)'-(ac)'-(bc)'+(abc)'.$$

And if in this expression we reject the terms into

which C enters, we have the Expectation of the last survivor of A & B,

$$= a' + b' - (ab)'.$$

By the same process the Expectation of the last survivor of any other number of lives may be determined.

But the analogy between the form of the Expectation, either curtate or complete, as herein deduced, and the expressions derived in Problem I. for the number of combinations in which one or more persons may be expected to survive t years, enables us to deduce the Expectation of the last survivor of two or more lives by a much easier process.

Thus, out of n combinations of three lives, the number which may be expected to have some two or all the three lives surviving at the end of t years, is in page 137 shown to be

$$= \{{}_t(ab) + {}_t(ac) + {}_t(bc) - 2{}_t(abc)\}n:$$

hence, of the last *two* survivors of three lives, the

Curtate Expectation is $ab + ac + bc - 2abc$

Complete ditto - $(ab)' + (ac)' + (bc)' - 2(abc)'$

Scholium.

In what has been advanced on the Expectation of Life, we have supposed the Table of Mortality employed to express the numbers living for each age, and the decrements during the year to be uniformly distributed throughout that period: but let

us further suppose, that a Table of Mortality were constructed for other intervals, either greater or less than a year. Thus let a, $'a$, ^{2z}a, ^{3z}a &c. represent the numbers living in such Table, opposite the age of an assigned life A, and opposite ages respectively z, $2z$, $3z$ &c. years older than A; then, supposing the decrements uniform throughout each interval of z years, and dividing that interval into m smaller ones, it is manifest that at the end of $\frac{x}{m}$th part of the first interval the number living is $\frac{1}{m}\{(m-x)a+x.'a\}$; and if in this expression we make m indefinitely great, so as to represent the number of instants in the proposed interval, take x successively $=1, 2, 3 \text{ - - - } m$, and add the results as in the foregoing pages, the quantity of existence which a persons now living may, among them, expect to enjoy, during the next interval of z years, is

$$\frac{1}{m}\left\{\frac{m^2}{2}.\ a+\frac{m^2}{2}.'a\right\}=\frac{m}{2}(a+'a) \text{ instants,}$$

whereof m constitute z years, or $\frac{m}{z}$ are equivalent to one year; hence, dividing $\frac{m}{2}(a+'a)$ by $\frac{m}{z}$, we have $\frac{z}{2}(a+'a)$ for the number of years' existence which a persons now living may, among them, expect to enjoy in the next z years: in like manner it may be shown that the quantity of existence which they may expect to enjoy, during the second interval of z years, is $\frac{z}{2}('a+^{2z}a)$, the third interval $\frac{z}{2}(^{2z}a+^{3z}a)$ and so on. The quantity of existence which they may expect to enjoy henceforth is therefore

$$= \tfrac{1}{2}(a + {}^{1}a) + \tfrac{1}{2}({}^{1}a + {}^{2z}a) + \tfrac{1}{2}({}^{2z}a + {}^{3z}a) \&c.$$
$$= z(\tfrac{1}{2}a + {}^{1}a + {}^{2z}a + {}^{3z}a + {}^{4z}a + {}^{5z}a \&c.),$$

and this divided by a, the number of lives proposed, $= \frac{1}{a}(\frac{1}{2}a + {}^{1}a + {}^{2z}a + {}^{3z}a \&c.)$ or $z(\frac{1}{2} + {}_{z}a + {}_{2z}a + {}_{3z}a \&c.)$ for the average duration of each.

And if, in either of these expressions, we make z = the number of years between the age of A and the limiting age in the Table, we have $a' = \frac{1}{2}z$.

Thus referring to Table x. age 18, and taking

z =	1 year - - - -	a' = 42.522
z =	5 years - - - -	a' = 42.535
z =	10 ditto - - - -	a' = 42.559
z =	80 ditto - - - -	a' = 40.000

Problem iv.

A certain number (n) of combinations of lives being proposed, each consisting of a like number of persons of the respective ages of A, B, C, *&c.*, and subject to some given law of mortality; it is required to determine how many of them may be expected to fail by an assigned order of Survivorship?

Case 1.—Suppose each combination to consist of two persons, designated by A and B?

Then, if it be required to determine the number in each of which the A may be expected to die before B, it is manifest by the Scholium to Problem 1, page 137 that, out of the proposed combinations, $\frac{({}^{1}a_{,} - {}^{1}a){}^{1}b.n}{ab}$ may be expected to have the A in each

dying within the tth year, and the B surviving that year; and if we subdivide this period into smaller intervals of $\frac{1}{m}$th of a year each, upon the supposition of equal decrements during the year, it is evident that $\frac{1}{m}({}^{t}a_{,} - {}^{t}a) \times \frac{1}{m}\left\{(m-x).{}^{t}b_{,} + x.{}^{t}b\right\}.\frac{n}{ab}$, represents the number of the proposed combinations, in each of which the B may be expected to survive the $\frac{x}{m}$th part of the tth year, and the A to die off during the preceding mth part of a year. Hence, making t indefinitely great, taking x successively $= 1, 2, 3 \text{ - - } m$, and adding the results as in the last problem, we have the number of the proposed combinations in each of which the A may be expected to die before B during the tth year $= \frac{1}{m}({}^{t}a_{,} - {}^{t}a) \times \frac{1}{m}\left\{\frac{m^2}{2}{}^{t}b_{,} + \frac{m^2}{2}.{}^{t}b\right\}\frac{n}{ab}$

$= \frac{n}{2ab}({}^{t}a_{,} - {}^{t}a)({}^{t}b_{,} + {}^{t}b) = \frac{n}{2ab}\left\{{}^{t}(ab)_{,} + {}^{t}a_{,}b - {}^{t}a.{}^{t}b_{,} - {}^{t}(ab)\right\}$;

and if in this expression we make t successively $= 1, 2, 3$ &c. to the greatest tabular duration of the oldest of the proposed lives, we have the number of the proposed combinations, in each of which the A may be expected to die before the B with which it is connected during the

1st year	$= \frac{n}{2ab}\{ab + a.{}^{1}b - {}^{1}ab - {}^{1}(ab)\}$				
2nd —	$= \frac{n}{2ab}\{{}^{1}(ab) + {}^{1}a.{}^{2}b - {}^{2}a.{}^{1}b - {}^{2}(ab)\}$				
3d —	$= \frac{n}{2ab}\{{}^{2}(ab) + {}^{2}a.{}^{3}b - {}^{3}a.{}^{2}b - {}^{3}(ab)\}$				
4th —	$= \frac{n}{2ab}\{{}^{3}(ab) + {}^{3}a.{}^{4}b - {}^{4}a.{}^{3}b - {}^{4}(ab)\}$				
&c.		&c.	&c.	&c.	&c.

the sum of which evidently produces the number required. But it has already been shown in page 146, that the sum of the

1st vertical column $= ab + ab.\mathrm{ab}$
2nd — — $= a_{,}b.\mathrm{a_{,}b}$
3d — — $= ab_{,}.\mathrm{ab_{,}}$
4th — — $= ab.\mathrm{ab}$

hence, by substitution, the required number becomes

$$= \tfrac{n}{2ab}\{ab + ab.\mathrm{ab} + a_{,}b.\mathrm{a_{,}b} - ab_{,}.\mathrm{ab_{,}} - ab.\mathrm{ab}\}$$

$$\frac{n}{2}\left\{1 + \frac{\mathrm{a_{,}b}}{{}_{,}a_{,}} - \frac{\mathrm{ab_{,}}}{{}_{,}b_{,}}\right\} = \frac{n}{2}\left\{1 + \frac{{}_{,}\mathbb{N} - \mathbb{N}_{,}}{\mathbb{D}}\right\}$$

And by interchanging B for A, we have the number of the proposed combinations in each of which the B may be expected to die before A, within the tth year, $= \tfrac{n}{2ab}\{{}'(ab)_{,,} - {}'a_{,}b + {}'a.{}'b_{,} - {}'(ab)\}$:

$$\text{Or, } \frac{n}{2}\left\{1 + \frac{\mathrm{ab_{,}}}{{}_{,}b_{,}} - \frac{\mathrm{a_{,}b}}{{}_{,}a_{,}}\right\} = \frac{n}{2}\left\{1 + \frac{\mathbb{N}_{,} - {}_{,}\mathbb{N}}{\mathbb{D}}\right\}$$

for the number of such events which may be expected henceforth; and by adding this to the preceding expression, the result $= n$, the number of combinations proposed.

Again, suppose it were required to determine the number of the proposed combinations in each of which A may be expected to die after B?

Reasoning as before, we have the number of the proposed combinations in each of which the B may be expected to die prior to the end of the $\frac{r}{m}$th part of the tth year, and the A to die during the mth part of a year immediately succeeding that period

$$=[b-\tfrac{1}{m}\{(m-x).{}'b_{,}+x.{}'b\}]\times\tfrac{1}{m}({}'a_{,}-{}'a)\tfrac{n}{ab}$$

$$=\tfrac{n}{abm}({}'a_{,}-{}'a)-\tfrac{n}{abm^2}({}'a_{,}-{}'a)\times\{(m-x).{}'b_{,}+x.{}'b\}:$$

and if in this expression we make m indefinitely great, take x successively $=1, 2, 3$ - - - m, and add the results as before, we obtain

$$\tfrac{n}{a}({}'a_{,}-{}'a)-\tfrac{n}{2ab}({}'a_{,}-{}'a)({}'b_{,}+{}'b)$$

$$=\tfrac{n}{a}({}'a_{,}-{}'a)-\tfrac{n}{2ab}\{{}'(ab)_{,}+{}'a_{,}{}'b-{}'a.{}'b_{,}-{}'(ab)\}$$

for the number of the proposed combinations in each of which A may be expected to die after B during the tth year.

And, by making t in this expression successively $=1, 2, 3$ &c. to the greatest tabular duration of A's life, it will be found that the number of the required events which may be expected to occur in the

1st Year $=\tfrac{n}{a}(a-{}^1a)-\tfrac{n}{2ab}\{(ab)+a.{}^1b-{}^1ab-{}^1(ab)\}$

2nd - $=\tfrac{n}{a}({}^1a-{}^2a)-\tfrac{n}{2ab}\{{}^1(ab)+{}^1a.{}^2b-{}^2a.{}^1b-{}^2(ab)\}$

3d - $=\tfrac{n}{a}({}^2a-{}^3a)-\tfrac{n}{2ab}\{{}^2(ab)+{}^2a.{}^3b-{}^3a.{}^2b-{}^3(ab)\}$

4th - $=\tfrac{n}{a}({}^3a-{}^4a)-\tfrac{n}{2ab}\{{}^3(ab)+{}^3a.{}^4b-{}^4a.{}^3b-{}^4(ab)\}$

&c. &c. &c.

The sum of which, taken as before, produces

$$n(1+\mathrm{a}-\mathrm{a})-\frac{n}{2}\left\{1+\mathrm{ab}+\frac{\mathrm{a_{,}b}}{{}'a_{,}}-\frac{\mathrm{ab_{,}}}{{}'b_{,}}-\mathrm{ab}\right\}$$

$$=n-\frac{n}{2}\left\{1+\frac{\mathrm{a_{,}b}}{{}'a_{,}}-\frac{\mathrm{ab_{,}}}{{}'b_{,}}\right\}$$

$$=\frac{n}{2}\left\{1+\frac{\mathrm{ab_{,}}}{{}'b_{,}}-\frac{\mathrm{a_{,}b}}{{}'a_{,}}\right\}=\frac{n}{2}\left\{1+\frac{\mathrm{N_{,}}-{}'\mathrm{N}}{\mathrm{D}}\right\}$$

for the number required.

And by interchanging A for B, we deduce

$$\tfrac{n}{b}({}'b_{,}-{}'b)-\tfrac{n}{2ab}\{{}'(ab)_{,}+{}'a.{}'b_{,}-{}'a_{,}{}'b-{}'(ab)\}$$

for the number of the proposed combinations in each of which the B may be expected to die after A during the tth year,

$$\text{or } \frac{n}{2}\left\{1+\frac{a_{,}b}{{}'a_{,}}-\frac{ab_{,}}{{}'b_{,}}\right\} = \frac{n}{2}\left\{1+\frac{{}'N-N_{,}}{D}\right\}$$

for the number of such events which may be expected henceforth.

On these expressions we may remark, that henceforth the number of the proposed combinations, in each of which the A may be expected to die before B, is exactly the same as that in which the B may be expected to die after A; but not so if we take a limited period less than the greatest tabular duration of the lives proposed. The former circumstance is, in fact, self-evident; and the latter will be obvious when we reflect that, in some of the combinations where the A's pre-decease their companions, within any shorter interval than that included between the present age of the B's and the extremity of life, the B's may outlive that interval, and consequently not die after the A's during such interval.

Case 2nd.—Suppose each combination to consist of three persons, designated by A, B & C.

Then, proceeding as before, by dividing each year into m indefinitely small intervals, taking x to denote a variable number of such intervals, and putting $f.$ to denote the sum of the results arising

by making x successively $=1, 2, 3$ - - - m, and $\int.$ to represent the sum of these results again, by making t successively $=1, 2, 3$ &c. it follows, by the foregoing Problems, that the number of the proposed combinations in each of which

A may be expected to die first is,

$$\int.\int. \tfrac{1}{n}({}'a_{,}-{}'a)\times\tfrac{1}{m}\{(m-x).{}'b_{,}+x.{}'b\}\times\tfrac{1}{m}\{(m-x).{}'c_{,}+x.{}'c\}\times\tfrac{n}{abc}$$

A second and B first is,

$$\int.\int.[b-\tfrac{1}{m}\{(m-x).{}'b_{,}+x.{}'b\}]\times\tfrac{1}{m}({}'a_{,}-{}'a)\times\tfrac{1}{m}\{(m-x){}'c_{,}+x.{}'c\}\times\tfrac{n}{abc}$$

A second and C first is,

$$\int.\int.[c-\tfrac{1}{m}\{(m-x).{}'c_{,}+x.{}'c\}]\times\tfrac{1}{m}({}'a_{,}-{}'a)\times\tfrac{1}{m}\{(m-x).{}'b_{,}+x.{}'b\}\times\tfrac{n}{abc}$$

A third is,

$$\int.\int.[b-\tfrac{1}{m}\{(m-x).{}'b_{,}+x.{}'b\}]\times[c-\tfrac{1}{m}\{(m-x).{}'c_{,}+x'c\}]\times\tfrac{1}{m}({}'a_{,}-{}'a)\times\tfrac{n}{abc}$$

The same mode of proceeding is evidently applicable to any other number of lives.

If, in these expressions, the operations indicated by the smaller sign $\int.$ be performed, it will be found that, during the tth year, the number of the proposed combinations in each of which

A may be expected to die first is,

$$\tfrac{n}{6abc}\{2.{}'(abc)_{,}+{}'(a,b,c)+{}'(a,bc_{,})+2.{}'(a,bc)\}$$
$$-\tfrac{n}{6abc}\{2.{}'(ab,c_{,})+{}'(ab,c)+{}'(abc_{,})+2.{}'(abc)\}$$

A second and B first is,

$$\tfrac{n}{2ac}\{{}'(ac)_{,}+{}'(a,c)-{}'(ac_{,})-{}'(ac)\}$$
$$-\tfrac{n}{6abc}\{2.{}'(abc)_{,}+{}'(a,b_{,}c)+{}'(a,bc_{,})+2.{}'(a,bc)\}$$
$$+\tfrac{n}{6abc}\{2.{}'(ab,c_{,})+{}'(ab,c)+{}'(abc_{,})+2.(abc)\}$$

A second and C first is,

$$\frac{n}{2ab}\{'(ab)_{,}+'(a_{,}b)-'(ab_{,})-'(ab)\}$$
$$-\frac{n}{6abc}\{2.'(abc)_{,}+'(a_{,}b_{,}c)+'(a_{,}bc_{,})+2.'(a_{,}bc)\}$$
$$+\frac{n}{6abc}\{2.'(ab_{,}c_{,})+'(ab_{,}c)+'(abc_{,})+2.'(abc)\}$$

A may be expected to die third is,

$$\frac{n}{a}\{'a_{,}-'a\}$$
$$-\frac{n}{2ab}\{'(ab)_{,}+'(a_{,}b)-'(ab_{,})-'(ab)\}$$
$$-\frac{n}{2ac}\{'(ac)_{,}+'(a_{,}c)-'(ac_{,})-'(ac)\}$$
$$+\frac{n}{6abc}\{2.'(abc)_{,}+'(a_{,}b_{,}c)+'(a_{,}bc_{,})+2.'(a_{,}bc)\}$$
$$-\frac{n}{6abc}\{2.'(ab_{,}c_{,})+('ab_{,}c)+('abc_{,})+2.'(abc)\}$$

And if, in these expressions, the operations indicated by the greater sign $\int.$ be performed, we obtain the required number of the proposed combinations in each of which henceforth the

A may be expected to die first,

$$=\frac{n}{6}\left\{2+\frac{1}{,a_{,}}\left(\frac{a_{,}b_{,}c}{,b_{,}}+\frac{a_{,}bc_{,}}{,c_{,}}+2.a_{,}bc\right)\right\}$$
$$-\frac{n}{6}\left\{\frac{1}{,b_{,}}\left(\frac{2.ab_{,}c_{,}}{,c_{,}}+ab_{,}c\right)+\frac{abc_{,}}{,c_{,}}\right\}$$

A second and B first,

$$=\frac{n}{2}\left\{1+\frac{a_{,}c}{,a_{,}}-\frac{ac_{,}}{,c_{,}}\right\}-\frac{n}{6}\left\{2+\frac{1}{,a_{,}}\left(\frac{a_{,}b_{,}c}{,b_{,}}+\frac{a_{,}bc_{,}}{,c_{,}}+2.a_{,}bc\right)\right\}$$
$$+\frac{n}{6}\left\{\frac{1}{,b_{,}}\left(\frac{2.ab_{,}c_{,}}{,c_{,}}+ab_{,}c\right)+\frac{abc}{,c_{,}}\right\}$$

A second and C first,

$$=\frac{n}{2}\left\{1+\frac{a_{,}b}{,a_{,}}-\frac{ab_{,}}{,b_{,}}\right\}-\frac{n}{6}\left\{2+\frac{1}{,a_{,}}\left(\frac{a_{,}b_{,}c}{,b_{,}}+\frac{a_{,}bc_{,}}{,c_{,}}+2.a_{,}bc\right)\right\}$$
$$+\frac{n}{6}\left\{\frac{1}{,b_{,}}\left(\frac{2.ab_{,}c_{,}}{,c_{,}}+ab_{,}c\right)+\frac{abc_{,}}{,c_{,}}\right\}$$

A may be expected to die third,

$$n-\frac{n}{2}\left\{1+\frac{a,b}{,a,}-\frac{ab,}{,b,}\right\}-\frac{n}{2}\left\{1+\frac{a,c}{,a,}-\frac{ac,}{,c,}\right\}$$
$$+\frac{n}{6}\left\{2+\frac{1}{,a,}\left(\frac{a,b,c}{,b,}+\frac{a,bc,}{,c,}+2.a,bc\right)\right\}$$
$$-\frac{n}{6}\left\{\frac{1}{,b,}\left(\frac{2.ab,c,}{,c,}+ab,c\right)+\frac{abc,}{,c,}\right\}$$

The sum of the second and third of these expressions represents the number of the proposed combinations in each of which the A may be expected to die second, without regarding whether the B or C dies first; and the sum of all the four gives n, the number proposed.

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5. *Construction of a Table of Mortality.*

In order to acquire a clear understanding of the method of constructing a Table of Mortality, the reader is again referred to what we have stated in pages 94 and 96; and to observe the relation which obtains between the decrements given in a Table of Mortality, and the number of deaths happening at the different ages in *one year* among a stationary population. Thus, in a stationary population, subject to the law of mortality, exhibited by the Northampton Table, kept up by 11650 annual births, and balanced by a like number of annual deaths, it is evidently immaterial whether the decrements be regarded as representing the deaths in the successive years of one and the same class of 11650

persons born at the same time, or as the deaths in one and the same year out of successive classes of 11650 persons born annually. On the latter supposition it is manifest that if in such stationary population a register of the deaths were carefully kept for *one year only*, (but more accurately by taking the average number of annual deaths at each age for several years) the results would at once enable us to form a Table of Mortality.

Thus, suppose it were found from such register that in a stationary population

1 person died annually between 96 and 97
3 persons - - - - 95 - 96
5 —— - - - - 94 - 95
7 —— - - - - 93 - 94

and so on, as represented by the decrements in the Table, page 95; by taking the successive sums we should find the

number living at 96= - 1
— — — 95=1+3=4
— — — 94=4+5=9

and so on, to the younger periods of life.

Again, were it possible to fix upon a community consisting of a stationary population kept up by a certain number of annual births produced *simultaneously at the beginning of each year*, we should have nothing more to do than to cause an enumeration to be made at the beginning of any year, so as to determine the numbers living at the respective ages of 1, 2, 3, 4, &c. years, to the extremity of life,

and the results would at once give the column designated by Number living in a Table of Mortality; and the decrements might then be found by deducting from the number living opposite any age the number living at the next greater age. Thus the decrement at the age of 20 is found by deducting from the number living at 20, the number living at 21.

In point of fact, however, it is manifest that no community can be found wherein the annual births are simultaneously produced; on the contrary, such births generally take place at different intervals throughout the year. This circumstance renders it impracticable, even in a stationary population, to fix upon any period of the year at which the persons living complete the different years of their age. Such enumerations must therefore be confined to determine the numbers living within certain intervals of age, as

between the age of 1 and 2
— — — 2 - 3 } and so on;
— — — 3 - 4

and the numbers thus determined might evidently be regarded as corresponding to the numbers living in a community formed by simultaneous births at the respective ages of $\frac{1}{2}$, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$, &c. years.

But in a community formed by a constant number of simultaneous births, it is manifest that upon the supposition of equal decrements during the year, the number living in the middle of any year from

birth is an arithmetical mean between the numbers living at the beginning and at the end of that year. Thus, by the Northampton Table, the number living at the age of $20\frac{1}{2}$ is $\frac{1}{2}(5132+5060)=5096$. Similarly, the number living at the middle of the tth year from the age of an assigned life A, is represented by $\frac{1}{2}({}^{t}a_{,}+{}^{t}a)$. Hence, in a community formed by a constant number of annual births distributed in equal proportions over equal intervals throughout the year, let the number living, as determined by actual enumeration, in the tth year from the age of an assigned life be put $={}^{t}l$, then $\frac{1}{2}({}^{t}a_{,}+{}^{t}a)={}^{t}l$, or ${}^{t}a_{,}=2.{}^{t}l-{}^{t}a$, from which we have $a=2.{}^{1}l-{}^{1}a$.

To apply this expression to the formation of a Table of Mortality, we have only to begin at the oldest age and proceed downwards from age to age to the younger periods of life. Thus, suppose it were found by enumeration that the numbers living in a stationary population were as under, viz.

between 96 and 97	=	1	
95 - 96	=	5	
94 - 95	=	13	
93 - 94	=	25	
&c.		&c.	

then, for a Table of Mortality we should have,

number living at 96	=	$2.{}^{1}l-{}^{1}a$	=	2 −	0	= 2
— — — 95	=	-	=	10 −	2	= 8
— — — 94	=	-	=	26 −	8	= 18
— — — 93	=	-	=	50 −	18	= 32

and so on, to the younger periods of life. When the column representing the numbers living at the different ages is thus formed, that representing the decrement is immediately deducible, by taking the differences of the numbers living. Thus the decrement at 93 is the difference between the numbers living at 93 and 94.

Both the number living, and the Expectation of life at any age, may, however, be deduced in a more direct manner from the results determined by enumeration; for, by making t successively = 1, 2, 3, &c. in the general expression ${}^{t}a = 2.{}^{t}l - {}^{t}a$ we have

$$a = 2.{}^{1}l - {}^{1}a$$
$${}^{1}a = 2.{}^{2}l - {}^{2}a$$
$${}^{2}a = 2.{}^{3}l - {}^{3}a$$
&c. &c.

whence substituting the value of ${}^{1}a$ in the expression $a = 2.{}^{1}l - {}^{1}a$, and the value of ${}^{2}a$ in the result, and so on, we have

$$a = 2({}^{1}l - {}^{2}l + {}^{3}l - {}^{4}l + \&c.)$$

Also from the expression $\frac{1}{2}({}^{t}a + {}^{t}a) = {}^{t}l$, we obtain $f.\frac{1}{2}({}^{t}a + {}^{t}a) = f.{}^{t}l$: but it has been shown in page 149, that $f.\frac{1}{a}\{\frac{1}{2}{}^{t}a + \frac{1}{2}{}^{t}a) = a'$, consequently $f.\frac{1}{2}({}^{t}a + {}^{t}a) = a.a'$: and $f.{}^{t}l = {}^{1}l + {}^{2}l + {}^{3}l + {}^{4}l$&c. by the notation: hence $a.a' = {}^{1}l + {}^{2}l + {}^{3}l + {}^{4}l$&c. or $a' = \frac{1}{a}({}^{1}l + {}^{2}l + {}^{3}l$&c.); from which, by substituting the value of a as already determined, we deduce

$$a = \frac{1}{2}\left\{\begin{matrix}{}^{1}l + {}^{2}l + {}^{3}l + {}^{4}l\&c. \\ {}^{1}l - {}^{2}l + {}^{3}l - {}^{4}l\&c.\end{matrix}\right\}$$

$$a = 2.{}^{1}l - \{{}^{2}l - ({}^{3}l - a)\}$$
$$= 2({}^{1}l - {}^{2}l + {}^{3}l - {}^{4}l + \ldots)$$

Enumerations are, however, seldom made so minute as to determine the numbers living between the ages of one and two, two and three, three and four, and so on, for every year of human life; nor does it appear necessary, except in the younger periods of childhood, to aim at greater accuracy in a Table of Mortality than what may be acquired by subdividing human life into longer intervals of five or ten years each.

Let us therefore further suppose that, in a stationary population, a register were kept, or an enumeration made, so as to determine the number of deaths, or the number living, in each of these longer intervals; then a Table of Mortality may be formed, either on the supposition of equal decrements during the different years of which each interval is composed, or on the supposition of the decrements increasing or decreasing uniformly during that interval, so as to be conterminous with those in the adjacent intervals.

On the first supposition, the mode of forming a Table of Mortality from a register of deaths is sufficiently obvious without the aid of symbols. Thus admitting it were found by the mortuary registers that in a stationary population

42	died annually between				90	and	95
140	-	-	-	-	85	-	90
283	-	-	-	-	80	-	85
363	-	-	-	-	75	-	80

and so on,

then the decrement in each year from 95 to 90 = 8.4
— — — — — 90 - 85 = 28.0
— — — — — 85 - 80 = 56.6
— — — — — 80 - 75 = 72.6

and so on, to the younger periods of life.

Hence assuming 97 as the limiting age, and taking the number living at 96 = 1,

that at - - - - 95 = 4, we have
the number living at - 94 = 4 + 8.4 = 12.4
— — — — 93 = 12.4 + 8.4 = 20.8
— — — — 92 = 20.8 + 8.4 = 29.2
— — — — 91 = 29.2 + 8.4 = 37.6
— — — — 90 = 37.6 + 8.4 = 46.0
— — — — 89 = 46.0 + 28 = 74.0
— — — — 88 = 74.0 + 28 = 102
— — — — 87 = 102 + 28 = 130
— — — — 86 = 130 + 28 = 158
— — — — 85 = 158 + 28 = 186

and thus deduce a Table of Mortality sufficiently accurate to determine the values of Life Annuities, or even the premiums of Assurances for the whole period of life. But in the application of a Table thus formed to determine the premiums for short period Assurances, the leaps which might possibly happen in the decrements at the junction of the different intervals would not unfrequently cause the premiums at some ages to be greater than those produced by like calculations at more advanced periods of life.

In order to avoid this incongruity, let the number of deaths deduced from the supposed register during the first interval of z years from the age of an assigned life be put - $= {}^{z}\delta$
ditto during the second interval $= {}^{2z}\delta$ &c.
— — third - - - $= {}^{3z}\delta$

and let the decrement at the assigned age $= d$
that at an age z years older - - - - - $= {}^{z}d$ &c.
— — $2z$ — — - - - - - $= {}^{2z}d$

then putting the decrement opposite an age
1 year older than the given life $= d + y$
2 years - - - - $= d + 2y$
3 —— - - - - $= d + 3y$
z —— - - - - $= d + zy$

we have $d + (d+y) + (d+2y) - - - + (d+\overline{z-1}.y) = {}^{z}\delta$,
or $zd + \frac{z(z-1)}{2}y = {}^{z}\delta$ - - - - - - - **A**
and $d + zy = {}^{z}d$, from which $y = \frac{1}{z}({}^{z}d - d)$;
whence substituting this value of y in equation **A**, there arises

$zd + \frac{z-1}{2}({}^{z}d - d) = {}^{z}\delta$, or $(z+1).\ d + (z-1).\ {}^{z}d = 2.{}^{z}\delta$

similarly it may be shown that - - $(z+1).\ {}^{z}d + (z-1).{}^{2z}d = 2.{}^{2z}\delta$
$(z+1).{}^{2z}d + (z-1).{}^{3z}d = 2.{}^{3z}\delta$,

and so on; from which by putting $\frac{2}{z+1} = p$ and $\frac{z-1}{z+1} = q$ we obtain

$$\left.\begin{aligned} d &= p.\ {}^{z}\delta - q.\ {}^{z}d \\ {}^{z}d &= p.{}^{2z}\delta - q.{}^{2z}d \\ {}^{2z}d &= p.{}^{3z}\delta - q.{}^{3z}d \end{aligned}\right\} \quad - \quad - \ \mathbf{B},$$

and so on, the sum of which produces

$d + {}^{z}d + {}^{2z}d$&c. $= p({}^{z}\delta + {}^{2z}\delta + {}^{3z}\delta$&c.$) - q({}^{z}d + {}^{2z}d + {}^{3z}d$&c.$)$

or $d = p({}^{z}\delta + {}^{2z}\delta + {}^{3z}\delta$&c.$) - (1+q)({}^{z}d + {}^{2z}d + {}^{3z}d$&c.$)$

or since $1+q = 1 + \frac{z-1}{z+1} = \frac{2z}{z+1} = zp$, we obtain

$$d = p\{{}^{z}\delta + {}^{2z}\delta + {}^{3z}\delta\text{&c.} - z({}^{z}d + {}^{2z}d + {}^{3z}d\text{&c.}\}$$

Hence, beginning at the oldest age and proceeding downwards, the values of ${}^{4z}d$, ${}^{3z}d$, ${}^{2z}d$, ${}^{z}d$, and d, may be successively determined. And this done, the intermediate decrements from d to ${}^{z}d$ become known by taking $x = 1, 2, 3$&c. in the expression $d + \frac{x}{z}({}^{z}d - d)$ or in $\frac{1}{z}\{(z-x)d + x.{}^{z}d\}$, those from ${}^{z}d$ to ${}^{2z}d$ are determined in like manner from the expression $\frac{1}{z}\{(z-x).{}^{z}d + x.{}^{2z}d\}$ and so on.

Otherwise, if in the first of the equations marked B we substitute the value of ${}^{z}d$ as determined by the second, and, in the result substitute the value of ${}^{2z}d$ as determined from the third, and so on, we obtain

$$d = p\{{}^{z}\delta - q.{}^{2z}\delta + q^2.{}^{3z}\delta - q^3.{}^{4z}\delta + q^4.{}^{5z}\delta\text{&c.}\};$$

and, when the decrements for all ages are determined, the numbers living become known by the process described in page 165.

Again, let the number living, as determined by enumeration on a community consisting of a stationary population, between the age of an assigned life A, and another age z years older than A, be denoted by ${}^{z}l$, the number living, as determined in like manner, between this latter age and another age $2z$ years older than $A = {}^{2z}l$, and so on. Or in other words, let the number living, as determined by enumeration, within the

1st interval of z years from the assigned age $= {}^{z}l$
2nd - - - - - - $= {}^{2z}l$
3rd - - - - - - $= {}^{3z}l$
&c. - - - &c.

then it is manifest that the aggregate number living, during the first interval from the given age, may be considered as made up of the numbers living

at $\frac{1}{2}$ year older than the assigned life
— $1\frac{1}{2}$ years - - ditto.
— $2\frac{1}{2}$ - - - ditto.
— $3\frac{1}{2}$ - - - ditto.
to $z-\frac{1}{2}$ - - - ditto.

But, on the supposition of equal decrements during the next z years, it follows, from what has been stated in the foregoing pages, that in a community formed by a constant number of annual births, produced simultaneously, the number living at the end of the $\frac{x}{m}$th part of the proposed interval, is $\frac{1}{m}\{(m-x)a+x.{}^{z}a\}$, from which, by making $m=2z=$ the number of half years in the proposed interval, and taking x successively $= 1, 3, 5, - - - -$ to $2z-1$, we have the number living at

an age $\frac{1}{2}$ year older than A $= \frac{1}{2z}\{(2z-1)a+{}^{z}a\}$
— $1\frac{1}{2}$ years - - $= \frac{1}{2z}\{(2z-3)a+3.{}^{z}a\}$
— $2\frac{1}{2}$ ditto - - $= \frac{1}{2z}\{(2z-5)a+5.{}^{z}a\}$

and so on to z terms, the sum of which is

$$\frac{1}{2z}\{(2z^2-z^2)a+z^2.{}^{z}a\} = \frac{z}{2}(a+{}^{z}a)$$

for the aggregate number living in a stationary population, between the age of A and another age z.

years older than A. In like manner it may be shown that the number living at the same period, between the latter of the ages above stated, and another age $2z$ years older than A is $\frac{z}{2}({}^{z}a + {}^{2z}a)$; or in other words the number living within the

1st interval of z years from the age of A is		$\frac{z}{2}(a + {}^{z}a)$
2nd	- - - - - -	$\frac{z}{2}({}^{z}a + {}^{2z}a)$
3rd	- - - - - -	$\frac{z}{2}({}^{2z}a + {}^{3z}a)$
&c.	- - - - - -	&c.

and these being compared with the results of the supposed enumeration, we obtain

$$\frac{z}{2}(a + {}^{z}a) = {}^{z}l, \text{ or } a = \frac{2}{z}.{}^{z}l - {}^{z}a$$

$$\frac{z}{2}({}^{z}a + {}^{2z}a) = {}^{2z}l, \text{ or } {}^{z}a = \frac{2}{z}.{}^{2z}l - {}^{2z}a$$

$$\frac{z}{2}({}^{2z}a + {}^{3z}a) = {}^{3z}l, \text{ or } {}^{2z}a = \frac{2}{z}.{}^{3z}l - {}^{3z}a$$

and if in the first of these expressions we substitute the value of ${}^{z}a$ as determined by the second, and in the result the value of ${}^{2z}a$, as determined by the third and so on, there arises

$$a = \frac{2}{z}\{{}^{z}l - {}^{2z}l + {}^{3z}l - {}^{4z}l + \&c.\}$$

And, by adding the above equations, we further obtain

$$z\{\tfrac{1}{2}a + {}^{z}a + {}^{2z}a + {}^{3z}a \&c.\} = {}^{z}l + {}^{2z}l + {}^{3z}l + \&c.$$

or, $\frac{z}{a}\{\frac{1}{2}a + {}^{z}a + {}^{2z}a + {}^{3z}a \&c.\} = \frac{1}{a}\{{}^{z}l + {}^{2z}l + {}^{3z}l + \&c.\}$:

the left hand member of this equation is represented by a′, as we have shown in page 157, and by substituting the value of a in the right hand member, and comparing the results, we deduce

$$a' = \frac{z}{2}\left\{\frac{{}^{z}l + {}^{2z}l + {}^{3z}l + {}^{4z}l \text{ \&c.}}{{}^{z}l - {}^{2z}l + {}^{3z}l - {}^{4z}l \text{ \&c.}}\right\}$$

a formula which enables us at once to deduce the Expectation of Life from the results of an enumeration made on a stationary population.

Again, suppose the numbers living in the proposed intervals to be determined by enumeration, and the decrements to be equal in equal portions of the same year, but progressively increasing or decreasing from year to year, so as to form a conterminous series at the juncture of the adjacent intervals.

Then it is manifest that the number living at

an age	$\frac{1}{2}$ year	older than A	is	$a - \frac{1}{2}.d$
—	$1\frac{1}{2}$ years	-	-	${}^{1}a - \frac{1}{2}.{}^{1}d$
—	$2\frac{1}{2}$	-	-	${}^{2}a - \frac{1}{2}.{}^{2}d$
—	$z - \frac{1}{2}$	-	-	${}^{z}a - \frac{1}{2}.{}^{z}d_{,}$

The sum of these, and the sums of the like results deduced from the succeeding intervals, give

$$a + {}^{1}a + {}^{2}a - - - - {}^{z}a_{,} - \tfrac{1}{2}(d + {}^{1}d + {}^{2}d - - - - {}^{z}d_{,}) = {}^{z}l$$

$${}^{z}a + {}^{z+1}a + {}^{z+2}a - - - {}^{2z}a_{,} - \tfrac{1}{2}({}^{z}d + {}^{z+1}d + {}^{z+2}d - - - {}^{2z}d_{,}) = {}^{2z}l$$

$${}^{2z}a + {}^{2z+1}a + {}^{2z+2}a - - - {}^{3z}a_{,} - \tfrac{1}{2}({}^{2z}d + {}^{2z+1}d + {}^{2z+2}d - - - {}^{3z}d_{,}) = {}^{3z}l$$

&c. &c. &c.

the sum of these continued to the extremity of life gives $a + {}^{1}a + {}^{2}a + {}^{3}a + \text{\&c.} - \frac{1}{2}(d + {}^{1}d + {}^{2}d + {}^{3}d \text{\&c.}) = {}^{z}l + {}^{2z}l + {}^{3z}l \text{\&c:}$ and from the nature of a Table of Mortality it is evident that $\frac{1}{2}(d + {}^{1}d + {}^{2}d + {}^{3}d \text{\&c.}) = \frac{1}{2}a$, consequently $a + {}^{1}a + {}^{2}a + {}^{3}a \text{\&c.} = \frac{1}{2}a + {}^{z}l + {}^{2z}l + {}^{3z}l \text{\&c.}$

But, upon the supposition of progressive decrements during the proposed interval, we have

$^1d = d + y$
$^2d = d + 2y$
$^3d = d + 3y$
$\left.\begin{matrix}\\\\\end{matrix}\right\}$ y being positive, negative, or = 0 according as the decrements increase, decrease, or remain constant.
$^zd = d + zy$, from which $y = \frac{1}{z}(^zd - d)$.

Also
$$a = a = a$$
$$^1a = a - d = a - d$$
$$^2a = {}^1a - {}^1d = a - 2d - y$$
$$^3a = {}^2a - {}^2d = a - 3d - 3y$$
$$^4a = {}^3a - {}^3d = a - 4d - 6y$$
$$^5a = {}^4a - {}^4d = a - 5d - 10y$$

and so on, to z terms, the sum of which gives

$$a + {}^1a + {}^2a \cdots {}^za_{,} = za - \tfrac{z(z-1)}{2}.d - \tfrac{z(z-1)(z-2)}{2\cdot 3}y;$$

or by substituting the value of y, and proceeding in like manner for the succeeding intervals, we have

$$a + {}^1a + {}^2a \cdots {}^za_{,} = za - \tfrac{z(z-1)}{2}.d - \tfrac{(z-1)(z-2)}{6}.({}^zd - d)$$
$${}^za + {}^{z+1}a + {}^{z+2}a \cdots {}^{2z}a_{,} = z.{}^za - \tfrac{z(z-1)}{2}.{}^zd - \tfrac{(z-1)(z-2)}{6}.({}^{2z}d - {}^zd)$$
$${}^{2z}a + {}^{2z+1}a + {}^{2z+2}a \cdots {}^{3z}a_{,} = z.{}^{2z}a - \tfrac{z(z-1)}{2}.{}^{2z}d - \tfrac{(z-1)(z-2)}{6}.({}^{3z}d - {}^{2z}d)$$

the sum of these again, continued to the extremity of life, gives

$$a + {}^1a + {}^2a + {}^3a \&c. = z(a + {}^za + {}^{2z}a \&c.) - m(d + {}^zd + {}^{2z}d \&c.) + nd$$

m being here put for $\frac{z(z-1)}{2}$, and n for $\frac{(z-1)(z-2)}{6}$.

But it has been already shown that

$$a + {}^1a + {}^2a + {}^3a \&c. = \tfrac{1}{2}a + {}^zl + {}^{2z}l + {}^{3z}l \&c.;$$ wherefore

$$z(a + {}^za + {}^{2z}a \&c.) - m(d + {}^zd + {}^{2z}d \&c) + nd = \tfrac{1}{2}a + {}^zl + {}^{2z}l + {}^{3z}l \&c.$$

or $$(z - \tfrac{1}{2})a - (m - n)d = {}^zl + {}^{2z}l \&c. - z({}^za + {}^{2z}a \&c.) + m({}^zd + {}^{2z}d \&c.)$$

Again, by continuing the progressions which exhibit the respective values of ^2a, ^3a, ^4a, &c. in terms of a, d and y, we further obtain

$$^za = a - zd - \frac{z(z-1)}{2}y = a - zd - \frac{z-1}{2}(^zd - d)$$

that is $$^za = a - z.d - \frac{z-1}{2}(^zd - d)$$

similarly $$^{2z}a = {}^za - z.{}^zd - \frac{z-1}{2}(^{2z}d - {}^zd)$$

$$^{3z}a = {}^{2z}a - z.{}^{2z}d - \frac{z-1}{2}(^{3z}d - {}^{2z}d)$$

&c. &c. &c.

the sum of which continued to the extremity of life, gives

$$^za + {}^{2z}a + {}^{3z}a\&c = a + {}^za + {}^{2z}a\&c - z(d + {}^zd + {}^{2z}d\&c) + \frac{z-1}{2}.d$$

or $$a = \frac{z+1}{2}d + z(^zd + {}^{2z}d + {}^{3z}d\&c.) \quad - - - - - \quad \mathbf{C.}$$

Similarly it may be shown that

$$^za = \frac{z+1}{2}.{}^zd + z(^{2z}d + {}^{3z}d + {}^{4z}d\&c.)$$

$$^{2z}a = \frac{z+1}{2}.{}^{2z}d + z(\quad {}^{3z}d + {}^{4z}d\&c.)$$

$$^{3z}a = \frac{z+1}{2}.{}^{3z}d + z(\quad\quad {}^{4z}d\&c.)$$

the sum of which taken to the extremity of life gives

$$^za + {}^{2z}a + {}^{3z}a\&c. = \frac{z+1}{2}(^zd + {}^{2z}d + {}^{3z}d\&c.) + z(^{2z}d + 2.{}^{3z}d + 3.{}^{4z}d\&c.),$$

this value of $^za + {}^{2z}a + {}^{3z}a$&c. being substituted in the foregoing equation produces

$$(z - \tfrac{1}{2})a - (m-n)d = {}^zl + {}^{2z}l_{\&c.} - z(^zd + {}^{2z}d_{\&c.}) - z^2.(^{2z}d + 2.{}^{3z}d + 3.{}^{4z}d_{\&c.})$$

from which, by restoring the values of m and n, and clearing the equation from fractions, we derive

$$3(2z-1)a - 2(z-1)(z+1)d = 6(^zl + {}^{2z}l_{\&c.}) - 6z(^zd + {}^{2z}d_{\&c.}) - 6z^2(^{2z}d + 2.{}^{3z}d + 3.{}^{4z}d_{\&c.})$$

and multiplying equation C by $3(2z-1)$ we have

$$3(2z-1)a - \tfrac{3}{2}(2z-1)(z+1)d = \quad -3z(^zd + {}^{2z}d_{\&c.}) + 6z^2(^zd + {}^{2z}d + {}^{3z}d + {}^{4z}d_{\&c.})$$

the difference of these two equations again gives

$$\tfrac{1}{2}(2z+1)(z+1)d = 6(^zl + {}^{2z}l_{\&c.}) - 3z(^zd + {}^{2z}d_{\&c.}) - 6z^2(^zd + 2.{}^{2z}d + 3.{}^{3z}d_{\&c.})$$

from which, by putting $\frac{6}{(2^z+1)(z+1)}=p$, we deduce

$$d=p\{2({}^1l+2^z l+3^z l_{\&c.})-z({}^1d+2^z d+3^z d_{\&c.})-2z^2({}^1d+2.2^z d+3.3^z d_{\&c.})\}$$

a general formula, by means of which the decrements at the commencement of the several intervals may be successively determined by beginning at the last interval and proceeding downwards to the younger periods of life.

Thus referring the assigned age to the commencement of the last z years of human life ${}^1d$, ${}^2d$ &c. each $= 0$, and $d=2p.{}^1l$: then referring to the commencement of the preceding interval, the d before determined now becomes ${}^1d=2p.{}^2l$ a known quantity, from which $d=p\{2({}^1l+{}^2l)-z.{}^1d-2z^2.{}^1d\}$ also becomes known, and so on from one interval to another.

The decrements at the commencement of each interval being thus found, the intermediate decrements may be determined by the process detailed in page 172, and the numbers living at each age, then deduced as in page 170. Or when the values of d, ${}^1d$ &c. shall have been determined, the numbers living at the commencement of each interval may be found by the formula

$$a={}^{z+1}_{2}d+z({}^1d+{}^2d+{}^3d\&\text{c.})$$

Moreover, instead of confining our investigations to a stationary population kept up by a constant number of annual births distributed uniformly throughout the year, it may sometimes be found convenient to suppose the births produced simul-

taneously, in which case it is manifest that, if an enumeration were made at the beginning of any year, the number living at

the age of an assigned life would be $= a$
that an age 1 year older - - - $= {}^1a$
— — 2 years — - - - $= {}^2a$
— — 3 — — - - - $= {}^3a$
— — $z-1$ — - - - $= {}^{z}a$,

these being made equal to ${}^{z}l$, and the like process continued to the extremity of life, we should find that in such case

$$a+{}^1a+{}^2a+{}^3a+\&c.={}^{z}l+{}^{2z}l+{}^{3z}l+\&c.$$

But, upon the supposition of progressive decrements during each interval of z years, we have shown, in page 176, that

$$a+{}^1a+{}^2a+{}^3a\&c.=z(a+{}^{z}a+{}^{2z}a\&c)-m({}^{z}d+{}^{2z}d\&c.)+nd;$$

hence, by comparing these equals we obtain

$$za-(m-n)d={}^{z}l+{}^{2z}l\&c-z({}^{z}a+{}^{2z}a\&c)+m({}^{z}d+{}^{2z}d\&c.)$$

which, by substituting the value of ${}^{z}a+{}^{2z}a\&c.$ as determined in page 177, and restoring the values of m and n as before, gives

$$6za-2(z-1)(z+1)d=6({}^{z}l+{}^{2z}l\&c)-6z({}^{z}d+{}^{2z}d\&c)-6z^2({}^{2z}d+2.{}^{3z}d+3.{}^{4z}d\&c)$$

and from equation C, page 177, it may be shown that,

$$6za-3z(z+1)d \text{ - - - - - - - - - } =6z^2({}^{z}d+{}^{2z}d+{}^{3z}d+{}^{4z}d\&c.)$$

Then taking the difference of these two equations and making $P=\frac{6}{(z+1)(z+2)}$ we deduce

$$d=P\{{}^{z}l+{}^{2z}l+{}^{3z}l\&c.-z({}^{z}d+{}^{2z}d+{}^{3z}d\&c.)-z^2({}^{z}d+2.{}^{2z}d+3.{}^{3z}d\&c.)\},$$

a general formula, by means of which the decrements at the beginning of the several intervals may be determined as before directed in page 178.

Again, if instead of finding the decrements of life at the different intervals from the enumeration, and deducing the numbers living from them in the manner already described, it were required at once to determine the numbers living, at the commencement of the various intervals, from the results of the enumeration, we should proceed as follows:

By equation C, page 177, we find that

$$a = \tfrac{z+1}{2}d + z({}^{z}d + {}^{2z}d + {}^{3z}d \&c.)$$

$$\text{or } {}^{z}d + {}^{2z}d + {}^{3z}d \&c = \tfrac{1}{z}.a - \tfrac{z+1}{2z}.d \quad - \quad - \quad - \quad \mathbf{D},$$

from which it is manifest that

$${}^{2z}d + {}^{3z}d + {}^{4z}d \&c. = \tfrac{1}{z}.{}^{z}a - \tfrac{z+1}{2z}.{}^{z}d$$

$${}^{3z}d + {}^{4z}d \&c. = \tfrac{1}{z}.{}^{2z}a - \tfrac{z+1}{2z}.{}^{2z}d$$

$${}^{4z}d \&c. = \tfrac{1}{z}.{}^{3z}a - \tfrac{z+1}{2z}.{}^{3z}d$$

$$\&c. \qquad \&c. \qquad \&c.$$

and the sum of these continued to the end of life is

$${}^{2z}d + 2.{}^{3z}d + 3.{}^{4z}d \&c. = \tfrac{1}{z}({}^{z}a + {}^{2z}a + {}^{3z}a \&c.) - \tfrac{z+1}{2z}.({}^{z}d + {}^{2z}d + {}^{3z}d \&c.),$$

which, by substituting the value of ${}^{z}d + {}^{2z}d + {}^{3z}d \&c.$ as already determined in equation D gives

$${}^{2z}d + 2.{}^{3z}d + 3.{}^{4z}d \&c = \tfrac{1}{z}({}^{z}a + {}^{2z}a + \&c) - \frac{z^2-1}{2z^2}.a + \frac{(z+1)^2}{4z^2}.d$$

But it has already been shown, in page 176, that

$$(z - \tfrac{1}{2})a - (m-n)d = {}^{z}l + {}^{2z}l \&c. - z({}^{z}a + {}^{2z}a \&c.) + m({}^{z}d + {}^{2z}d \&c.)$$

hence, by transposing and restoring the values of m and n,

$$ {}^{z}d + {}^{2z}d + {}^{3z}d\&c. = \begin{cases} \frac{2}{z-1}({}^{z}a. + {}^{2z}a + {}^{3z}a\&c.) + \frac{2z-1}{z(z-1)}.a \\ -\frac{2}{z(z-1)}.({}^{z}l + {}^{2z}l + {}^{3z}l\&c.) - \frac{2(z+1)}{3z}.d \end{cases} $$

then referring this expression to the succeeding intervals, adding the results as before, and substituting the value of ${}^{z}d + {}^{2z}d + {}^{3z}d\&c.$ as determined by equation D, we derive

$$ {}^{2z}d + 2.{}^{3z}d + 3.{}^{4z}d\&c. = $$

$$ \begin{cases} \frac{2}{z-1}.({}^{2z}a + 2.{}^{3z}a + 3.{}^{4z}a\&c. + \frac{2z-1}{z(z-1)}.({}^{z}a + {}^{2z}a + {}^{3z}a\&c.) \\ -\frac{2}{z(z-1)}.({}^{2z}l + 2.{}^{3z}l + 3.{}^{4z}l\&c.) - \frac{2(z+1)}{3z^2}d + \frac{(z+1)^2}{3z^2}.d \end{cases} $$

Whence comparing the two values of ${}^{z}d + {}^{2z}d + {}^{3z}d\&c.$ and likewise the two values of ${}^{2z}d + 2.{}^{3z}d + 3.{}^{4z}d\&c.$, d becomes exterminated, and we obtain

$$ a = p \begin{cases} 2z({}^{z}l + 2.{}^{2z}l + 3.{}^{3z}l\&c) - 2z^2({}^{z}a + 2.{}^{2z}a + 3.{}^{3z}a\&c) \\ -(z-1)({}^{z}l + {}^{2z}l + {}^{3z}l\&c) - z({}^{z}a + {}^{2z}a + {}^{3z}a\&c.) \end{cases} $$

$$ p \text{ being as before } = \frac{6}{(2z+1)(z+1)} $$

But if we suppose, as in pages 178—9, the numbers living taken at the beginning of the different years, instead of the middle, we should find in like manner

$$ a = P \times \begin{cases} z({}^{z}l + 2.{}^{2z}l + 3.{}^{3z}l\&c.) - z^2({}^{z}a + 2.{}^{2z}a. + 3\,{}^{3z}a\&c.) \\ -\frac{z-1}{2}({}^{z}l + {}^{2z}l + {}^{3z}l\&c.) - z({}^{z}a + {}^{2z}a + {}^{3z}a\&c.) \end{cases} $$

$$ P \text{ being here put for } \frac{6}{(z+1)(z+2)} $$

Having endeavoured to trace a few of the principal relations between the decrements and the numbers living in a stationary population, and to

deduce some general formulæ applicable to the construction of mortality Tables, either from the registers of deaths, or enumerations made to determine the numbers living at the several periods of life, we shall add a few remarks on the mode of determining the rate of mortality from observations on a community consisting of

6. *Increasing or decreasing Population.*

In this case it is clear that neither a register of the deaths, nor an enumeration of the numbers living alone, would enable us to determine the rate of mortality, because the relations subsisting between the numbers living and the decrements in a Table of Mortality do not obtain in such community. For instance, in an increasing population subject to a constant law of mortality at each interval, but formed by an increasing number of births, there must of necessity exist a greater number living at the younger periods of life, as compared with those of more advanced ages than what would have existed in a stationary population kept up by a constant number of annual births ; and the like disproportion must equally apply to the number of annual deaths, at the different periods from childhood to old age. Nor is it less manifest that, in a decreasing population the like disproportion must obtain in an inverted order. But if, in addition to

the results obtained from the mortuary registers, or enumerations, the proportionate number of annual births were given for a period not less than the greatest tabular duration, prior to the time at which the observation is made, such allowances might be made in the decrements, or the numbers living at the different ages, as to render them proportionate to what would have obtained with the same law of mortality in a stationary population. For example,—let us suppose that in a community subject to a constant law of mortality the number of births in the year

1824 had been	-	-	-	=	10,000
1804	-	-	-	-	-	= 9,000
1784	-	-	-	-	-	= 8,000
1764	-	-	-	-	-	= 7,000

then it follows, that in order to reduce the numbers living at the different ages, in such community, to the corresponding numbers living in a stationary population, subject to the same law of mortality, and kept up by 10,000 annual births, the number now living between the ages of 20 and 21, survivors of the children born in 1804, ought to be increased in the ratio of 9 to 10; those now living between the ages of 40 and 41, survivors of the children born in 1784, ought, in like manner, to be increased in the ratio of 8 to 10, and so on.

But, even with this mode of proceeding, our results must necessarily be affected by the various

starts of migration or emigration which may have happened, or any season of more or less than common mortality, which may have occurred within the life-time of the oldest person now living. These circumstances evidently tend to circumscribe the practical application of the formulæ deduced in the foregoing pages; to throw a degree of uncertainty on any Table of mortality formed thereby; and to render it necessary to have recourse to some other mode by which these uncertainties may be lessened, if not entirely obviated.

This object may, however, be accomplished by classifying any large community according to the ages of its members, when divided into short intervals; and deducing the law of mortality peculiar to each class, by comparing the number living with the number of deaths which may happen among the members of that class in one year. Thus referring to the Table in page 95, it is evidently immaterial, as to the law of mortality, whether we regard 40475 persons living between 30 and 40, and 750 as the number of deaths happening among that class in one year, or any other number living, greater or less than 40475, provided the number of deaths be increased or diminished in the same proportion: the law of mortality being equally represented by $\frac{40475}{750}$ or $\frac{8095}{150}$ or $\frac{1619}{30}$ &c. or by $\frac{750}{40475}=\frac{150}{8095}=\frac{30}{1619}$ &c. each of the first three fractions equally representing the proportion of the number living to the number of deaths, and each of the last three fractions the converse.

Observing, however, that a Table of mortality necessarily refers to simultaneous births, and not to a community formed by a number of births distributed throughout the year, which circumstance ought always to be regarded in adapting the latter to the former.

Let us, therefore, suppose that in any community the number living, as determined by enumeration, within the

1st interval of z years from an assigned age $= {}^{z}L$
2nd ditto - - $= {}^{2z}L$
3rd ditto - - $= {}^{3z}L$
&c. &c. &c.

and that the deaths in the corresponding intervals, as determined by the mortuary registers, were found to be respectively $= {}^{z}D$, ${}^{2z}D$, ${}^{3z}D$&c.* then will the law of mortality in the

1st class be represented by $\frac{{}^{z}L}{{}^{z}D}$ which put $= {}^{z}r$
2nd — — — — $\frac{{}^{2z}L}{{}^{2z}D}$ — — $= {}^{2z}r$
3rd — — — — $\frac{{}^{3z}L}{{}^{3z}D}$ — — $= {}^{3z}r$
&c. — — — — &c. — — &c.

* In these cases, to ensure as much accuracy as possible, the number of deaths in each interval ought to be taken in the same year as the enumeration is made, or else from an average of a certain period consisting of a given number of years before the time the enumeration is taken, and as many years after that time.

Hence putting ${}^{1}l$, ${}^{2}l$&c. for the numbers which would be living, at the like intervals, in a community consisting of a stationary population, which has been unaffected by migration &c. for about a century past, and subject to the same laws of mortality, at the respective periods of life, as those which now obtain at the corresponding periods in the community proposed, and ${}^{1}\delta$, ${}^{2}\delta$&c. for the deaths in the like intervals, it is manifest that if the community proposed be not affected by very sudden starts of emigrations &c, no material error can arise by supposing,

$${}^{1}L : {}^{1}D :: {}^{1}l : {}^{1}\delta, \quad \text{or } \frac{{}^{1}L}{{}^{1}D} = \frac{{}^{1}l}{{}^{1}\delta} = {}^{1}r$$

$${}^{2}L : {}^{2}D :: {}^{2}l : {}^{2}\delta, \quad \text{or } \frac{{}^{2}L}{{}^{2}D} = \frac{{}^{2}l}{{}^{2}\delta} = {}^{2}r$$

&c. &c. &c. &c.

Upon this supposition it therefore follows that

$${}^{1}l = {}^{1}r.{}^{1}\delta$$

$${}^{2}l = {}^{2}r.{}^{2}\delta$$

$${}^{3}l = {}^{3}r.{}^{3}\delta$$

&c. &c.

wherefore ${}^{1}l + {}^{2}l + {}^{3}l$&c. $= {}^{1}r.{}^{1}\delta + {}^{2}r.{}^{2}\delta + {}^{3}r.{}^{3}\delta$&c.

But from the nature of a Table of Mortality

${}^{1}\delta = a - {}^{1}a$, ${}^{2}\delta = {}^{1}a - {}^{2}a$, ${}^{3}\delta = {}^{2}a - {}^{3}a$, ${}^{4}\delta = {}^{3}a - {}^{4}a$&c.;

and, upon the supposition of equal decrements during the several years of which each interval is composed, it has also been shown in page 174, that

$$^{t}l + {}^{2t}l + {}^{3t}l \&c. = z\{\tfrac{1}{2}a + {}^{t}a + {}^{2t}a + {}^{3t}a \&c.\},$$

hence $z\{\frac{1}{2}a + {}^{t}a + {}^{2t}a \&c.\} = {}^{t}r(a - {}^{t}a) + {}^{2t}r({}^{t}a - {}^{2t}a) \&c.$

or $({}^{t}r - \frac{1}{2}z)a = (z + {}^{t}r - {}^{2t}r).{}^{t}a + (z + {}^{2t}r - {}^{3t}r).{}^{2t}a + (z + {}^{3t}r - {}^{4t}r).{}^{3t}a \&c.$

from which, by making $z + {}^{t}r - {}^{2t}r = {}^{t}A$, $z + {}^{2t}r - {}^{3t}r = {}^{2t}A \&c.$ and $\frac{1}{{}^{t}r - \frac{1}{2}z} = {}^{t}B$, we obtain

$$a = {}^{t}B\{{}^{t}A.{}^{t}a + {}^{2t}A.{}^{2t}a + {}^{3t}A.{}^{3t}a \&c.\}$$

a general formula, from which the numbers living at the commencement of the several intervals may be successively determined, by beginning at the oldest age and proceeding downwards, from one interval to another; the number living at the commencement of the last interval of human existence being assumed at pleasure. And when the numbers living at the commencement of the several intervals are thus determined, those at the intermediate ages from ${}^{t}a_{t}$ to ${}^{t}a$ become known by making x successively $= 1, 2, 3 \&c.$ in the expression $\frac{t}{t}\{(z-x).{}^{t}a_{t} + x.{}^{t}a\}$.

Otherwise, since $\frac{t}{2}(a + {}^{t}a) = {}^{t}l$, and $a - {}^{t}a = {}^{t}\delta$, by substituting these values in the expression ${}^{t}l = {}^{t}r.{}^{t}\delta$, we obtain

$$\frac{t}{2}(a + {}^{t}a) = {}^{t}r.(a - {}^{t}a)$$

$$\text{or} \qquad a = \frac{(2.{}^{t}r + z).{}^{t}a}{2.{}^{t}r - z}$$

And if the numbers living be referred to the beginning of the different years, we have

$$a + {}^{1}a + {}^{2}a - - - - {}^{t}a_{,} = {}^{t}l \text{ and } a - {}^{t}a = {}^{t}\delta.$$

$$\left.\begin{aligned} \text{Also } a &= a \\ {}^1a &= a-d \\ {}^2a &= a-2d \\ {}^3a &= a-3d \\ {}^za_{,} &= a-(z-1)d \end{aligned}\right\} \begin{aligned} &\text{and } {}^za = a - zd \\ &\text{or } d = \frac{a-{}^za}{z} \end{aligned}$$

the sum of which gives

$$a + {}^1a + {}^2a \text{ - - - - } {}^za_{,} = za - \tfrac{z(z-1)}{2}.d;$$

hence ${}^zl = za - \frac{z-1}{2}.(a - {}^za) = \frac{1}{2}\{(z+1)a + (z-1).{}^za\}$,

and by substitution we further obtain

$$\tfrac{1}{2}\{(z+1)a + (z-1).{}^za\} = {}^zr(a - {}^za) = \tfrac{z}{2}(d + {}^za).$$

See 194. $\quad$ or $a = \dfrac{\{2.{}^zr + z - 1\}.{}^za}{2.{}^zr - (z+1)}$ $\quad$ $* = {}^zr\,{}^zd$

By a similar mode of proceeding, upon the supposition of progressive decrements, general formulæ might be deduced for determining, in succession, either the numbers living or the decrements at the commencement of the several intervals from old age to the younger periods of life. But the unavoidable complexity of such formulæ would render them less convenient in their practical application than those subjoined, which are derived as before from the relation $\dfrac{{}^zL}{{}^zD} = \dfrac{{}^zl}{{}^z\delta}$.

7. It has been shown in the foregoing pages, that $a + {}^1a + {}^2a \text{ - - - - } {}^za_{,} - \frac{1}{2}(d + {}^1d + {}^2d \text{ - - - - } {}^zd_{,}) = {}^zl$, and it is evident that $d + {}^1d + {}^2d \text{ - - - - } {}^zd_{,} = {}^z\delta$, hence dividing the former of these equations by the latter we obtain

$$\frac{a + {}^1a + {}^2a \text{ - - - - } {}^za_{,}}{d + {}^1d + {}^2d \text{ - - - - } {}^zd_{,}} - \tfrac{1}{2} = \frac{{}^zl}{{}^z\delta} = \frac{{}^zL}{{}^zD}$$

See 199

Beg of year $\quad \dfrac{a + {}^1a + {}^2a + \dots + {}^za_{,}}{d + {}^1d + {}^2d + \dots + {}^zd_{,}} = \dfrac{{}^zl}{{}^z\delta} = \dfrac{{}^zL}{{}^zD} = q$

or $\frac{a+{}^1a+{}^2a\ldots{}^za}{d+{}^1d+{}^2d\ldots{}^zd}=\frac{{}^zL}{{}^zD}+\frac{1}{2}$ which put $=q$,

then $a+{}^1a+{}^2a\ldots{}^za=q.{}^z\delta=q(a-{}^za)$,

or by substituting the value of $a+{}^1a+{}^2a\ldots{}^za$ as already determined in page 176, we have

$q(a-{}^za)=za-\frac{z(z-1)}{2}.d-\frac{(z-1)(z-2)}{6}({}^zd-d)$ from which

$6(q-z)a+2(z-1)(z+1)d=6q.{}^za-(z-1)(z-2).{}^zd\ldots$ **E.**

But, on the supposition of progressive decrements, we have shown in page 177 that

${}^za=a-zd-\frac{z-1}{2}({}^zd-d)$, from which we deduce

$2a-(z+1)d=2.{}^za+(z-1).{}^zd\ldots$ **F,**

hence referring this equation to the beginning of the last interval of human life, wherein ${}^za$ and ${}^zd$ both vanish, we find that $a=\frac{z+1}{2}d$, as may be otherwise derived from equation C, page 177; and from this it is manifest that if d be assumed at pleasure a becomes known, and *vice versâ*.

Again, referring the two equations marked E and F to the commencement of the preceding interval, ${}^za$, and ${}^zd$, may be considered as known, and a and d as the quantities required to be found; hence by comparing the two equations involving these quantities, we obtain

$$a=\frac{(6q+4z-4).{}^za+z(z-1).{}^zd}{6q-2z-4}$$

$$d=\frac{6z.{}^za-(z-1)(3q-2z-2).{}^zd}{(z+1)(6q-2z-4)}$$

By these expressions the number living and the

decrement may be determined, by proceeding from one interval to another, from old age towards the younger period of life; nor is it here necessary that all the intervals should be equal, the formulæ being alike applicable, however unequal the intervals may be, provided the quantity z be varied accordingly.

To these may be added the following expressions, which, on some occasions, may be found useful:—

$$q = \frac{2(z+2)a + 4(z-1).^{z}a + z(z-1).^{z}d}{6(a - {}^{z}a)}$$

$$q.^{z}\delta = \frac{2(z+2)a + 4(z-1).^{z}a + z(z-1).^{z}d}{6}$$

$$d = \frac{2(a - {}^{z}a) - (z-1).^{z}d}{z+1}$$

Thus if z be taken equal ten years we have

$$a = \frac{(q+6).^{z}a + 15.^{z}d}{q-4}, \text{ and } d = \frac{20.^{z}a - 3(3q-22).^{z}d}{11(q-4)}$$

$$q = \frac{4a + 6.^{z}a + 15.^{z}d}{a - {}^{z}a}, \text{ and } d = \frac{2(a - {}^{z}a) - 9.^{z}d}{11}$$

And if z be taken equal five years, we obtain

$$a = \frac{(3q+8).^{z}a + 10.^{z}d}{3q-7}, \text{ and } d = \frac{5.^{z}a - 2(q-4).^{z}d}{3q-7}$$

$$q = \frac{7a + 8.^{z}a + 10.^{z}d}{3(a - {}^{z}a)}, \text{ and } d = \frac{a - {}^{z}a - 2.^{z}d}{3}.$$

And if in the foregoing expressions we refer the number living to the beginning, instead of the middle, of the year, we have $a + {}^{1}a + {}^{2}a - - - - {}^{z}a_{,} = {}^{z}l$; and we have already shown in page 189, that $a + {}^{1}a + {}^{2}a - - - - {}^{z}a_{,} = q.^{z}\delta$; it therefore follows, that when the numbers living are so taken

$q.{}^z\delta = {}^z l$, or $q = \frac{{}^z l}{{}^z\delta} = \frac{{}^z L}{{}^z D}$, and not $= \frac{{}^z L}{{}^z D} + \frac{1}{2}$

hence, substituting ${}^z l$ for $q.{}^z\delta$ in the foregoing expressions, we deduce, upon the supposition here made

$$ {}^z l = \frac{2(z+2)a + 4(z-1).{}^z a + z(z-1).{}^z d}{6} $$

And if in this formula there be substituted

$z = 10$, then ${}^z l = 4a + 6.{}^z a + 15.{}^z d$,

$z = 5$, — ${}^z l = \frac{7a + 8.{}^z a + 10.{}^z d}{3}$

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8. *Of Interpolation.*

When the numbers living at the commencement of the different intervals shall have been determined, those at the intermediate ages may be otherwise interpolated as follows.—Thus in the general equation $y = A - {}^1Ax + {}^2Ax^2 - {}^3Ax^3$ &c. let y denote the number living at an age x years older than that of any assigned life A, and suppose that when

$x = 0$, $y = a$
$x = q$, $y = b$
$x = r$, $y = c$
$x = s$, $y = d$
&c. &c. &c. &c.

Then by substituting these values in the general equation, we obtain

$$1.\ a = A$$
$$2.\ b = A - {}^1Aq + {}^2Aq^2 - {}^3Aq^3 \&c.$$
$$3.\ c = A - {}^1Ar + {}^2Ar^2 - {}^3Ar^3 \&c.$$
$$4.\ d = A - {}^1As + {}^2As^2 - {}^3As^3 \&c.$$
$$\&c. \qquad \&c. \qquad \&c.$$

as so many independent equations from which the values of the coefficients ${}^1A$, ${}^2A$, ${}^3A$&c. may be determined; the value of A being already found by equation 1st $= a$.

For example,—let us suppose four terms a, b, c and d to be given, then by deducting the second, third, and fourth equations severally from the first, and dividing the remainders respectively by q, r and s, we have

$$B = \frac{a-b}{q} = {}^1A - {}^2Aq + {}^3Aq^2 \quad - - - - - \mathbf{G}$$
$${}^1B = \frac{a-c}{r} = {}^1A - {}^2Ar + {}^3Ar^2$$
$${}^2B = \frac{a-d}{s} = {}^1A - {}^2As + {}^3As^2$$

Also deducting the second of these equations from the first, the third from the second, and dividing the remainders respectively by $r-q$ and $s-r$ we obtain

$$C = \frac{B - {}^1B}{r-q} = {}^2A - {}^3A(r+q) \quad - - - - \mathbf{H}$$
$${}^1C = \frac{{}^1B - {}^2B}{s-r} = {}^2A - {}^3A(s+r)$$

Finally, deducting the second of these equations from the first, and dividing the remainder by $s-q$, we derive $D = \frac{C - {}^1C}{s-q} = {}^3A$

And this value of ${}^3A$ being substituted in equation H, gives ${}^2A = C + {}^3A(r+q) = C + D(r+q)$; also substituting the values of ${}^3A$ and ${}^2A$ in equation G, we have ${}^1A = B + {}^2Aq - {}^3Aq^2 = B + q(C + Dr)$.

The same mode of proceeding is evidently applicable to any other number of terms; but the increased labour of raising the quantity x, for each age, to the several powers contained from unity to the number of intervals included between the extreme terms, renders it inexpedient in practice to embrace more than three or four intervals at a time.

$${}^1A = B + {}^2Aq - {}^3Aq^2 = B + qC + q^3A(r+q) - q^3A$$
$$= B + qC + rq^3A = B + q(C + r^3A)$$
$$= B + q(C + r$$

9. *Construction of the Equitable Table.*

To show the practical application of the foregoing Theories, let us again refer to Mr. Morgan's statements, given at the beginning of this Chapter. By these statements it appears, that the mortality among the members of the Equitable, for a period exceeding half a century, has been to that expressed by the Northampton Table,

from	10	to	20	as 1	to 2,	or as 5 to 10,
	20	to	30	as 1	to 2,	or as 5 to 10,
	30	to	40	as 3	to 5,	or as 6 to 10,
	40	to	50	as 3	to 5,	or as 6 to 10,
	50	to	60	as 5	to 7,	or as 7 to 10,
	60	to	80	as 4	to 5,	or as 8 to 10,
	80 and upwards progressively as 9 to 10.					

The easiest mode of deducing the Rate of Mortality from these data, is by means of the Formula 188. $a=\frac{(2.{}^{z}r+z-1).{}^{z}a}{2.{}^{z}r-(z+1)}$ by referring the numbers living to the beginning of the year, and making $z=1$, in which case the general formula is transformed into

$a=\frac{2.{}^{z}r.{}^{z}a}{2.{}^{z}r-2}$, which by dividing both numerator and denominator by $2.{}^{z}r$, and making $1-\frac{1}{{}^{z}r}=u$ is further simplified into $a=\frac{{}^{1}a}{u}$.

But when $z=1$, ${}^{z}r=\frac{a}{d}$ and consequently $\frac{1}{{}^{z}r}=\frac{d}{a}$; hence, supposing a Table, analagous to that numbered XII. at the end of this work, to be formed from the Northampton Table, by dividing the decrement at each age by the number living at that age, we should find that, according to the Northampton Table, at the age of

96, - - - $\frac{d}{a}$	$=1.00000$, $\frac{9}{10}$ of which is	$=.900000=\frac{1}{{}^{z}r}$
95 - - - —	$=.750000$, —	—	$=.675000=$ —
94 - - - —	$=.555556$, —	—	$=.500000=$ —
93 - - - —	$=.437500$, —	—	$=.393750=$ —
92 - - - —	$=.333333$, —	—	$=.300000=$ —
91 - - - —	$=.294118$, —	—	$=.264706=$ —
90 - - - —	$=.260869$, —	—	$=.234782=$ —
&c.	&c.		&c.

Then deducting each of the numbers in the last column from 1, we should find that at the age of

96,	$u = 1-\frac{1}{r}$	$= .100000$
95	- - - -	$= .325000$
94	- - - -	$= .500000$
93	- - - -	$= .606250$
92	- - - -	$= .700000$
91	- - - -	$= .735294$
90	- - - -	$= .765218$
&c.		&c.

whence, assuming the number living at $97 = v = {}^1a$, we should find the number living at the age of

96	$= a = \frac{v}{.100000}$	$= 10.00v$	
95	- - $= \frac{10v}{.325000}$	$= 30.77v$	
94	- - $= \frac{30.77v}{.500000}$	$= 61.54v$	
93	- - $= \frac{61.54v}{.606250}$	$= 101.51v$	
92	- - $= \frac{101.51v}{.700000}$	$= 145.01v$	
91	- - $= \frac{145.01v}{.735294}$	$= 197.21v$	
90	- - $= \frac{197.21v}{.765218}$	$= 257.72v$	
&c.	&c.	&c.	

This process, continued from age to age, to the

younger periods of life, and the value of v assumed at pleasure, would form a Table very nearly agreeing with the data stated by Mr. Morgan, as may be proved by any one who may choose to complete the operations.

In the next place, let the formula $\frac{(2.{}^{z}r+z-1).{}^{z}a}{2.{}^{z}r-(z+1)}$ be referred to the intervals for which the value of ${}^{z}r$ has been given in page 97; then representing the number living at 80, the commencement of the last interval, by v, we have the number living at

$$60 = \frac{(2\times 20.306+19)v}{2\times 20.306-21} = 3.040v$$

$$50 = \frac{2\times 42.545+9}{2\times 42.545-11}\times 3.040v = 3.861v$$

$$40 = \frac{2\times 70.407+9}{2\times 70.407-11}\times 3.861v = 4.455v$$

$$30 = \frac{2\times 89.940+9}{2\times 89.940-11}\times 4.455v = 4.982v$$

$$20 = \frac{2\times 128.438+9}{2\times 128.438-11}\times 4.982v = 5.386v$$

$$10 = \frac{2\times 200.530+9}{2\times 200.530-11}\times 5.386v = 5.661v$$

Hence assuming the value of v at pleasure, the numbers living at 10, 20, 30, 40, 50, 60 and 80 become known; and the numbers living at the intermediate ages in each interval may be found, by making x successively $= 1, 2, 3, - - - - z$ in the expression $a-\frac{x}{z}(a-{}^{z}a)$ or in $\frac{1}{z}\{(z-x)a+x.{}^{z}a)\}$

Thus if $5.661v$ be made $= 2844$, the number living at 10, in Table x, we have $v = \frac{2844}{5.661} = 502.27$.

And from this value we derive the number living at the age of

20	= 2705	50	= 1939
30	= 2502	60	= 1527
40	= 2237	80	= 502

all, excepting the last, very nearly agreeing with the numbers given at the like ages in the Table showing the Rate of Mortality among the members of the Equitable.

But if the latter period of life be divided into shorter intervals, and the value of q be deduced for each interval from the proportions stated in page 193, the Rate of Mortality may be approximated still nearer.* Thus, by comparing the numbers living in the Northampton Table with the decrements in the same, we have the Rate of Mortality

from 85 to 90 = 4.2, $\frac{10}{9}$ of which is $4.66 = q$

80 to 85 = 6.16, $\frac{10}{9}$ of which is $6.85 = q$

70 to 80 = 11.47, $\frac{5}{4}$ of which is $14.34 = q$

60 to 70 = 20.76, $\frac{5}{4}$ of which is $25.95 = q$

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* We should here remark that the proportions above referred to, as obtaining for a considerable interval, are not mathematically correct when referred to the several portions of that interval; but the error arising from such supposition is unimportant in calculations where some degree of latitude must necessarily be allowed.

These numbers being substituted for q in the formula $a = \frac{(2.r+z-1).a}{2.r-(z+1)}$, and the number living at 90 being assumed $= v$, we have the number living at the age of

$$90 = \text{- - - - - - - - } v$$

$$85 = \frac{(9.32+4)v}{9.32-6} = \text{- - - } 4.01v$$

$$80 = \frac{13.70+4}{13.70-6} \times 4.01v = 9.22v$$

$$70 = \frac{28.68+9}{28.68-11} \times 9.22v = 19.65v$$

$$60 = \frac{51.90+9}{51.90-11} \times 19.65v = 29.26v$$

$$50 = \frac{85.09+9}{85.09-11} \times 29.26v = 37.16v$$

$$40 = \frac{140.81+9}{140.81-11} \times 37.16v = 42.89v$$

$$30 = \frac{179.88+9}{179.88-11} \times 42.89v = 47.97v$$

$$20 = \frac{256.88+9}{256.88-11} \times 47.97v = 51.87v$$

$$10 = \frac{401.06+9}{401.06-11} \times 51.87v = 54.53v$$

And if in these expressions the value of v be assumed at pleasure, the numbers living at the beginning of the several intervals become known. Thus if $54.53v$ be made $= 2844$ we have $v = 52.155$,

and substituting this value of v, we find as before, the number living at the age of

$$10 = 54.53 \times 52.155 = 2844$$
$$20 = 51.87 \times 52.155 = 2705$$
$$30 = 47.97 \times 52.155 = 2502$$
$$40 = 42.89 \times 52.155 = 2237$$
$$50 = 37.16 \times 52.155 = 1938$$
$$60 = 29.26 \times 52.155 = 1526$$
$$70 = 19.65 \times 52.155 = 1025$$
$$80 = 9.22 \times 52.155 = 481$$
$$85 = 4.01 \times 52.155 = 209$$
$$90 = 1 \times 52.155 = 52$$

now agreeing up to the age of 80, as nearly as possible with the numbers given in Table x.

Again, upon the supposition of progressive decrements, assuming the limiting age at 97, and referring to the age of 90, we have $z=7$; and by the formula $a=\frac{z+1}{2}.d$, putting d = any arbitrary quantity v, we have $a=4v$; also the aggregate number living from 90 upwards found by the formula

$$^{z}l=\tfrac{1}{6}\{2(z+2)a+4(z-1).{}^{z}a+z(z-1).{}^{z}d\}$$

wherein ^{z}a and ^{z}d each $= 0$, and $a = 4v$, becomes $= 3a = 12v$.

In the next place, referring to the age of 85, we have q, already found in page 197, $= 4.66 = 4\frac{2}{3}$.

Hence - - - - $a=\frac{(3q+8).{}^{\cdot}a+10.{}^{\cdot}d}{3q-7}=14v$;

and consequently - $d=\frac{a-{}^{\cdot}a-2.{}^{\cdot}d}{3}=2\frac{2}{3}v$.

Also the aggregate numbers living from 85 to 90 $=\frac{7a+8.{}^{\cdot}a+10.{}^{\cdot}d}{3}=\frac{140v}{3}=46\frac{2}{3}v$; and the aggregate numbers living from 80 to 85, found by the same formula, referred to the age of 80, is $\frac{7a}{3}+46.22v$; to which adding the $46\frac{2}{3}v$ living from 85 to 90, and the $12v$ living from 90 upwards, we have the total number living from 80 upwards

$$=\tfrac{7a}{3}+46.22v+46.67v+12v=\tfrac{7a}{3}+105v.$$

Hence in the equation ${}^{\cdot}l=q.{}^{\cdot}\delta$, we have ${}^{\cdot}l=\frac{7a}{3}+105v$, $q=5.84$, as stated in page 98, and ${}^{\cdot}\delta=a$; consequently $5.84a=\frac{7a}{3}+105v$, an equation from which $a=30v$, and $d=\frac{a-{}^{\cdot}a-2.{}^{\cdot}d}{3}=\frac{30v-14v-5.3v}{3}=3.5v$

Again, referring to the age of 70, we have the value of q from 70 to 80 found in page 197, $=14.34$ or $14\frac{1}{4}$. Hence $a=\frac{(q+6).{}^{\cdot}a+15.{}^{\cdot}d}{q-4}=64\frac{1}{4}v$: and

$$d=\frac{2(a-{}^{\cdot}a)-9.{}^{\cdot}d}{11}=\frac{2(64\frac{1}{4}v-30v)-31.5v}{11}=3.4v.$$

Also the aggregate number living from 70 to $80=4a+6.{}^{\cdot}a+15.{}^{\cdot}d=257v+180v+53v=490v$; and the number living from 60 to 70, found by the same formula, referred to the age of 60, is $4a+437v$; which, being added to the former result, gives the aggregate numbers living at all ages from 60 to 80

$= 4a + 927v$: hence making this aggregate $= {}^{z}l$, $a - {}^{z}a = a - 30v = {}^{z}\delta$, and $q = 20.306$, or $20\frac{1}{4}$, as stated in page 97, we have $4a + 927v = 20\frac{1}{4}(a - 30v)$, an equation from which a is found $= 95\frac{1}{4}v$, and consequently $d = \frac{2(a - {}^{z}a) - 9.{}^{z}d}{11} = 2.8v$.

Moreover, referring to the age of 50 we have $q = 42.545$, or $42\frac{1}{2}$, as stated in page 97, ${}^{z}a = 95\frac{1}{4}v$, and ${}^{z}d = 2.8v$; whence $a = \frac{(q+6).{}^{z}a + 15.{}^{z}d}{q-4} = 121.1v$, and $d = \frac{2(a - {}^{z}a) - 9.{}^{z}d}{11} = 2.4v$.

Referring in like manner to the ages of 40, 30, 20, and 10, and taking the successive values of $q = 70.407$, 89.940, 128.438 and 200.530, as stated in page 97, or more convenient for calculation and sufficiently near for any practical purpose, taking for the values of q, $70\frac{1}{2}$, 90, $128\frac{1}{2}$, and $200\frac{1}{2}$, we find by the same formula, that at the age of

40,	$a = 139\frac{3}{4}v$,	$d = 1.5v$
30,	$a = 156.3v$,	$d = 1.8v$
20,	$a = 169.1v$,	$d = .9v$
10,	$a = 177\frac{3}{4}v$,	$d = .8v$

Having thus determined the numbers living at the commencement of the different intervals in terms of the arbitrary quantity v, let the value of that quantity be assumed at pleasure. Thus if $v = 16$, then the number living at the age of

10	=	$177\frac{3}{4} \times 16$	=	2844
20	=	169.1×16	=	2705
30	=	156.3×16	=	2501
40	=	$139\frac{3}{4} \times 16$	=	2236
50	=	121.1×16	=	1937
60	=	$95\frac{1}{4} \times 16$	=	1524
70	=	$64\frac{1}{4} \times 16$	=	1028
80	=	30.0×16	=	480
85	=	14.0×16	=	224
90	=	4.0×16	=	64

The numbers living at the commencement of the different intervals having been thus determined, in order to ensure as much regularity as possible in the decrements, so as to avoid the incongruities mentioned in page 170, it has been found necessary to deduce the numbers living at the intermediate ages, by the method of interpolation laid down in section VIII, page 191, rather than to deduce them from the decrements already found in terms of v.

Thus taking the first three intervals from the age of 10, we have

$a = 2844, \quad q = 10, \quad B = \frac{a-b}{q} = 13.90$

$b = 2705, \quad r = 20, \quad {}^1B = \frac{a-c}{r} = 17.15$

$c = 2501, \quad s = 30, \quad {}^2B = \frac{a-d}{s} = 20.27$

$d = 2236$

$$C = \frac{B - {}^1B}{r-q} = \frac{13.90 - 17.15}{10} = -.325$$

$${}^1C = \frac{{}^1B - {}^2B}{s-q} = \frac{17.15 - 20.27}{10} = -.312$$

$$D = \frac{C - {}^1C}{s-q} = \frac{-.325 + .312}{20} = -.00065$$

Hence ${}^3A = D = -.00065$

$${}^2A = C + D(r+q) = -.3445$$

$${}^1A = B + q(C + Dr) = 10.52$$

And by substituting these values for ${}^1A$, ${}^2A$ & ${}^3A$, and also 2844 for A, in the general equation

$y = A - {}^1Ax + {}^2Ax^2 - {}^3Ax^3$, we have

$y = 2844 - 10.52x - .3445x^2 + .00065x^3$,

an equation by which the numbers living at the several intermediate ages from 10 to 40 have been determined. Similarly by putting a, b, c & d for the numbers living at 40, 50, 60 & 70, and finding therefrom the coefficients ${}^1A$, ${}^2A$ & ${}^3A$ in the foregoing equation, the numbers living at the several intermediate ages from 40 to 70 have been deduced. This done, and the like process applied to the numbers living at 70, 80, 85 & 90, the numbers living up to the age of 90 have been determined, while those above the age of 90 have been assumed, in a great measure, arbitrarily: nor ought it to be concealed that, after the work was done, and the decrements deduced for all ages, some trifling modifications were made therein so as to render them more progressive, but not in any case altering the

numbers living above 1 or 2, from what had been obtained by calculation.

It may also be considered necessary to remark, that the numbers living have been here referred to the beginning of the different years, merely for the purpose of facilitating the calculations, as it is evidently indifferent, when one Table of Mortality is deduced from the relation it bears to another, whether the numbers living be referred to the beginning or to the middle of the year, provided the same principle be applied to both. In other cases, however, we must not forget, that the numbers living ought to be referred to the middle of the year, and consequently q taken $= \frac{{}^{z}L}{{}^{z}D} + \frac{1}{2}$ and not $= \frac{{}^{z}L}{{}^{z}D}$.

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10. *On the Rate of Mortality in Sweden and Finland.*

As an exercise to the learner, in the construction of Tables of Mortality, and, probably, the best data hitherto furnished for determining the comparative longevity of males and females, the following abstract, taken from the Supplement to the Encyclopædia Britannica, has been subjoined.

By this abstract it appears that in all Sweden and Finland, for the five years ending with 1805,

Between the Ages of	Mean number of the living.		Annual average number of deaths.		That is, one out of 'r.	
	Males.	Females.	Males.	Females.	Males.	Females.
0 and 1	44536	43847	11132	9238	4.00	4.74*
1 — 3	85548	86533	4113	3752	20.79	23.06
3 — 5	84854	85909	1857	1771	45.69	48.57
5 — 10	170878	171343	1919	1743	89.04	98.30
10 — 15	161613	160777	872	797	185.33	201.72
15 — 20	140467	144782	799	795	175.80	182.11
20 — 25	132414	143012	1018	927	130.07	154.27
25 — 30	120349	130183	977	978	123.18	133.11
30 — 35	100293	118978	982	1056	110.79	112.67
35 — 40	108804	111158	1078	1150	93.03	96.06
40 — 45	94497	103711	1293	1324	73.08	78.33
45 — 50	82258	91932	1442	1255	57.04	73.25
50 — 55	71899	81265	1811	1582	39.70	51.36
55 — 60	54543	64127	1768	1666	30.85	38.49
60 — 65	42847	51938	1931	2015	22.19	25.77
65 — 70	30923	40414	1942	2242	15.92	18.02
70 — 75	20945	28615	2138	2620	9.79	10.92
75 — 80	11009	15660	1627	2135	6.76	7.33
80 — 85	4452	6817	994	1452	4.47	4.69
85 — 90	1214	1988	352	561	3.45	3.54
90, &c.	268	468	102	207	2.62	2.26

From the last two columns, by adding $\frac{1}{2}$ to each of the several numbers therein given, and using the result in each case as the value of q, Tables may be formed by the formulæ $a = \frac{(3q+8).^{r}a + 10.^{r}d}{3q-7}$ and $d = \frac{a - {}^{r}a - 2.^{r}d}{3}$, given in page 190, without any modification of the results.

Valuable Tables for Males and Females sepa-

* Were an Insurance Office to classify the persons insured according to their dates of births, and compare the decrements at each age with the number living for some years, valuable data might thereby be acquired for determining the rate of mortality among assured lives.

rately, and for both collectively, have been constructed by Mr. Milne from these data, which Tables will be found in the work here referred to; and any person disposed to undertake the labour of constructing Annuity Tables, grounded upon them, would thereby form an important addition to the Tables already published.

By the Tables which Mr. Milne constructed from the foregoing data, it appears that the Expectation of life in Sweden and Finland is

At the age of	Among Males.	Among Females.	Among Both.
0	37.82	41.02	39.38
5	48.99	51.05	50.01
10	46.68	48.57	47.63
15	42.89	44.73	43.81
20	39.05	40.90	39.98
25	35.49	37.17	36.33
30	31.85	33.49	32.68
35	28.21	29.90	29.06
40	24.62	26.35	25.49
45	21.19	22.92	22.07
50	17.90	19.37	18.65
55	14.97	16.09	15.55
60	12.17	12.98	12.60
65	9.61	10.22	9.93
70	7.25	7.70	7.50
75	5.51	5.78	5.66
80	4.09	4.22	4.16
85	3.23	3.23	3.23
90	2.55	2.26	2.36
95	1.70	1.70	1.70 †

† By this Table it appears, that Female Life is considerably better than Male; and by comparing the Expectation here given, with that deduced from the Swedish Tables found in the Works of Price and Milne, it will be discovered that, up to about the age of 40, a gradual improvement has taken place in human life, and that after that period the Expectation remains nearly the same as it was about 60 years ago.

11. *On the Rate of Mortality among the general mass of population in England and Wales.*

By the Population Abstract of 1821, it will be found, that in England and Wales the number of persons whose ages were returned was as follows;—

	Males.	Females.
Under the Age of 5 Years.	791579	774689
Between 5 and 10 —	693858	682457
—— 10 — 15 —	603613	569366
—— 15 — 20 —	509586	535569
—— 20 — 30 —	755780	901338
—— 30 — 40 —	593662	649507
—— 40 — 50 —	482329	500977
—— 50 — 60 —	342204	352160
—— 60 — 70 —	231509	249184
—— 70 — 80 —	115032	124648
—— 80 — 90 —	29587	36315
—— 90 — 100 —	2253	3280
100 and upwards.	60	129
	5,151,052	5,379,619

making an aggregate of 10,530,671 persons.

But even this enumeration, valuable as it is, will scarcely enable us to deduce any certain conclusion concerning the Rate of Mortality; for it is manifest, by the statements made in pages 87 and 88, that the population of England and Wales has, at least for some time, been rapidly increasing, and the rate of mortality at the earlier stages of life has, for some years, gradually decreased. Added to these circumstances, a considerable deficiency in the number returned of the male sex, must necessarily have

arisen from the exclusion of those employed in the Army and Navy at the time the enumeration was made.

And by the above abstract it would seem that, either from the disposition of the Female sex to conceal their ages, if above 30, or from some other cause, the number living of that sex between 20 & 30, is considerably greater in proportion to the number of males than what might be expected from the proportions which obtain among the two sexes in the contiguous intervals.

The number between 20 & 30 being stated at	901338
— — — 30 & 40 - - - - -	649507
Total in the 20 years - - - - - - -	1550845
Making an average for each year of - -	77542
For half a year - - - - - - - - -	38771

Supposing, therefore, only those that were between the age of 30 and $30\frac{1}{2}$ years, to have represented themselves as under 30, by deducting 38771 from the number living between 20 & 30, and adding it to those living between 30 & 40, we shall find the number living between 20 & 30 = 862567, and the number living between 30 & 40 = 688278. Nor is it less perceptible in these returns that some of the males, really above 15, must, from fear of being drawn for the militia, have represented themselves under that age. The error from this source is, however, not sufficiently important for us to attempt a correction, as our object is more to form a rough

guess, than to deduce an accurate Table, from data confessedly insufficient for that purpose.

Again, by the general summary in page 542 of the work referred to, it is stated, that the total number of persons in Great Britain, in 1821, was in

England - - - - - -	11,261,437
Wales - - - - - - -	717,438
Scotland - - - - - -	2,093,456
	14,072,331
In the Army, Navy, &c.	319,300

Hence, supposing one-eighth of the persons employed in the "army, navy, marines, and seamen in registered vessels" to be Irish and Foreigners, the remaining 279,388 considered as belonging to Great Britain, amount to about one-fiftieth of the resident inhabitants.

The like proportion of those whose ages were returned being taken, gives about 210,000 males to be apportioned amongst the different classes given in page 207. And if for the sake of diminishing the number of figures we take a hundredth part of this number, and the like proportion of the numbers given by the enumeration, classifying them into decades, correcting the number of females as in page 208, and regarding that most striking law of nature which prescribes the relation between the number of males and that of females, it is possible that we shall not materially err in allotting the

above number, now reduced to 2100, in the following proportions, viz. 100 between 10 and 20

1000	——	20	-	30
800	——	30	-	40
150	——	40	-	50
50	——	50	-	60

This being assumed, the proportionate numbers living in England and Wales, in 1821, will stand thus :

	Males.	Females.	Both.
Under 10 - -	14854	14571	29425
between 10 and 20	11232	11049	22281
—— 20 - 30	8558	8626	17184
—— 30 - 40	6737	6883	13620
—— 40 - 50	4973	5010	9983
—— 50 - 60	3472	3522	6994
—— 60 - 70	2315	2492	4807
—— 70 - 80	1150	1246	2396
—— 80 - 90	296	363	659
—— 90 - 100	23	33	56
Totals - -	53610	53795	107405

The next correction to be applied arises from the unequal number of births from which these different classes are produced. Thus it is manifest that all those whose ages were under 10 about May, 1821, must have been born between May, 1811, and May, 1821; while those whose ages at the same period were between 10 and 20, must have been born between May, 1801, and May, 1811. In the same manner it may be shown that those whose ages were between 20 and 30 in May, 1821, must have been born between May, 1791, and May, 1801, and so on.

But by classifying the average annual baptisms (given in page 87) into decades, and supposing those baptisms to be in a constant ratio to the births, it will be found that

for - 325,506	births between	- 1810 & 1820
only 287,890	— took place between	1800 - 1810
— 261,776	- - - -	1790 - 1800
— 239,714	- - - -	1780 - 1790
&c.		&c. &c.

Supposing, therefore, that the rate of mortality had remained constant, it follows, that in order to reduce the several classes before stated to their corresponding numbers, upon the supposition that the average number of annual births from 1720 to 1820 was equal to that which obtained from 1810 to 1820, the numbers living in the several classes must be increased as follows:

287890 : 325506 : : 11232 : 12700

261776 : 325506 : : 8558 : 10641

239714 : 325506 : : 6737 : 9147

and so on. It therefore appears, that had the number of annual births throughout the last century been constantly the same as the annual average between 1810 and 1820, and the rate of mortality continued invariable during that period, the proportionate number of males living in 1821 would have been

under 10 - - - 14854
between 10 and 20 - 12700 } and so on.
—— 20 — 30 - 10641

The same process applied to the different classes both of males and females, would lead us to conclude that the proportionate numbers living at the different ages in the year 1821, had the number of annual births been constant for the preceding 100 years, would have been

	Males.	Females.
Under 10 - -	14854	14571
between 10 and 20	12700	12493
—— 20 - 30	10641	10725
—— 30 - 40	9147	9344
—— 40 - 50	7327	7383
—— 50 - 60	5563	5642
—— 60 - 70	3979	4282
—— 70 - 80	2079	2252
—— 80 - 90	565	694
—— 90 - 100	46	66

Again referring the formula $\frac{z}{2}(a+{}^{z}a)={}^{z}l$ to birth, and making $z=5$ years, we have

$\frac{5}{2}(a+{}^{5}a)={}^{5}l$ = number living from 0 to 5

and $\frac{5}{2}({}^{5}a+{}^{10}a)={}^{10}l=$ ditto - - 5 to 10

Hence $\frac{5}{2}(a+2.{}^{5}a+{}^{10}a)=$ ditto from - - 0 to 10

$\frac{5}{2}(a-{}^{10}a)={}^{5}l-{}^{10}l$

and $a-{}^{10}a=\frac{2}{5}({}^{5}l-{}^{10}l)$ = the number of deaths under the age of 10. This result compared with the number living under 10 years gives $\frac{\frac{2}{5}({}^{5}l-{}^{10}l)}{{}^{5}l+{}^{10}l}$

$$=\frac{2}{5}\left\{\frac{15663-13763}{15663+13763}\right\}=\frac{2}{5}\left\{\frac{1900}{29426}\right\}=.02582$$

for the rate of mortality among children under 10, during the ten years preceding 1821, say from the

end of 1810 to the end of 1820. Hence putting this number $= q$, and supposing the rate of mortality at this period of life to have varied during the 100 years preceding 1821, in the ratio expressed by the numbers deduced in page 88, we have

$$\tfrac{1}{2}(623+612) : \tfrac{1}{2}(659+697)$$

or 618 : 678 : : q : 1.097q for the rate of mortality, under the age of 10, from 1800 to 1810,

618 :	747 :	: q :	1.209.q	= do. from	1790	to 1800
618 :	787 :	: q :	1.273.q	—	1780	- 1790
618 :	857 :	: q :	1.387.q	—	1770	- 1780
618 :	840 :	: q :	1.359.q	—	1760	- 1770
618 :	848 :	: q :	1.372.q	—	1750	- 1760
618 :	924 :	: q :	1.495.q	—	1740	- 1750
618 :	1043 :	: q :	1.688.q	—	1730	- 1740
618 :	1068 :	: q :	1.728.q	—	1720	- 1730

But, upon the supposition of equal decrements for the whole 10 years, the expression $\frac{5}{2}(a+2.{}^{5}a+{}^{10}a)$ becomes $=5(a+{}^{10}a)={}^{5}l+{}^{10}l$, and $q=\frac{\frac{2}{5}({}^{5}l-{}^{10}l)}{{}^{5}l+{}^{10}l}=\frac{a-{}^{10}a}{5(a+{}^{10}a)}$

wherefore $5q(a+{}^{10}a)=a-{}^{10}a$, from which we derive ${}^{10}a=\frac{a(1-5q)}{1+5q}$ which by substituting the value of q becomes $=\frac{a(1-\cdot1291)}{1+\cdot1291}=\frac{a(.8709)}{1.1291}=a(.77132)$,

for the proportion of the children born between 1810 and 1820, which may be computed to attain the age of 10 years. And by substituting 1.097q, 1·209q 1.273q &c. in succession for q in the formula

$^{10}a = \frac{a(1-5q)}{1+5q}$ we obtain the proportion which attained the age of 10 years, of a children born

between	1800	and 1810	=	$a \times .75208$
	1790	- 1800	=	$a \times .73010$
	1780	- 1790	=	$a \times .71777$
	1770	- 1780	=	$a \times .69650$
	1760	- 1770	=	$a \times .70155$
	1750	- 1760	=	$a \times .69909$
	1740	- 1750	=	$a \times .67660$
	1730	- 1740	=	$a \times .64217$
	1720	- 1730	=	$a \times .63518$

In order therefore to compensate for this variable law of mortality, and to deduce the proportionate numbers which would have been living at the different ages in the year 1821, had the law of mortality, under the age of ten, been the same at all periods from 1720 as it appears to have been from 1810 to 1820, the results given in page 212, must be further increased as the reciprocals of the decimals expressing the proportions of the numbers born in the several decades, which attained the age of ten years.

Thus, if the numbers living under the age of ten be divided by .77132, those between ten and twenty must be divided by .75208; those between twenty and thirty by .73010; and so on. These operations being performed on the numbers of males and of

females given in page 212, we find that the proportionate numbers living in 1821 would have been

	Males.	Females.	Both.
Under the age of 10	19258	18891	38149
between 10 and 20	16886	16611	33497
—— 20 - 30	14575	14690	29265
—— 30 - 40	12744	13018	25762
—— 40 - 50	10520	10600	21120
—— 50 - 60	7929	8042	15971
—— 60 - 70	5692	6127	11819
—— 70 - 80	3073	3330	6403
—— 80 - 90	880	1081	1961
—— 90 - 100	72	104	176

Hence, applying to these, the formula given in page 175, the Expectation of Life is found to be as follows :

Age.	Males.	Females.	Both.
10	40.2	42.7	41.6
20	33.2	35.6	35.5
30	30.6	31.6	31.0
40	23.2	23.1	23.2
50	19.7	21.8	20.7
60	14.0	14.1	14.1
70	9.0	9.6	9.3
80	5.9	6.0	6.0

It would appear by this result, (if any dependence can be placed upon it,) that the Rate of Mortality among the general mass of population in England and Wales is considerably higher, at the younger periods of life, than that which obtained among the members of the Equitable; but approximates pretty nearly, at all ages, to the average between the Equitable and Northampton Tables.

CHAPTER IV.

ON LIFE ANNUITIES.

SECTION I.

General Observations.

Life Annuities are periodical payments, depending upon the existence of some assigned Life or Lives.

The *Value* of a Life Annuity is an average quantity deduced upon the supposition of the life or lives proposed being associated with a number of other lives of the same ages and prospect of longevity, upon which similar Annuities are conceived to depend: the mortality among the whole mass being assumed equal to that represented by some given Table.*

* The value of an Annuity on an assigned life, considered abstractedly, without reference to others, is totally indeterminable: and the idea entertained by those who regard the value of an Annuity on a life of any given age as the value of an Annuity Certain for as many years as a person of that age may expect to live, is altogether erroneous; because the operation of interest necessarily renders the average value of Annuities independent of the average duration of the periods for which they are made payable. Thus suppose A. owes B. £200, of

Life Annuities are of different denominations, according to the number of lives on which they depend, and to the periods at which they commence, or for which they continue ; and their values also vary according to the rate of interest employed in the calculation, and to the intervals (as yearly, half-yearly, quarterly, &c.) at which they are made payable.

Annuity on a *Single Life* is that which depends on one life only.

Annuity on *Joint Lives* is that which depends on two or more lives, but ceases with the *first* life that fails.

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which, by agreement, £100 is to be paid at the end of six years, and the other £100 at the end of ten years, any reflecting mind will perceive the injustice of making the whole of this debt payable at the end of eight years (the average of six and ten,) for in such case A. would be allowed the use of the first £100 from the end of six to the end of eight years, and in return be called upon to give to B. the use of the second £100 for the two years included between the end of eight and the end of ten years, which is evidently of less value than the use of the other £100 for the two preceding years. Similarly it may be shown, that the *average* of five Annuities of £1 each, to continue respectively for 10, 20, 30, 40, and 50 years, is considerably less than the value of a like Annuity made payable for 30 years, (the average of 10, 20, 30, 40, and 50 years,) the former value, as found by Table VIII., being at 5 per cent. only = 14.194; while the latter is 15.372. Upon the same principle it will be found, that the value of a Life Annuity on a given age, is generally less than the value of an Annuity Certain, taken for as many years as a person of that age may expect to live.

Annuity on the *Last Survivor* is that which depends on two or more lives, but ceases with the extinction of the last of them; or, in other words, that which continues until all the lives become extinct.

Survivorship Annuity is that which commences at the extinction of an assigned life, or at the extinction of the joint existence, or of the last survivor, of two or more assigned lives, and ceases with the extinction of another nominated life, or with the extinction of the joint existence, or of the last survivor, of two or more other nominated lives.

Contingent Annuity is that which commences at the extinction of an assigned life, provided it fails before or after another nominated life, and is then made payable during the remainder of a third life named.

Successive Annuity is that which commences at the extinction of some life or lives, and is afterwards made payable during the existence of some other life or lives to be *then* nominated.

Temporary Annuity is that which ceases at the expiration of a given term, although the life or lives on which it depends should survive beyond that term; but subject to cease sooner, in case such life or lives should previously drop.

Deferred Annuity is that which commences at the end of a given term, and is made payable during the remainder of an assigned life, or until the extinction of the joint existence or last survivor, &c. of two or more nominated lives.

A Life Annuity may also be considered either as *Curtate* or *Complete*, according as it is made to cease with the last payment which may become due prior to the death of the party, or continued up to the day of the death.*

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2. *Construction of Annuity Tables.*

From the definition already given (in page 216,) of the value of a Life Annuity, the reader will immediately perceive that the principal difficulty to be encountered in determining such value consists in the quantity of mechanical labour necessary to perform the calculation.

For example, suppose a number of children, just born, were proposed, and it were required to find the value of £1 annuity, payable during the life of each of them; reckoning interest at 3 per cent. per annum, and assuming the mortality among the whole to be correctly expressed by the Northampton Table?

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* In the following pages an Annuity is understood to be *Curtate*, if not otherwise expressed.

Referring to the Table given in page 95, it will be found that, according to the rate of mortality here assumed, out of

11650 children just born,

8650	may be expected to survive	1 year
7283	- - -	2 years
6781	- - -	3 do.
6446	- - -	4 do.
&c.		&c.

Conceiving, therefore, that a Fund were now formed, sufficient to provide £1 per annum, during life, for each of these 11650 children, it is manifest that such Fund must be adequate to pay

£8650	at the end of the - -	1st year
7283	- - - - - - - - -	2nd do.
6781	- - - - - - - - -	3rd do.
6446	- - - - - - - - -	4th do.
&c.		&c.

Hence, by discounting these sums respectively for 1, 2, 3, 4, &c. years, it will be found, that at 3 per cent., the present value of the

1st payment is -	$8650 \times .970874$	=	8398.060
2nd do. - - - -	$7283 \times .942596$	=	6864.926
3rd do. - - - -	$6781 \times .915142$	=	6205.575
4th do. - - - -	$6446 \times .888487$	=	5727.187
&c.	&c.		&c.

The same process continued to the extremity of life, gives the present values of the several payments due at the end of the 5th, 6th, 7th, &c. years, to the end of life, as in column D of the following

Showing the Values of Annuities on Single Lives, according to the Northampton Rate of Mortality, reckoning Interest at 3 per Cent.

Age.	D	N	A	Age.	D	N	A
0	11650.0000	142947.3507	12.2702	49	689.8140	8756.2453	12.6937
1	8398.0582	134549.2925	16.0215	50	651.7019	8104.5434	12.4360
2	6864.9260	127684.3665	18.5995	51	614.7818	7489.7616	12.1828
3	6205.5755	121478.7910	19.5758	52	579.2444	6910.5172	11.9303
4	5727.1875	115751.6035	20.2109	53	545.2557	6365.2615	11.6740
5	5390.4421	110361.1614	20.4735	54	512.7556	5852.5059	11.4138
6	5079.3420	105281.8194	20.7275	55	481.6859	5370.8200	11.1500
7	4817.5673	100464.2521	20.8537	56	451.9914	4918.8286	10.8826
8	4590.4147	95873.8374	20.8857	57	423.6179	4495.2107	10.6115
9	4395.4000	91478.4374	20.8123	58	396.5138	4098.6969	10.3369
10	4222.7329	87255.7045	20.6633	59	370.6292	3728.0677	10.0588
11	4062.1747	83193.5295	20.4800	60	345 9160	3382.1517	9.7774
12	3908.7899	79284.7399	20.2838	61	322.3281	3059.8236	9.4929
13	3760.8943	75523.8456	20.0814	62	299.8206	2760.0038	9.2055
14	3618.2977	71905.5479	19.8728	63	278.5063	2481.4967	8.9100
15	3480.8174	68424.7305	19.6577	64	258.1793	2223.3174	8.6115
16	3348.2759	65076.4546	19.4358	65	238.9464	1984.3710	8.3047
17	3218.6875	61857.7671	19.2183	66	220.6149	1763.7561	7.9948
18	3090.8704	58766.8967	19.0131	67	203.1485	1560.6076	7.6821
19	2964.9170	55801.9797	18.8208	68	186.5124	1374.0952	7.3673
20	2841.4640	52960.5157	18.6385	69	170.6733	1203.4219	7.0510
21	2719.9993	50240.5164	18.4708	70	155 5984	1047.8235	6.7342
22	2601.6342	47638.8822	18.3112	71	141.2568	906.5667	6.4179
23	2487.8564	45151.0258	18.1486	72	127.6188	778.9479	6.1037
24	2378.4997	42772.5261	17.9830	73	114.6552	664.2927	5.7939
25	2273.4024	40499.1237	17.8144	74	102.3387	561.9540	5.4912
26	2172.4097	38326.7140	17.6425	75	90.6425	471.3115	5.1997
27	2075.3715	36251.3425	17.4674	76	79.5405	391.7710	4.9254
28	1982.1431	34269.1994	17.2890	77	69.3167	322.4543	4.6520
29	1892.5847	32376.6147	17.1070	78	60.0196	262.4347	4.3725
30	1806.5618	30570.0529	16.9217	79	51.6893	210.7454	4.0772
31	1723.9446	28846.1083	16.7326	80	44.0751	166.6703	3.7815
32	1644.6073	27201.5010	16.5398	81	37.0434	129.6269	3.4994
33	1568.4292	25633.0718	16.3432	82	30.6495	98.9774	3.2294
34	1495.2933	24137.7785	16.1425	83	24.8547	74.1227	2.9823
35	1425.0874	22712.6911	15.9378	84	19.5384	54.5843	2.7938
36	1357.7027	21354.9884	15.7288	85	15.0781	39.5062	2.6202
37	1293.0340	20061.9544	15.5154	86	11.4121	28.0941	2.4619
38	1230.9810	18830.9734	15.2976	87	8.4817	19.6124	2.3124
39	1171.4457	17659.5277	15.0750	88	6.1574	13.4550	2.1852
40	1114.3341	16545.1936	14 8476	89	4.4655	8.9895	2.0131
41	1059.2580	15485.9356	14.6196	90	3.2166	5.7729	1.7948
42	1006.1560	14479.7796	14.3912	91	2.3083	3.4846	1.5010
43	954.9682	13524.8114	14.1626	92	1.5819	1.8827	1.1903
44	905.9084	12618.9030	13.9296	93	1.0238	.8589	0.8390
45	858.8965	11760.0065	13.6920	94	.5591	.2998	0.5362
46	813.8548	10946.1517	13.4498	95	.2413	.0585	0.2427
47	770.7080	10175.4437	13.2028	96	.0585	.0000	.0000
48	729.3844	9946.0593	12.9508				

Having thus determined the present value of the 1st, 2nd, 3rd, &c. payments to the 96th, by adding them together, their sum £142947.3507, gives the amount of the Fund, which when improved at 3 per cent. would just provide for the several payments as they respectively become due; or, in other words, provide a Curtate Annuity of £1 for each of the 11,650 children, supposing them to die off, as expressed by the Northampton Table. Hence, dividing the whole Fund by the number of lives proposed, we have the average value of each annuity $= \frac{142947.3507}{11650} = 12.2702 =$ £12 5s 5d, as required.

Similarly, if it were required to find the value of an annuity of £1 on the life of a child aged one year, we should, as before, refer to the Table of Mortality, and conceive the life proposed to be one of 8650 children all of the same age and prospect of longevity, and upon whose lives similar annuities are made payable. This assumption being made, it is manifest that a fund sufficient to provide for all these annuities must be adequate to pay

£7283	at the end of the			1st year
6781	-	-	-	2nd —
6446	-	-	-	3rd —
6249	-	-	-	4th —
&c.				&c.

These being respectively multiplied by the present value of £1 to be received at the end of 1, 2, 3, 4, &c. years, and the operations continued to the

extremity of life; the results, being added together, would produce £138585.77 for the present sum, which when improved, would just provide for the annuities proposed. Hence, dividing this sum by the proposed number of lives, we have $\frac{138585.77}{8650}$ $=16.0215=$£16 0s 5d for the value of each annuity as was required to be found.

By the same process, the value of an annuity on any other age may be determined. Nor does it appear that any shorter method can be applied to determine the value of an annuity on any isolated age. To construct a Table for all ages by this method would, however, be excessively laborious; but it fortunately happens, that when this is our object, the operations performed in the calculation of the value of an annuity on one age may be made subservient to determine the values of similar annuities on other ages, by which means it will be found that the construction of a Table, to represent the values of annuities on all ages, requires but little more labour than that which must be performed to determine the value of an annuity on the life of a child just born.

To point out the method by which this advantage is gained, let it be observed, that in theory it matters not from what number of lives our average is deduced. Let us therefore suppose, that from the age of one year upwards, the numbers living at the different ages in the Table of Mortality are reduced in the ratio of 1 to the present value of

£1 due one year hence, which at 3 per cent. is as 1 to .970874, so that the number living at the age of

1 year may be	$= 8650 \times .970874 =$	8398.058
2 years - -	$= 7283 \times .970874 =$	7070.867
3 — - -	$= 6781 \times .970874 =$	6583.497
4 — - -	$= 6446 \times .970874 =$	6258.254
&c.	&c.	&c.

This done, it is manifest that when referred to the age of one year, the number of lives from which the average is taken is reduced to 8398.058, out of which it is also manifest that

7070.867 may be expected to survive	1 year
6583.497 - - - - -	2 years
6258.254 - - - - -	3 —
&c.	&c.

Hence, supposing a Fund to be raised sufficient to provide annuities of £1 each for the 8398.058 lives now proposed, it is manifest that such Fund must be adequate to the payment of

£7070.867	at the end of the 1st year
6583.497 - - -	2nd —
6258.254 - - -	3rd —
&c.	&c.

These sums being discounted, as before, for 1, 2, 3, &c. years, it will be found that the present value of the payments to be made at the end of the

1st year	$= 7070.867 \times .970874 =$	6864.92
2nd —	$= 6583.497 \times .942596 =$	6205.57
3rd —	$= 6258.254 \times .915142 =$	5727.19
&c.	&c.	&c.

where it may be observed that the number of lives and the present values of the several payments all agree with the numbers already given in column D of the foregoing Table.

In like manner it may be shown, that if the number living from two years upwards be reduced in the ratio of 1 to the present value of £1 due two years hence, column D will be alike applicable to represent the number living at the age of two years, and the present values of the several payments due at the end of 1, 2, 3, &c. years from that age.

The same principle is equally applicable to any other age: thus, referring to the age of 21, and supposing a fund raised sufficient to provide £1 per annum for 2720 lives of that age, the present value of the payment becoming due at the end of the

1st year will be, at 3 per cent. - -	£2601,6342
2nd - - - - - -	2487,8564
3rd - - - - - -	2378,4997
&c.	&c.

And, consequently, the present value of the whole fund must be the sum of the several numbers given in that column, at all ages, from 22 upwards.

This principle being established, it is manifest that if we begin at the oldest age, and form another column N, by taking the successive sums of the numbers given in column D, as directed in page 120, the result opposite any age will denote the

present value of the fund necessary to provide £1 per annum each for the number of persons living at that age as expressed in column D. Thus referring again to the age of 21, it will be found that a present sum of £50240.5164, laid out and improved at 3 per cent. will be just sufficient to provide a Curtate Annuity of £1 each, for 2720 persons living at that age, supposing them to die off in the proportion expressed by the Northampton Table. From this view of the subject, it follows that the value of an Annuity on any age is found by dividing the number in column N. opposite that age, by the corresponding number in column D. Thus the value of an Annuity on the life of a child aged

1 year	= 134549.2925 ÷ 8398.0582	=	16.0215
2 years	= 127684.3665 ÷ 6864.9260	=	18.5995
3 —	= 121478.7910 ÷ 6205.5755	=	19.5758
4 —	= 115751.6035 ÷ 5727.1875	=	20.2109
&c.	&c.	&c.	&c.

which values are found registered in column A of the foregoing Table.

In this manner columns D. and N. of Table XXI. have been formed from the rate of mortality among the members of the Equitable; and, for the purpose of determining the values of increasing or decreasing annuities, column S. has been deduced by taking the successive sums of the numbers given in

column N. Columns M. and R. relate to Life Assurances, their construction and application will therefore be explained in the next Chapter. The XXIV. Table has also been formed from Table XXI., by dividing the numbers given in column N. by the corresponding numbers in column D.

Again, let us suppose it were required to determine the value of an annuity of £1 on two *joint lives*, of the respective ages of 0 and 19, reckoning interest at 3 per cent., and assuming the rate of mortality to be correctly expressed by the Northampton Table?

Referring to the Table given in page 95, and to the principle deduced in pages 109-10, it will be found that out of

Pairs of Joint lives.		
11650 × 5199	existing at the ages of 0 and 19	
8650 × 5132	may be expected jointly to survive	1 year
7283 × 5060	- - - - -	2 yrs.
6781 × 4985	- - - - -	3 —
6446 × 4910	- - - - -	4 —
&c.		&c.

Conceiving, therefore, the proposed pair of joint lives to be one of 11650 × 5199 pairs of like ages, and subject to similar laws of mortality; and supposing, as before, a fund were now formed sufficient to provide an annuity of £1 for each pair,

until they respectively become extinct, it is manifest that such fund must be adequate to pay

£8650 × 5132	at the end of the 1st year
7283 × 5060 - - -	2nd —
6781 × 4985 - - -	3rd —
6446 × 4910 - - -	4th —
&c.	&c.

Hence, by discounting these respective sums for 1, 2, 3, 4, &c. years, it will be found that the present value of the payments becoming due at the end of the

1st year = 8650 × 5132 × .970874 = £43098841

2nd — = 7283 × 5060 × .942596 = 34736523

3rd — = 6781 × 4985 × .915142 = 30934806

and so on, to the end of life, the sum of which, divided by 11650 × 5199, gives the value required.

But as any given proportion of these numbers will equally answer the end in view, provided the number of combinations be reduced in the same ratio, let the proposed combinations of lives, and the present values of the several payments, be each reduced in the ratio of 1 to the present value of £1 due as many years hence as are equal to the age of the older of the proposed lives, which is here in the ratio of 1 to .570286, then will the number living at the proposed ages become = 11650 × 5199 × .570286 = 34541283, and the present value of the payment due at the end of the

1st year = 8650 × 5132 × .553676 = 24578664
2nd — = 7283 × 5060 × .537549 = 19809754
3rd — = 6781 × 4985 × .521893 = 17641681
4th — = 6446 × 4910 × .506692 = 16036722
&c. &c. &c.

This reduction being effected, the product of the last two factors in each case will be found already determined in column D. of the Table given in page 221, opposite the respective ages of 19, 20, 21, 22, &c., thus

5199 × .570286 = 2964.917
5132 × .553676 = 2841.464
5060 × .537549 = 2719.999
4985 × .521893 = 2601.634
4910 × .506692 = 2487.856
&c. &c.

Hence, supposing the number of combinations proposed to be represented by 11650 × 2984.917 = 34541283, we have the present value of the payment due at the end of the

1st year = 8650 × 2841.464 = 24578664
2nd — = 7283 × 2719.999 = 19809754
3rd — = 6781 × 2601.636 = 17641681
4th — = 6446 × 2487.856 = 16036722
&c. &c. &c.

Proceeding in like manner from age to age to the greatest Tabular duration of the older life, the present values of the several payments will be found to agree with the numbers given in column D. of the following

FORM OF TABLE

For determining the Values of Annuities on Two Joint Lives, according to the Rate of Mortality at Northampton, reckoning Interest at 3 per Cent.

Difference of Age 19 *years.*

Ages.	D	N	AB	Ages.	D	N	AB
		341503625					
0 & 19	34541283	306962342	8.8868	39 & 58	1471066	12401030	8.4300
1 - 20	24578664	282383679	11.4890	40 - 59	1347237	11053793	8.2048
2 - 21	19809755	262573924	13.2548	41 - 60	1231115	9822678	7.9787
3 - 22	17641681	244932242	13.8837	42 - 61	1122346	8700332	7.7519
4 - 23	16036722	228895520	14.2732	43 - 62	1020589	7679743	7.5248
5 - 24	14863244	214032276	14.4001	44 - 63	926312	6753431	7.2997
6 - 25	13788185	200244090	14.5229	45 - 64	838566	5914864	7.0535
7 - 26	12871527	187372563	14.5571	46 - 65	757460	5157404	6.8088
8 - 27	12068285	175304277	14.5260	47 - 66	682141	4475263	6.5606
9 - 28	11367591	163936687	14.4214	48 - 67	612290	3862974	6.3001
10 - 29	10740418	153196269	14.2635	49 - 68	547601	3315373	6.0544
11 - 30	10158297	143037972	14.0809	50 - 69	487614	2827760	5.7992
12 - 31	9607543	133430428	13.8881	51 - 70	431941	2395818	5.5466
13 - 32	9083166	124347262	13.6899	52 - 71	380546	2015273	5.2957
14 - 33	8584013	115763249	13.4859	53 - 72	333340	1681933	5.0457
15 - 34	8108976	107654274	13.2760	54 - 73	290078	1391855	4.7982
16 - 35	7656995	99997279	13.0596	55 - 74	250525	1141329	4.5557
17 - 36	7222978	92774301	12.8443	56 - 75	214460	926869	4.3219
18 - 37	6803945	85970356	12.6354	57 - 76	181671	745199	4.1019
19 - 38	6399870	79570486	12.4331	58 - 77	152635	592563	3.8822
20 - 39	6011859	73558626	12.2356	59 - 78	127242	465322	3.6570
21 - 40	5638531	67920096	12.0457	60 - 79	105343	359979	3.4172
22 - 41	5280401	62639695	11.8627	61 - 80	86211	273768	3.1755
23 - 42	4940226	57699469	11.6795	62 - 81	69419	204349	2.9437
24 - 43	4617271	53082198	11.4964	63 - 82	54955	149394	2.7185
25 - 44	4312124	48770074	11.3100	64 - 83	42551	106843	2.5109
26 - 45	4023930	44746143	11.1200	65 - 84	31887	74956	2.3507
27 - 46	3751871	40994273	10.9264	66 - 85	23401	51555	2.2031
28 - 47	3495161	37499112	10.7289	67 - 86	16799	34756	2.0690
29 - 48	3253054	34246058	10.5274	68 - 87	11807	22950	1.9438
30 - 49	3024834	31221223	10.3216	69 - 88	8078.6	14871	1.8408
31 - 50	2808835	28412388	10.1154	70 - 89	5501.6	9369.4	1.7030
32 - 51	2603601	25808787	9.9127	71 - 90	3705.6	5663.8	1.5284
33 - 22	2409657	23399131	9.7106	72 - 91	2474.5	3189.3	1.2889
34 - 53	2227370	21171761	9.5053	73 - 92	1569.3	1620.0	1.0323
35 - 54	2056150	19115611	9.2968	74 - 93	933.8	686.2	.7349
36 - 55	1895434	17220177	9.0851	75 - 94	465.2	221.0	.4750
37 - 56	1744687	15475490	8.8701	76 - 95	181.4	39.5	.2179
38 - 57	1603394	13872097	8.6517	77 - 96	39.5	0	0

FORM OF TABLE

For determining the Values of Annuities on Two Joint Lives, according to the Rate of Mortality at Northampton, reckoning Interest at 3 per Cent.

Difference of Age 20 *Years.*

Ages.	D	N	AB	Ages.	D	N	AB
		325161569		38 & 58	1500805	12723227	8.4776
0 & 20	33103055	292058514	8.8227	39 - 59	1375034	11348193	8 2530
1 - 21	23527994	268530520	11.4132	40 - 60	1257405	10090788	8.0251
2 - 22	18947702	249582818	13.1722	41 - 61	1147166	8943623	7.7963
3 - 23	16870154	232712664	13.7943	42 - 62	1043975	7899647	7.5669
4 - 24	15331809	217380855	14.1784	43 - 63	948035	6951612	7.3326
5 - 25	14206492	203174363	14.3015	44 - 64	858704	6092908	7.0955
6 - 26	13175665	189998699	14.4204	45 - 65	776098	5316810	6.8507
7 - 27	12296576	177702122	14.4514	46 - 66	699349	4617461	6.6025
8 - 28	11526162	166175960	14.4173	47 - 67	628135	3989326	6.3511
9 - 29	10853973	155321987	14.3102	48 - 68	562148	3427177	6.0966
10 - 30	10252238	145069749	14.1501	49 - 69	501097	2926081	5.8394
11 - 31	9693741	135376008	13.9653	50 - 70	444544	2481536	5.5822
12 - 32	9165396	126210612	13.7703	51 - 71	392129	2089407	5.3284
13 - 33	8662434	117548177	13.5699	52 - 72	343805	1745602	5.0773
14 - 34	8183740	109364437	13.3636	53 - 73	299480	1446123	4.8288
15 - 35	7728249	101636188	13.1513	54 - 74	258917	1187206	4.5853
16 - 36	7294936	94341252	12.9324	55 - 75	221893	965313	4.3504
17 - 37	6878941	87462311	12.7145	56 - 76	188193	777120	4.1294
18 - 38	6477422	80984889	12.5026	57 - 77	158319	618801	3.9086
19 - 39	6090346	74894543	12.2973	58 - 78	132163	486638	3.6821
20 - 40	5718763	69175780	12.0963	59 - 79	109581	377056	2.4409
21 - 41	5359845	63815935	11.9063	60 - 80	89825	287231	3.1977
22 - 42	5015688	58800247	11.7233	61 - 81	72457	214774	2.9641
23 - 43	4688894	54111353	11.5403	62 - 82	57437	157337	2.7393
24 - 44	4380067	49731286	11.3540	63 - 83	44564	112772	2.5305
25 - 45	4088347	45642939	11.1642	64 - 84	33450	79323	2.3714
26 - 46	3812910	41830029	10.9706	65 - 85	24608	54715	2.2235
27 - 47	3552964	38277065	10.7733	66 - 86	17712	37004	2.0892
28 - 48	3307758	34969307	10.5719	67 - 87	12485	24518	1.9637
29 - 49	3076570	31892736	10.3663	68 - 88	8571	15947	1.8605
30 - 50	2857713	29035024	10.1602	69 - 89	5859	10088	1.7218
31 - 51	2649709	26385314	9.9578	70 - 90	3963	6125	1.5455
32 - 52	2453100	23932214	9.7559	71 - 91	2659	3466	1.3032
33 - 53	2268264	21663950	9.5509	72 - 92	1696	1770	1.0440
34 - 54	2094607	19569344	9.3427	73 - 93	1016	754.7	.7431
35 - 55	1931561	17637783	9.1314	74 - 94	510.0	244.8	.4800
36 - 56	1778586	15859197	8.9167	75 - 95	200.7	44.04	.2194
37 - 57	1635165	14224032	8.6988	76 - 96	44.04	0	0

FORM OF TABLE

For determining the Values of Annuities on Two Joint Lives, according to the Rate of Mortality at Northampton, reckoning Interest at 3 per Cent.

Difference of Age 21 *Years.*

Ages.	D	N	AB	Ages.	D	N	AB
		309440174					
0 & 21	31687991	277752183	8.7652	38 & 59	1402831	11641053	8.2983
1 - 22	22504135	255248048	11.3423	39 - 60	1283348	10357704	8.0708
2 - 23	18119058	237128989	13.0873	40 - 61	1171663	9186042	7.8402
3 - 24	16128606	221000383	13.7024	41 - 62	1067061	8118980	7.6087
4 - 25	14654352	206346031	14.0809	42 - 63	969759	7149221	7.3722
5 - 26	13575388	192770643	14.2000	43 - 64	878842	6270379	7.1348
6 - 27	12587128	180183515	14.3149	44 - 65	794736	5475643	6.8899
7 - 28	11744198	168439317	14.3424	45 - 66	716557	4759086	6.6416
8 - 29	11005380	157433937	14.3052	46 - 67	643981	4115105	6.3901
9 - 30	10360632	147073305	14.1954	47 - 68	576696	3538409	6.1356
10 - 31	9783386	137289919	14.0330	48 - 69	514409	3024000	5.8786
11 - 32	9247627	128042293	13.8460	49 - 70	456837	2567163	5.6194
12 - 33	8740856	119301437	13.6487	50 - 71	403571	2163592	5.3611
13 - 34	8258505	111042932	13.4459	51 - 72	354270	1809323	5.1072
14 - 35	7799503	103243428	13.2372	52 - 73	308881	1500441	4.8577
15 - 36	7362822	95880607	13.0223	53 - 74	267309	1233132	4.6131
16 - 37	6947472	88933135	12.8008	54 - 75	229325	1003807	4.3772
17 - 38	6548819	82384316	12.5800	55 - 76	194715	809092	4.1552
18 - 39	6164147	76220169	12.3651	56 - 77	164003	645088	3.9334
19 - 40	5793423	70426746	12.1563	57 - 78	137085	508004	3.7058
20 - 41	5436112	64990634	11.9554	58 - 79	113820	394184	3.4632
21 - 42	5091149	59899485	11.7654	59 - 80	93440	300744	3.2186
22 - 43	4760516	55138968	11.5826	60 - 81	75494	225250	2.9837
23 - 44	4448010	50690958	11.3963	61 - 82	59950	165300	2.7573
24 - 45	4152765	46538193	11.2066	62 - 83	46578	118722	2.5489
25 - 46	3873949	42664244	11.0131	63 - 84	35032	83690	2.3889
26 - 47	3610767	39053478	10.8158	64 - 85	25814	57876	2.2420
27 - 48	3362462	35691016	10.6146	65 - 86	18625	39251	2.1075
28 - 49	3128306	32562709	10.4091	66 - 87	13164	26087	1.9818
29 - 50	2906590	29656119	10.2031	67 - 88	9063.8	17024	1.8782
30 - 51	2695818	26960300	10.0008	68 - 89	6216.1	10808	1.7386
31 - 52	2496543	24463757	9.7990	69 - 90	4220.3	6587.3	1.5609
32 - 53	2309158	22154599	9.5942	70 - 91	2843.8	3743.5	1.3163
33 - 54	2133063	20021536	9.3863	71 - 92	1822.4	1921.1	1.0542
34 - 55	1967687	18053849	9.1752	72 - 93	1097.6	823 5	.7502
35 - 56	1812485	16241364	8.9608	73 - 94	554.7	268.8	.4845
36 - 57	1666936	14574427	8.7432	74 - 95	220.0	48.72	.2214
37 - 58	1530543	13043884	8.5224	75 - 96	48.72	0	0

The explanation already given in pages 225-6, applies to a Table thus formed for the purpose of determining the values of Annuities on Two Joint Lives. Thus referring to difference of age, 19 years, it may be shown that if 6011859 pairs of Joint Lives were proposed, each pair consisting of individuals of the respective ages of 20 and 39, and a fund were formed sufficient to provide an annuity of £1 for each pair, the present value of the payments becoming due at the end of the

1st year would be - -	£5638531
2nd - - - -	5280401
3rd - - - -	4940226

and so on, to the greatest Tabular duration of the older life, the sum of which, £73558626, as shown by column N opposite the proposed ages, gives the present value of the whole fund. Hence, dividing this fund by the number of pairs proposed, we have $\frac{73558626}{6011859} = 12.2356$ for the average present value of each annuity, as registered in the column headed AB.

Similarly it may be shown, that if a Table of this kind were constructed for every difference of age, the quotient arising by dividing the number in column N opposite any proposed ages by the corresponding number in column D would be the value of an annuity of £1 on the joint lives of two persons of those ages.

Moreover, let it be required to find the present value of an Annuity of £1 on *Three Joint Lives*, of the respective ages of 0, 20 & 40, reckoning interest at 3 per Cent., and assuming the Rate of Mortality to be correctly expressed by the Northampton Table ?

Reasoning as in page 114, it may be shown that out of a number of combinations of three joint lives, expressed by $11650 \times 5132 \times 3635$

$8650 \times 5060 \times 3559$ will jointly survive	1 year
$7283 \times 4985 \times 3482$ - - -	2 years
$6781 \times 4910 \times 3404$ - - -	3 do.
&c.	&c.

Or if, in order to reduce the number of figures, we take a 10,000th part of each of these results, and conceive the proposed combination to be one of $1.1650 \times 5132 \times 3635$ similar combinations ; then suppose a fund to be formed, as before, sufficient to provide a curtate annuity of £1 for each, until the failure of the joint existence of the individuals of which it is composed, it is manifest that such fund must be adequate to pay

£.$8650 \times 5060 \times 3559$ at the end of the	1st year
.$7283 \times 4985 \times 3482$ - - -	2nd
.$6781 \times 4910 \times 3404$ - - -	3rd

and so on to the greatest tabular duration of the oldest of the proposed lives : the results thus obtained being respectively discounted for 1, 2, 3, &c. years, would evidently give the present values of the payments becoming due at the end of the 1st, 2nd,

3rd, &c. year, the sum of which continued to the extremity of life, and divided by the number of combinations from which the average is taken, would give the required value of the annuity proposed.

But if the number of combinations, and the present values of the payments becoming due at the end of the 1st, 2nd, 3rd, &c. year, be each of them further reduced in the ratio of 1 to the present value of £1, due as many years hence as are equal to the age of the oldest of the three lives proposed, here as 1 to the present value of £1 due 40 years hence, which at 3 per Cent. is as 1 to .306557, it will be found that the number of combinations will become

$$= 1.1650 \times 5132 \times 3635 \times .306557$$

and the present value of the payments becoming due at the end of the

$$\begin{aligned}
\text{1st year} &= .8650 \times 5060 \times 3559 \times .297628 \\
\text{2nd} &= .7283 \times 5985 \times 3482 \times .288959 \\
\text{3rd} &= .6781 \times 4910 \times 3404 \times .280543 \\
\text{4th} &= .6446 \times 4835 \times 3326 \times .272372
\end{aligned}$$

This reduction being effected, the product of the last three factors in each case will be found already determined in column D of the Table given in page 231, opposite the respective ages of

20	and	40	=	5718763
21	—	41	=	5359845
22	—	42	=	5015688
23	—	43	=	4688894
24	—	44	=	4380067

and so on.

so that the number of combinations will thus become = 1.1650 × 5718763 = 6662359, and the present value of the payments becoming due at the end of the

1st year	=	£.8650 × 5359845 =	£4636265
2nd	=	.7283 × 5015688 =	3652925
3rd	=	.6781 × 4688894 =	3179537
4th	=	.6446 × 4380067 =	2823390
&c.		&c.	&c.

By this mode of proceeding, it is manifest that from the results given in column D of a Two Joint Life Table, a similar column may be formed for Three Joint Lives, the successive sums of which being taken as before, gives the corresponding numbers for column N: this done, the value of an Annuity on three joint lives may be found, as before, by dividing the number given opposite the proposed ages in column N, by the corresponding number in column D. As a specimen of this kind, Table XXIII has been formed from the Rate of Mortality among the members of the Equitable.

In conclusion we may remark that, by a Table formed as directed in this section, for either single or joint lives, the value of an annuity, payable at the beginning of the year, is found by dividing the number given in column N opposite 1 year younger than the age given, by the number in column D at the age proposed. Thus, by the Table in page 221, the present value of an annuity of £1, payable at the beginning of the year, on a life of 20 is

$\frac{55801.9797}{2841.4640} = 19.6385$, as is evident from the construction of the Table.

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3. *Amount of Life Annuities Forborn.*

The method of taking the average from a number of persons, is not only applicable to determine the *Present Value* of an annuity on an assigned life, but to find the *Improved Amount* to which an annuity will accumulate by the time that life may become extinct.

Thus, suppose a number of children just born were proposed, and it were required to find the average amount to which an Annuity of £1 on the life of each child would accumulate, if improved at 3 per Cent. compound interest, until the end of the year in which that life may become extinct, assuming the rate of mortality among the whole mass to be correctly expressed by the Northampton Table?

By referring again to the Table in page 95, and conceiving a society formed of 11650 children of the proposed age, upon whose lives similar Annuities are made payable to a common fund, from which the required average is to be withdrawn for each life, at the end of the year in which that life may fail, it will be found that the numbers of contributors and claimants at the end of the several years will be as under, viz. at the end of the

1st year	8650	Contributors, and	3000	Claimants
2nd —	7283	—	1367	—
3rd —	6781	—	502	—
4th —	6446	—	335	—

and so on to the end of life, as shown by the Table.

Hence, by multiplying the number of contributors by the present value of £1 due at the end of 1, 2, 3, &c. years respectively, it will be found that the present values of the contributions due at the end of the

1st year	= 8650 × .970874	=	8398.060
2nd —	= 7283 × .942596	=	6864.926
3rd —	= 6781 × .915142	=	6205.575
4th —	= 6446 × .888487	=	5727.187

and so on to the greatest tabular duration, all agreeing with the numbers in column D of the Table given in page 221. Their sum is therefore found in column N of that Table, opposite age 0 = £142947.3507 = the present value of all the contributions to the supposed fund.

Again, referring to the column expressing the number of claims to be made on the supposed fund at the end of the 1st, 2nd, 3rd, &c. year, it is manifest that if only £1 were paid to each claimant, the present value of the claims becoming due at the end of the

1st year would be	3000 × .970874	=	2912.622
2nd — —	1367 × .942596	=	1288.529
3rd — —	502 × .915142	=	459.401
4th — —	335 × .888487	=	297.643

and so on as in column P of the subjoined

TABLE.

Age.	P	M	Age.	P	M
0	- - -	7147.166	49	18.3261	414.6865
1	2912.622	4234.544	50	18.0205	396.6660
2	1288.529	2946.015	51	17.9385	378.7275
3	459.401	2486.614	52	17.6310	361.0965
4	297.643	2188.971	53	17.1175	343.9790
5	169.934	2019.037	54	16.6190	327.3600
6	154.097	1864.940	55	16.1349	311.2251
7	113.833	1751.107	56	15.6650	295.5601
8	86.8350	1664.272	57	15.2087	280.3512
9	61.3133	1602.958	58	14.7657	265.5855
10	44.6456	1558.313	59	14.3357	251.2498
11	37.5659	1520.747	60	13.9181	237.3317
12	35.0690	1485.678	61	13.5127	223.8190
13	34.0476	1451.630	62	13.1192	210.6998
14	33.0559	1418.574	63	12.5817	198.1180
15	32.0931	1386.481	64	12.2153	185.9027
16	31.1583	1355.323	65	11.7131	174.1896
17	32.0659	1323.257	66	11.3719	162.8177
18	34.0689	1289.188	67	11.0407	151.7770
19	35.9280	1253.260	68	10.7191	141.0579
20	37.0963	1216.164	69	10.4069	130.6510
21	38.7035	1177.460	70	10.1038	120.5472
22	39.1419	1138.318	71	9.80950	110.7377
23	38.0019	1100.317	72	9.52379	101.2139
24	36.8950	1063.422	73	9.24640	91.9675
25	35.8204	1027.601	74	8.97709	82.9904
26	34.7771	992.8241	75	8.71562	74.2748
27	33.7642	959.0599	76	8.46176	65.8130
28	32.7808	926.2791	77	7.90723	57.9058
29	31.8260	894.4531	78	7.27812	50.6277
30	30.8990	863.5541	79	6.58217	44.0455
31	29.9990	833.5551	80	6.10851	37.9370
32	29.1253	804.4298	81	5.74811	32.1889
33	28.2770	776.1528	82	5.31495	26.8740
34	27.4534	748.6994	83	4.90213	21.9718
35	26.6538	722.0456	84	4.59236	17.3795
36	25.8774	696.1682	85	3.89114	13.4883
37	25.1237	671.0445	86	3.22688	10.2614
38	24.3920	646.6525	87	2.59801	7.66344
39	23.6815	622.9710	88	2.07722	5.58622
40	22.9918	599.9792	89	1.51254	4.07368
41	22.6197	577.3595	90	1.11884	2.95484
42	22.2499	555.1096	91	.814693	2.14015
43	21.8823	533.2273	92	.659136	1.48101
44	21.2450	511.9823	93	.511951	.969059
45	20.6262	491.3561	94	.434909	.534150
46	20.0254	471.3307	95	.301602	.232548
47	19.4422	451.8885	96	.175690	.056858
48	18.8759	433.0126	97	.056858	0

The present values of the several payments becoming due at the end of the 1st, 2nd, 3rd, &c. year to the extremity of life, being thus found, on the supposition that each claimant were to receive only £1, their sum £7147.166 shows the present fund which, when improved at 3 per Cent., would be sufficient to provide for the payment of £1 for each of the proposed lives, at the end of the year in which such life may become extinct. Hence, by the rule given in page 69 we have

$$\frac{142947.3507}{7147.166} = £20$$

for the required amount of the Annuity proposed.

If, instead of taking the sum of the numbers in column P at one operation, we begin at the oldest age, and take their successive sums, so as to form another column M, the results will be found applicable to other ages, by conceiving the number of lives from which the average is taken to be reduced as explained in pages 223–4.

Thus, supposing 2720 persons of the age of 21 to contribute £1 per Annum each to a common fund, from which the average amount is to be withdrawn for each life, at the end of the year in which that life may become extinct, it is manifest, from what has been already stated, that the present values of the several contributions becoming due at the end of the 1st, 2nd, 3rd, &c. year, will be denoted by the numbers in column D, page 221, opposite ages respectively 1, 2, 3, &c. years older than the age pro-

proposed; and their sum, or the total present value of all the contributions will be denoted by £50240.5164, the number given in column N of the same Table opposite the proposed age.

Similarly, on the supposition of only £1 being paid to each claimant, it may be shown that the present values of the claims becoming due at the end of the 1st, 2nd, 3rd, &c. year will be denoted by the numbers given in column P of the Table in page 239, opposite ages respectively 1, 2, 3, &c. years older than that proposed; and their sum, continued to the extremity of life, or the present value of a fund sufficient to provide for the payment of £1 for each claimant, is denoted by £1177.460, the number given in column M, opposite the age proposed. Hence $\frac{50240.5164}{1177.460} = 42.6686 =$ £42 13s 4d = the improved amount of an Annuity of £1 on a life of 21.

By dividing the number given in column N, by the corresponding number in column M, it may be shown, in like manner, that at 3 per Cent., according to the Northampton rate of mortality, the amount of an annuity of £1 on a life

aged 10 years is	87255.704 ÷ 1558.313	=	55.9937
— 20 —	52960.516 ÷ 1216.164	=	43.5472
— 30 —	30570.053 ÷ 863.554	=	35.4003
— 40 —	16545.194 ÷ 599.979	=	27.5763
— 50 —	8104.543 ÷ 396.666	=	20.4317

aged 60 years is $3382.152 \div 237.332 = 14.2507$

— 70 — $1047.823 \div 120.547 = 8.6922$

— 80 — $166.670 \div 37.937 = 4.3934$

But, if the contributions be made payable at the beginning of the year, the amount of the whole fund will be greater than if they were made payable at the end of the year, by the contributions made at the beignning of the first year, and therefore equal to the number in column N opposite one year younger than the age proposed, as evidently appears by the construction of the Table. It therefore follows that the improved amount of an annuity of £1 on an assigned life, payable at the beginning of the year, is found by dividing the number in column N opposite the next younger age by the number in column M opposite the age proposed. Thus the amount of such annuity on a life of 20 is $= \frac{55801.9797}{1216.164} = 45.8836.$

Columns M and N of Table XXI. may be applied in like manner, to determine the amount of an Annuity on any assigned life, (not under 10) according to the rate of mortality among the members of the Equitable.

Thus on a life of 90, the improved amount of an Annuity of £1, reckoning interest at 4 per Cent. is, by this rate of mortality $= \frac{3.7404}{1.6881} = 2.216.$

For the purpose of further illustrating this subject:—Suppose a society formed of 65 persons, of

the proposed age, each contributing £1 per annum during life to a common fund, for an equivalent average value, to be withdrawn by his survivors at the end of the year in which he may die: then, by referring to Table x, it will be found that at the end of the several years to the extremity of life, the numbers of contributors to, and claimants on, the supposed fund will be as under, viz.

1st year	49	Contributors, and	16 Claimants
2nd —	36	—	13 —
3rd —	25	—	11 —
4th —	16	—	9 —
5th —	9	—	7 —
6th —	4	—	5 —
7th —	1	—	3 —
8th —	0	—	1 —

Hence, from the 1st year's contributions	£49.000
deduct the 1st year's claims $16 \times 2.216 =$	35.456
balance remaining at the end of 1st year	13.544
improved amount of ditto, at 4 per Cent	14.086
add 2nd year's contributions - - -	36.000
	50.086
deduct 2nd year's claims $13 \times 2.216 =$	28.808
balance at the end of 2nd year - -	21.278
improved amount of ditto, at 4 per Cent,	22,129
add 3rd year's contributions - - -	25,000
	47.129

deduct 3rd year's claims 11×2.216 =	£24.376
balance at the end of 3rd year - - -	22.753
improved amount of ditto, at 4 per Cent.	23.663
add 4th year's contributions - - - -	16.000
	39.663
deduct 4th year's claims 9×2.216 =	19.944
balance at the end of 4th year - - -	19.719
improved amount of ditto at 4 per cent =	20.508
add 5th year's contributions - - - -	9.000
	29.508
deduct 5th year's claims 7×2.216 =	15.512
balance at the end of 5th year - - -	13.996
improved amount of ditto at 4 per Cent.	14.556
add 6th year's contributions - - - -	4.000
	18.556
deduct 6th year's claims 5×2.216 =	11.080
balance at the end of 6th year - - - -	7.476
improved amount of ditto, at 4 per Cent.	7.775
add 7th year's contributions - - - -	1.000
	8.775
deduct 7th year's claims 3×2.216 =	6.648
balance at the end of the 7th year - -	2.127
improved amount of ditto, at 4 per Cent.	2.212

which just provides for the remaining claim at the end of the 8th year.

This mode of illustration has been given to enable such readers as may be unacquainted with Mathematical investigations, to satisfy themselves as to the accuracy of the method here given for finding the amount of a Life Annuity Forborn; which method differs essentially, not only in form, but also in the results produced, from that given by Mr. Morgan in the VII Problem of the last edition of his work on Assurances, &c.

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4. *Present value of a Sum receivable on the extinction of an assigned Life.*

As a Corollary from what has been stated in the last two sections, we deduce an easy method of finding the average present value of £1, to be received at the end of the year in which an assigned life may fail, by simply dividing the number given in column M opposite the proposed age, by the corresponding number in column D. For it appears, from what was stated in page 241, that, if 2720 persons of the age of 21 were formed into a society, for the purpose of raising a common fund sufficient to pay £1 at the end of the year in which each of them may happen to die, such fund, in present money, must amount to £1177.460; hence, dividing this fund by the number of persons proposed, we have

$$\frac{1177.46}{2720} = .43289$$

for the average present value of £1 payable at the end of the year in which any one of these persons may happen to die, reckoning interest at 3 per Cent. and assuming the rate of mortality to be correctly expressed by the Northampton Table.

By the same mode of reasoning it may be shown, that the method of dividing the number given in column M opposite any other age, by the corresponding number in column D, will equally represent the present value of £1 payable at the end of the year in which a person of that age may happen to die; or, in other words, the Single Premium which ought to be paid for the Assurance of £1 on that age.

Having thus anticipated the subject of Life Assurances, we may further remark, that the improved amount of an Annuity on any assigned age, is identical with the sum which a person of that age ought to secure to his survivors as an equivalent for such annuity.

Thus, supposing money to be improved at 3 per cent., and the rate of mortality to be expressed by the Northampton Table, it is manifest that a person of 20 paying £1 per Annum, at the beginning of each year during life, to a common fund, ought to secure to his survivors the improved amount of his annuity; which amount we have already shown, in page 242, to be equal to the quotient arising by dividing the number given in column N opposite the age of 19, by the number in col. M opposite the age of 20 $= \frac{55801.9797}{1216.164} = 45.8836$; or in symbols $= \frac{N_{19}}{M_{20}}$.

Hence, by proportion we say, if to secure £$\frac{N_{,}}{M}$ payable at death, it requires a contribution of £1 per Annum, what annual premium should be required to secure £1 payable at death: or thus, as $\frac{N_{,}}{M} : 1 : : 1 : \frac{M}{N_{,}}$ From this it is manifest, that

The Annual Premium for the Assurance of £1, *on any age, may be found by dividing the number given in column* M *opposite that age, by the number in column* N, *opposite an age* 1 *year younger.*

Thus the annual premium for the Assurance of £1 on a life of

20	=	1216.164 ÷ 55801.98	=	.021794
— — 30	=	863.5541 ÷ 32376.61	=	.026672
— — 40	=	599.9792 ÷ 17659.53	=	.033975
— — 50	=	396.6660 ÷ 8756.25	=	.045301
— — 60	=	237.3317 ÷ 3728.07	=	.063661

which will be found to agree with the premiums charged by the Equitable Society.

5. *Method of determining the value of an Annuity on an isolated Life.*

In the foregoing sections we have supposed the value of an annuity on an assigned life, to be determined by taking the average from a number of persons of the same age and prospect of longevity; and

as far as we can judge of the future from the experience of the past, it is probable, that an individual or a public company purchasing a considerable number of such annuities, might rely with some confidence, that the losses sustained by the premature deaths of some of the nominees, would be compensated by the greater longevity of others; but in the purchase of a single annuity it is manifest that this average cannot obtain, and that the capital advanced by the purchaser, if not otherwise secured, must be placed in a state of extreme hazard.

To guard themselves against this uncertainty, most persons, who speculate in this kind of property, deem it advisable to secure the purchase money by effecting an assurance on the life on which the annuity is made payable, and, by that means, realize to themselves a given rate of interest for the money advanced, beside securing the purchase money in the event of the life failing: but no Table has hitherto been published by which this object may be effected, nor is it an easy task to form such a Table as would apply in all cases; because the annuity may be payable yearly, half-yearly, or quarterly, and made to cease with the last payment which may become due prior to the death of the party, or be continued up to the day of the death. The different rates of premiums charged by the various Life Assurance Offices, also tend to increase the difficulty of forming any Table that may be universally applicable.

In the following investigation we shall, therefore, suppose the annuity to be payable yearly, and to cease with the last payment that may become due prior to the death of the nominee: we shall also suppose the premium of insurance to be, in all cases, regulated by the rates charged by the Equitable Society; and that the money will be paid by the Insurance Office at the end of the year in which the life of the nominee may happen to fail.*

Upon these suppositions it is manifest, that the annuity purchased must be just adequate to pay the premium upon the sum insured, and to cover the given rate of interest on the sum advanced by the purchaser. But the sum advanced by the purchaser is made up of the amount of the purchase money and the first year's premium for the assurance; that premium being charged, in the first instance, before any payment of the annuity shall have become due. And the sum to be insured must be still greater than the sum advanced by the interest due at the end of the year in which the life may drop; that interest not being covered by the annuity which must have previously ceased. The sum to be insured must therefore be equal to the purchase money, the first year's premium, and the last year's interest, making together the purchase money, and one payment of the annuity. These relations being established,

* The practice of most of the Insurance Offices is to pay the money within three months after proof is rendered of the death of the party.

Let s = the Purchase Money,

a = the Annuity,

p = the Premium per £1 for the Assurance,

i = the Interest of £1 at the given rate,

v = the Present Value of £1 due 1 year hence,

and $d = 1-v$ = the Discount of ditto.

Then, disregarding the expences of stamps, &c. we have

$s+a$ = the sum to be insured,

$p(s+a)$ = the annual premium for ditto,

$s+p(s+a)$ = the purchase money, and the first year's premium, making together the sum advanced by the purchaser in the first instance, and on which interest must be computed. This interest is therefore expressed by $i\{s+p(s+a)\}$; to which adding the premium for insurance, the result becomes

$$p(s+a)+i\{s+p(s+a)\}=\{i+p(1+i)\}s+p(1+i)a,$$

which being made equal to the annuity we obtain

$$\{i+p(1+i)\}s+p(1+i)a=a \quad \text{or} \quad s=\frac{a\{1-p(1+i)\}}{i+p(1+i)}$$

or by dividing the numerator and denominator of this expression by $1+i$ (observing that $\frac{1}{1+i}=v$, and $\frac{i}{1+i}=1-\frac{1}{1+i}=1-v=d$) it becomes transformed into $s=\frac{a(v-p)}{d+p}$, which by further substituting $1-d$ for v becomes transformed into

$$s = \frac{a(1-d-p)}{d+p} = \frac{a\{1-(d+p)\}}{d+p} = a\left\{\frac{1}{d+p}-1\right\}$$

And if the annuity be taken = £1, we have

$$s = \frac{1}{d+p} - 1$$

a simple formula, by means of which a Table may be easily constructed.

Thus, reckoning interest at 5 per Cent., by Table III. we have $d = .047619$, and taking the values of p as determined in page 247, we find the present value of an annuity of £1 on a life of

$$20 = \frac{1}{.047619+.021794} - 1 = 13.034^*$$

$$30 = \frac{1}{.047619+.026672} - 1 = 12.461$$

$$40 = \frac{1}{.047619+.033975} - 1 = 11.256$$

$$50 = \frac{1}{.047619+.045301} - 1 = 9.762$$

$$60 = \frac{1}{.047619+.063661} - 1 = 7.986$$

In this manner Table XXV. has been constructed, giving the value of an annuity of £1, at the several rates of 5, 6 and 7 per Cent.

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* If we suppose the Purchaser satisfied with 3 per Cent. for his money, the rate at which the premium of Assurance is computed, the value of an Annuity of £1 on a Life of 20, found by this method, would be, $\frac{1}{.029126+.021794} - 1 = 18.6386$, the same as already given in column A. of the Table in page 221.

Another view in which this subject may be taken is, to determine what rate should be required for advancing a given sum on a Life Annuity, so as to allow the party advancing the money a given rate of interest, beside securing the repayment of his capital by a Life Assurance. This object may be effected by the foregoing Formulæ, by supposing s given, and a as the quantity required: for it follows that if

$$\frac{a(v-p)}{d+p} = s, \qquad a(v-p) = s(d+p), \text{ or}$$

$$a = \frac{s(d+p)}{v-p}$$

or, if in this expression we substitute $1-v$ for d, we have $a = \frac{s(1-v+p)}{v-p} = \frac{s\{1-(v-p)\}}{v-p} = s\left\{\frac{1}{v-p}-1\right\}$

and if s be taken $= £1$, we finally deduce

$$a = \frac{1}{v-p} - 1.$$

Thus, reckoning interest at 5 per Cent. as before, it will be found that $v = .952381$, and the rate of annuity to be charged for each £1 advanced on a life of

$$20 = \frac{1}{.952381 - .021794} - 1 = .07672 \text{ or } 7.672 \text{ for } £100$$

$$30 = \frac{1}{.952381 - .026672} - 1 = .08025 \text{ or } 8.025 \quad —$$

$$40 = \frac{1}{.952381 - .033975} - 1 = .08884 \text{ or } 8.884 \quad —$$

$$50 = \frac{1}{.952381 - .045301} - 1 = .10244 \text{ or } 10.244 \text{ for } £100$$

$$60 = \frac{1}{.952381 - .063661} - 1 = .12521 \text{ or } 12.521 \text{ —}$$

In this manner Table XXVI. has been constructed, showing the rate of annuity to be charged for every £100 advanced, at the several rates of 5, 6 and 7 per Cent.

The Formulæ deduced in this section for determining the value of a given annuity, or to find the annuity to be charged for a given sum, are equally applicable when the annuity is made payable during the joint existence, or last survivor of two or more lives, provided the value of p be varied accordingly.

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6. *Of the Legal Value of a Life Annuity.*

In the Assessment of Duties chargeable upon Legacies, according to the 36 and 55 of Geo. III, Life Annuities are directed to be valued by the Tables annexed to the first mentioned Statute, which Tables have been calculated from the rate of mortality exhibited by the Northampton Table, reckoning interest at 4 per cent: these give the estimated value of an Annuity of £100 on a Single Life of any age, or on any combination of Two Joint Lives. When such Tables are not at hand the value of an Annuity may be calculated sufficiently near to determine

the amount of Duty by the XXXVII. Table given in this work.*

To prevent misconstruction of the term *Legal Value*, here employed, we should remark that, although Government have found it necessary to define the Value of a Life annuity for the purposes we have alluded to, it does not appear that any Legislative enactments affect the market value of this species of property.

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7. *Practical Examples.*

Example I.

What is the present value of an Annuity of £50 on a life of 40, estimating the average duration of human life by the Equitable Table, and reckoning interest at 4 per cent.?

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* The Amount of Duty chargeable on Legacies varies with the degree of consanguinity according to the following Scale, viz.—when a Legacy is bequeathed to, or by intestacy devolves for the benefit of,

	£	s.	*Cent.*
A Child, or any descendant of a Child, or the Father or Mother, or any lineal ancestor, of the deceased	£1	0	0
A Brother or Sister, or any descendant of a Brother or Sister, of the deceased	3	0	0
A Brother or Sister of the Father or Mother of the deceased, or any of their descendants	5	0	0
A Brother or Sister of a Grandfather or Grandmother of the deceased, or any of their descendants	6	0	0
Any person not related, as above, to the deceased	10	0	0

Any Legacy under the value of £20, or such as may be left to, or devolve for the benefit of, a Husband or Wife of the deceased, is exempted from Duty.

By Table XXIV, opposite the given age, and under 4 per cent., it appears, that the value of an Annuity of £1 on the proposed life is 14.9390, and this multiplied by 50 gives $14.939 \times 50 = 746.950$ = £746 19, the present value required.

But if the rate of mortality be estimated by the Northampton register, it will be found by Table XXXVI. that the present value of a similar annuity, on the same life, at the same rate of interest, is only $13.197 \times 50 = 659.850$ = £695 17.

EXAMPLE II.

Required the present value of an Annuity of £40 on the joint lives of two persons whose ages are respectively 40 and 50 years, estimating the rate of mortality by the Equitable Table, and reckoning interest at 4 per cent.?

By Table XXVII, looking for the older age in the first column, and the younger age in the next, opposite the latter and under 4 per cent. the value of an Annuity of £1 on the proposed ages is 10.5471, this multiplied by £40, the Annuity proposed, gives $10.5471 \times 40 = 421.884$ = £421 17s 8d the Answer.

If we estimate the rate of mortality by the Northampton register, the value of a like annuity, on the same ages, and at the same rate of interest, as found by Table XXXVII. is only $8.834 \times 40 = 353.360$ = £353 7s 2d.

Example iii.

What is the present value of an Annuity of £75, on the joint lives of two persons, of the respective ages of 57 and 74, estimating the rate of mortality by the experience of the Equitable, and reckoning interest at 6 per cent.?

Here the ages proposed are not to be found in the Table, but if we seek for the *older age* in the first column, and take out the two numbers between which the *younger age* falls, the required value may be considered as one of four arithmetical means between the numbers so extracted; its place in the series corresponding with the distance of the younger age, from either of the two ages between which it falls.

Thus, by Table xxvii, the present value of an annuity of £1, on the joint lives of 74 & 54 is 4.7201
ditto on - - - - - - - - - 74 & 59 is 4.5588
between which the several combinations of 74 & 55, 74 & 56, 74 & 57, and 74 & 58 are manifestly included; the combination proposed being the third term from one extreme, or the second term from the other; on the supposition of the Annuity decreasing uniformly, as the younger age increases, it therefore follows that the value of an Annuity of £1, on the joint lives of 74 & 57, is less than the first of the above numbers by 3-5ths of the difference between the two, or greater than the second by 2-5ths of the

same difference. Hence, from the value of an Annuity of £1 on the joint lives of 74 & 54 = 4.7201
deduct ditto - - - - - - - 74 & 59 = 4.5588

difference for the 5 years diff. in younger age = .1613
One fifth of which = difference for each year = .0323
the double of it, = the difference for 2 years = .0646
and this added to the annuity on the joint lives of 74 and 59 (4.5588) gives - - } 4.6234

for the approximated value of £1 Annuity on the joint lives of 74 and 57; and this multiplied by £75, the amount of Annuity, gives $4.6234 \times 75 = 346.775$ = £346 15s 6d for the present value required.

By the same method the value of an Annuity on any other ages, not found in the Table, may be approximated; and if a greater degree of accuracy be required, the method of interpolation, pointed out in pages 191—3, may be successfully applied.

Example iv.

Required the present value of an annuity of £120, on the joint existence of three persons, each aged 60 years, estimating the rate of mortality by the experience of the Equitable, and reckoning interest at 3 per cent.?

Referring to Table xxiii, opposite the age of 60, and dividing the number in column N by that in column D, agreeably to what has been stated in pages 234—6, we find the present value of £1 annuity on

the proposed lives = 361832 ÷ 60078.8 = 6.0226, and this multiplied by 120, the amount of annuity, gives 6.0226 × 120 = 722.5120 = £722 10s 3d, the answer.

The value of an annuity on *Three Joint Lives* may also be approximated by the assistance of Single Life and Two Joint Life Tables, in the following manner, viz.—

*Find, as in the foregoing Examples, the value of £1 annuity on the joint existence of the two oldest lives, and seek for the age of a Single Life on which £1 annuity would be equivalent to the result; then find the value of the given annuity on the joint existence of this life and the youngest of the lives proposed, and the result will be the approximated value required.**

Thus, referring to the last Example, we find by Table XXVII that the value of £1 annuity on Two Joint Lives, each aged 60, is at 3 per cent. = 7.7082, which by Table XXIV, at the like rate of interest, is found to be equivalent to a similar annuity on a Single Life of $69\frac{3}{4}$. Hence, proceeding as directed in Example III, we find that the value of

£1 annuity on the Joint Lives of 69 & 60	= 6.1989
ditto on - - - - - - - - 70 & 60	= 6.0198
difference - - - - - - - - - - - -	.1791

* When the three Lives are of equal ages, any one of them may be considered as the youngest.

$\frac{3}{4}$ of this difference taken from the first, or $\frac{1}{4}$ of it added to the second, gives 6.0645 as the approximated value of £1 annuity on the *Three Joint Lives proposed*, and this multiplied by 120, produces $6.0645 \times 120 = 727.7400 =$ £727 14s 10d, which exceeds the correct value already found by little more than £5.

Example v.

Required the present value of £25 annuity on the Joint Lives of 40, 50, & 60, estimating the rate of mortality by the experience of the Equitable, and reckoning interest at 4 per cent.?

By Table xxvii, the present value of £1 annuity on the Joint Lives of 50 & 60 is, at 4 per cent. = 8.1651, which being referred to Table xxiv, under 4 per cent., is found to correspond to a Single Life of 66.6; hence, proceeding as directed in Example iii, the value of

£1 annuity on the Joint Lives of 66 & 40 = 7.5018
ditto on - - - - - - - - 67 & 40 = 7.2636
difference - - - - - - - - - - - - .2382

6-10ths of this difference, taken from the first, gives 7.3589 for the approximated value of £1 annuity on the lives proposed. Hence $7.3589 \times 25 = 183.9725$ = £183 19s 5d the answer.

Proceeding in like manner by Table xxxvii, it will be found that the value of the same annuity, by the Northampton rate of mortality, is only £150 1s 6d.

Example vi.

What is the present value of £50 annuity on the last survivor of 40 & 50, estimating the rate of mortality by the experience of the Equitable, and reckoning interest at $3\frac{1}{2}$ per cent.?

By the definitions given in the first section of this chapter, it is manifest that an annuity on the joint existence of two lives, and a like annuity on the last survivor of them, are together equivalent to the sum of similar annuities on the two lives taken separately; for, in either case, one annuity would cease with the first death, and the other with the second; it therefore follows that

The value of an annuity on the Last Survivor of two lives, is equal to the aggregate value of similar annuities on the two lives taken singly, diminished by the value of a like annuity on their joint existence.

Thus, by Table XXIV, the present value of £1 ann. on a life of 40, at $3\frac{1}{2}$ p. ct.	= 15.9082
ditto on a life of 50 - - - - - - -	= 13.2787
their sum - - - - - - - - - - -	29.1869
and by Table XXVII, the value of £1 per annum on joint lives of 40 & 50	= 11.0382
difference = the value of £1 per annum on the last survivor of 40 & 50 -	= 18.1487

which, multiplied by 50, the annuity given, produces 907.435 = £907 8s 9d, the answer.

Example vii.

What annuity should be granted on a life of 40 for £100 sunk, estimating the rate of mortality by the experience of the Equitable, and reckoning interest at $3\frac{1}{2}$ per cent. ?

By Table xxiv, the present value of £1 annuity on the proposed life is found = 15.9082.

Hence, 15.9082 : £1 : : 100 : 100÷15.9082 = 6.286 = £6 5s 9d, the answer.

In the same manner it may be shown that the annuity to be granted on any assigned life, or on any combination of lives, for a given sum paid down, is found by dividing the sum given by the present value of £1 annuity on the life or lives proposed.

Thus, by the Equitable Table, at $3\frac{1}{2}$ per cent. the annuity to be granted for £100 sunk,

on a life of 21 is	100÷19.6021 =	5.101 =	£	5	2	0
— — 30 is	100÷17.9517 =	5.570 =		5	11	5
— — 40 is	100÷15.9082 =	6.287 =		6	5	9
— — 50 is	100÷13.2787 =	7.531 =		7	10	8
— — 60 is	100÷10.4813 =	9.541 =		9	10	10
— — 70 is	100÷ 7.3894 =	13.533 =		13	10	8
— — 75 is	100÷ 5.8133 =	17.202 =		17	4	0

These annuities are supposed to be payable yearly, and to cease with the last payment that may

become due prior to the death of the nominee; but if it be required to make the annuity payable half-yearly, the divisor must be increased by $\frac{1}{4}$, or .25; if quarterly, by $\frac{3}{8}$ or .375; and if the annuity be made payable from day to day up to the death of the nominee, the divisor ought to be increased by $\frac{1}{2}$ or .5.*

If we suppose the average mortality among persons sinking money on life annuities, not to exceed the rate deduced from the experience of the Equitable,† by comparing the foregoing results with the "Rates of Government Annuities on Single Lives for the whole duration," as shown by Table II, it will be found that, when 3 per cent. stock is at 85, (the price when money bears $3\frac{1}{2}$ per cent. interest,) Government grants Life Annuities upon terms disadvantageous to itself, and more particularly so when it is considered that such annuities are payable half-yearly, with a fraction equivalent, on an average, to the time intervened between the last payment and the death of the nominee.

The observations here made with respect to Go-

* These increments are, however, nothing more than approximations; the correct quantities to be added on account of the annuities being payable at shorter periods than 1 year, will be shown in Section 9.

† Instead of the rate of mortality among life annuitants exceeding that which obtains among assured lives, it is not improbable that the former falls short of the latter, because few persons, conscious of possessing weak or diseased constitutions, would sink money on life annuities.

vernment, apply, with increased force, to some of the Insurance Offices, which hold out to the public much more liberal terms; and such, in fact, as would soon swallow up their Subscribed Capitals, if they could persuade any considerable proportion of the public to avail themselves of their liberality.

Example viii.

Required the annuity which ought to be granted on the last survivor of two persons aged 40 & 50, for £85 paid down, supposing the annuity made payable from day to day up to the second death; the rate of mortality to be estimated by the experience of the Equitable, and the interest of money to be computed at $3\frac{1}{2}$ per cent.?

In Example vi it has been shown that the present value of an annuity of £1, payable yearly, and ceasing with the last payment which may become due prior to the second death among the proposed nominees, is 18.1487, to which adding $\frac{1}{2}$ or .5, on account of the annuity being made payable from day to day, the result is 18.6487. Hence, $85 \div 18.6487 = 4.558 =$ £4 11s 2d.

By the "Rates of Government Annuities on Two Joint Lives and the Life of the Survivor," given in Table ii, it will be found that Government would grant an annuity of £4 16s on the proposed lives for £100 three per cent. stock, which would cost £85 money, when interest is $3\frac{1}{2}$ per cent.

Example ix.

What is the present value of a Survivorship annuity of £60 on the life of *A*, aged 40, after the extinction of another life *B*, aged 50; estimating the rate of mortality by the experience of the Equitable, and computing interest at $3\frac{1}{2}$ per cent.?

Here it is manifest, that an annuity on the joint lives of *A* and *B*, and a similar annuity during what may remain of *A*'s life after *B*'s death, are, together, equivalent to a like annuity on *A*'s life for the whole duration. Hence it follows that

The present value of a Survivorship Annuity to A after the decease of B, is equal to the value of a similar annuity on the life of A, diminished by the value of a like annuity on the joint lives of A and B.

Thus by Table xxiv, £1 an. on a life of 40 = 15.9082
and by Table xxvii, £1 annuity on the joint lives of 40 & 50 - - = 11.0382

difference - - - - - - - - - 4.8700

showing the value of £1 per annum on the life of *A* after the decease of *B*; this multiplied by 60, the annuity proposed, gives $4.87 \times 60 = 292.20 = £292\ 4s$, the answer.

Similarly it may be shown that the present value of a survivorship annuity of £100, on the life of *A*, aged 40, after the decease of another person *B* aged

40, is by the Equitable Table at 4 per cent. = 100(14.9390 − 11.9071) = 303.190 = £303 3s 10d. But if the rate of mortality be estimated by the Northampton Table, the present value of the like annuity, on the same contingency, and at the same rate of interest, found by Tables XXXVI and XXXVII, is 100(13.197 − 9.820) = 337.7 = £337 14s.

In this case, as well as in many others which might be selected, the value of the annuity is found to be greater by the Northampton Table than by that deduced from the experience of the Equitable. This example tends to confirm an observation made by Mr. Benjamin Gompertz, (Actuary to the Alliance Assurance Company) in an ingenious paper read before the ROYAL SOCIETY, June 29, 1820, wherein he states, that the proper mode of regulating the premiums for life assurances, &c. is by employing what may appear to be the correct average rates of interest and mortality, to form the calculation upon, and to the results make such additions as may leave an adequate portion for the security, profit and expences of the Insurance Companies; assigning as a reason—"For it does not seem possible, in "the various beneficial applications which can be "made from a proper knowledge of this branch of "the mathematics, to judge uniformly how to adapt "tables of mortality, which are not correct in themselves, connected with a rate of interest which is "not the average rate made in reality, so that the "advantage may tend to any one direction."

The conductors of Insurance Offices ought to be aware of this circumstance, lest the advantage which the Northampton Table affords them in some contingencies, should happen to turn to their disadvantage in others.

Example x.

What annual premium, payable at the beginning of each year during the joint lives of two persons, *A* & *B*, should be required for securing a survivorship annuity of £60 to *A*, aged 40, after the decease of *B*, aged 50, estimating the rate of mortality by the experience of the Equitable, and reckoning interest at $3\frac{1}{2}$ per cent. ?

By Table xxvii it will be found that the present value of £1 per annum, payable at the *end* of the year during the joint lives of *A* & *B*, is 11.0382, to which adding the £1 to be paid down at the beginning of the first year, we have 12.0382 for the present value of £1 per annum, payable at the *beginning* of the year during the joint lives of *A* & *B*. And the present value of £1 per annum to *A* after the death of *B*, as found by the last example, is 4.8700. Hence, by the Rule given in page 69, we have $\frac{4.8700}{12.0382} \times 60 = .40454 \times 60 = 24.27240 = £24$ 5s 5d, the answer.

These two Examples point out the construction of Table xxx.

Example xi.

Find, by the assistance of Table xxx, what consideration should be required, either by Single or Annual payment, for a Survivorship Annuity of £100 to *A*, aged 50, after the decease of *B*, aged 40?

Here, multiplying the numbers extracted from the Table, by the amount of annuity proposed, we find

The Single payment = $2.2405 \times 100 =$ £224 1 0
The Annual payment = $.18611 \times 100 =$ £ 18 12 3

Example xii.

What is the present value of a Survivorship Annuity of £100 to *A*, aged 50, after the decease of *B*, aged 43; estimating the rate of mortality by the experience of the Equitable, and reckoning the interest of money at $3\frac{1}{2}$ per cent.?

£1 annuity on a life of 50, by Table xxiv = 13.2787
ditto on the joint lives of 50 & 43 found as in Example iii. - - - - - = 10.7594
difference = £1 per annum, to = 50 after 43 = 2.5193
and this multiplied by £100 gives £251 18s 7d, the answer.

The required value may also be approximated by Table xxx in the following manner. Thus,

£1 per annum to 50 after 45 - - - = 2.7051
ditto — — to 50 after 40 - - - = 2.2405

difference - - - - - - - - - - - - .4646

2-5ths of this difference taken from the first of these numbers, or 3-5ths of it added to the second, gives 2.5193 for the value of each £1 annuity, the same as before.

By the same mode of proceeding either the Single or Annual premium for a Survivorship Annuity may be determined, provided either of the ages be found in the Table.

Example xiii.

Find, by the assistance of Table xxx, what annual premium should be required for a Survivorship Annuity of £70 to *A*, aged 52, after the decease of *B*, aged 43?

Proceeding as directed in the last Example, we find the Annual premium for £1 annuity to

50 after 43 - - - - - - - = .21468
55 after 43 - - - - - - - = .18135

difference - - - - - - - = .03333

2-5ths of this difference taken from the first of the above numbers, or 3-5ths of it added to the second, gives .20135, the approximated premium for each £1 of the annuity proposed. This multiplied by £70 gives £14 1s 11d, the answer.*

* In calculating the premium to be required for a Survivorship Annuity to *A*, after the death of *B*, it is not very material whether that

On further examination of Table xxx we find, that in some cases the value of an annuity to a younger life, after the extinction of an older one, increases with the increased ages of the parties, and in other cases diminishes: but if the annuity be made payable to an older life, after the extinction of a younger one, the value generally diminishes as the parties grow older. Thus, taking two lives of 10 & 20, and supposing them jointly to survive, we find the present value of £1 annuity to

10 after 20 =	4.0089,	ditto to	20 after 10 =	2.3250
20 — 30 =	4.3356,	—	30 — 20 =	2.5144
30 — 40 =	4.5452,	—	40 — 30 =	2.5017
40 — 50 =	4.8700,	—	50 — 40 =	2.2405
50 — 60 =	4.8159,	—	60 — 50 =	2.0185
60 — 70 =	4.6200,	—	70 — 60 =	1.5281
70 — 80 =	4.1141,	—	80 — 70 =	.9288

On the contrary, it will be found that the annual premium for an annuity to a younger life after the extinction of an older one, always increases with the increased ages of the parties; but if the annuity be made payable to an older life after the extinction of a younger one, the premium generally increases for

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Annuity be made payable yearly, or by two or more instalments; for when the annuity is payable yearly, the calculation supposes the first payment to become due at the end of the year in which *B*'s life may fail, and not at the end of one year from *B*'s death. It is however manifest that an Annuity *to be entered upon at B's death* is of greater value if payable half-yearly or quarterly, than if the same were payable yearly.

some years, and afterwards decreases towards the more advanced periods of life.

Thus the annual premium for a Survivorship Annuity of £1 to

10	after	20	= .21730,	ditto to	20	after	10	= .12603
20	—	30	= .26376,	—	30	—	20	= .15297
30	—	40	= .31549,	—	40	—	30	= .17365
40	—	50	= .40454,	—	50	—	40	= .18611
50	—	60	= .50893,	—	60	—	50	= .21331
60	—	70	= .67334,	—	70	—	60	= .22271
70	—	80	= .96230,	—	80	—	70	= .21725
10	after	40	= .43760,	ditto to	40	after	10	= .08240
20	—	50	= .59103,	—	50	—	20	= .09363
30	—	60	= .80811,	—	60	—	30	= .09539
40	—	70	= 1.1878,	—	70	—	40	= .08554
50	—	80	= 1.9424,	—	80	—	50	= .07237
10	after	60	= 1.0420,	ditto to	60	after	10	= .04401
20	—	70	= 1.5857,	—	70	—	20	= .04426
30	—	80	= 2.7711,	—	80	—	30	= .03554
10	after	70	= 1.7514,	ditto to	70	after	10	= .02787
20	—	80	= 3.0891,	—	80	—	20	= .02441

The circumstance here noticed relative to the decrease of the annual premium, is of some importance to Institutions granting survivorship annuities.

For instance, suppose a society formed of a considerable number of combinations of lives, each consisting of one person of 60 and another of 30,

for the purpose of securing to each combination a Survivorship Annuity of £1, in the event of the person of 60 outliving that of 30; charging an annual premium of .09539, then supposing the money received by the society to be improved without loss or delay at $3\frac{1}{2}$ per cent., without any incidental expences incurred, and the rate of mortality among the whole mass to be correctly expressed by the experience of the Equitable, it is manifest that such society would have just enough to answer the claims that would come upon it, provided each of the proposed combinations were to continue to subscribe during the joint lives of the persons of which it is composed.

But if, at the end of 10 years, those combinations which survive entire were to leave the former society, and unite to form a new one for securing to themselves the same benefits, they might accomplish their object for the reduced premium of .08554 for each combination. It therefore follows, that the original society must lose just as much by their leaving, as they gain by the formation of the new society.

And if the original society can do no more than fulfil its engagements, when all the combinations continue to subscribe, until their respective claims are determined, it necessarily must become insolvent by the surviving combinations leaving it to its fate, although they get not a farthing back of what they shall have previously contributed.

From what has been stated, it plainly appears an

unsafe transaction for an Insurance Office to grant a Survivorship Annuity to a person of a given age, after the decease of another person much younger, for a constant equated annual premium, (unless that premium be considerably more than equivalent to the average annual risk,) for such equated premium must be less than the risk for the first few years, and afterwards greater than the risk; the parties contracting with the Office would evidently have an interest to pay the premium, so long as the risk covered would be of greater value than the premium paid; but, some years after, finding that their object might be effected at a cheaper rate by a new Insurance, they would not scruple to discontinue their payments, leaving the Office in the situation which we have pointed out relative to the supposed society, and probably dissatisfied that they were not allowed some part of the premium paid as a consideration for surrendering the policy.

Example xiv.

What is the present value of £1 per annum for the next five years, dependent on a life of 40, estimating the rate of mortality by the experience of the Equitable, and computing the interest of money at 4 per cent.?

Referring to Table xxi, folio headed 4 per cent., and supposing a fund formed sufficient to provide for 465.733 similar annuities, by reasoning as in

page 220, it is evident that the present value of the payments becoming due at the end of the

1st year	= £442.214	as given in column D opposite	41
2nd -	= 419.814	— — — —	42
3rd -	= 398.481	— — — —	43
4th -	= 377.992	— — — —	44
5th -	= 358.316	— — — —	45
Total -	1996.817		

making the aggregate value of the supposed fund, and this divided by 465.733, the number of annuities supposed, gives, for the average value of each, $\frac{1996.817}{465.733} = 4.2875 =$ £4 5s 9d, the answer.

The present value of the supposed fund may however be obtained in a simpler manner by deducting from the number in column N, opposite the given age (40), the number in the same column opposite the advanced age (45), making $6957.623 - 4960.806 = 1996.817$, the same as before.

Were the annuity payable at the beginning of each year for five payments, by taking an average of 465.733 similar annuities, as before, it may be shown, that the present value of the

1st payment	= 465.733	found in col. D opposite	40
2nd —	= 442.214	— — —	41
3rd —	= 419.814	— — —	42
4th —	= 398.481	— — —	43
5th —	= 377.992	— — —	44
Total —	2104.234,	which may otherwise be	

found by deducting the number in column **N**, opposite the age of 44, from that in the same column opposite the age of 39. This total divided by 465.733, the number of annuities proposed, gives, for the average value of each $\frac{2104.234}{465.733} = 4.5181 =$ £4 10s 4d.*

The same explanation is equally applicable to any other age, or period of payment, it is therefore manifest that

The present value of a *Temporary Annuity* of £1 on any assigned age is found by the following

RULE.—*From the number in column* **N** *opposite the given age, deduct the number in the same column opposite the advanced age,* (i. e. *the age of the assigned life when the annuity ceases,) and divide the remainder by the number in column* **D** *opposite the age given.*

If the annuity be made payable at the beginning of the year, take the numbers from column **N** *opposite ages respectively* **1** *year younger than the given and advanced ages, and proceed as before.*

The same Rule is equally applicable to two or three Joint Lives, provided Tables be formed as directed in pages 227—236.

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* If, instead of £1, the annuity proposed were £60, its present value, if payable at the end of the year, would be 4.2875×60=£257 5s; or if payable at the beginning of the year 4.5181×60=£271 1s 9d.

Example xv.

Suppose £40 per annum, in Government Long Annuities, to be left to *A*, aged 30, during life, with remainder to *B*, or his heirs, in the event of *A* dying before the expiration of those annuities, (in January 1860): it is required to determine the respective interests of *A* and *B*, estimating the rate of mortality by the experience of the Equitable, and the improvement of money at $3\frac{1}{2}$ per cent.?

Supposing the Long Annuities to continue for 35 years, and disregarding the circumstance of their being payable half-yearly, we find by Table viii, that the collective shares of *A* and *B*, in each £1 annuity, is - - - - - - - - - - - 20.0007

share of *A*, found as in last Example $= \frac{15995.845 - 1232.513}{891.051} = 16.5684$

difference for *B*'s share - - - - - = 3.4323

Hence $16.5684 \times 40 =$ £662 14s 9d = *A*'s share.

and $3.4323 \times 40 =$ £137 5s 10d = *B*'s share.

The legal value of *A*'s share, found in like manner by Table xxxv, at 4 per cent. is £562 19s. 0d.

that of *B* - - - - - - - - - £183 12s. 8d.

Example xvi.

What sum, paid down, should be required for securing to a person, now 30 years of age, an annuity of £1 to commence at the age of 60, the first pay-

ment becoming due when he attains the age of 61, and to be continued during the remainder of his life, estimating the rate of mortality by the experience of the Equitable, and the improvement of money at $3\frac{1}{2}$ per cent.?

Referring to Table XXI, folio headed $3\frac{1}{2}$ per cent, and supposing 891.051* persons of the proposed age to join to form a fund, sufficient when improved at $3\frac{1}{2}$ per cent., to provide a Deferred Annuity of £1 to such as may survive the age of 60, by reasoning as in page 220, it is manifest that the present value of the

1st payment = £181.265 found in col. D opposite 61
2nd — = 169.685 — — — 62
3rd — = 158.563 — — — 63
4th — = 147.893 — — — 64

and so on to the end of life, the sum of which, found in column N, opposite the age of 60, = 2027.573 = the aggregate value of the supposed fund; and this, divided by the number of contributors, gives $\frac{2027.573}{891.051} = 2.2755$ = £2 5s 6d for the average value of each Annuity, or the sum which ought to be paid down by each, as an equivalent to the annuity proposed. If, instead of £1, the annuity be taken £100, its present value will be 2.2755×100 = £227 11s 0d.

From what has been stated, it is obvious that

* The number in column D, opposite the age of 30.

The present value of a Deferred Annuity of £1, *on any assigned life, is found by dividing the number in column* N *opposite the age at which the annuity is to commence, by the number in column* D *opposite the present age.*

If the annuity be payable half yearly, or quarterly, let the dividend be increased by $\frac{1}{4}$, *or* $\frac{3}{8}$, *of the number in column* D *opposite the age at which the annuity is to commence, and proceed as before.*

Thus the present value of the annuity proposed in the foregoing Example, supposing it were made payable half-yearly, is $\frac{2027.573 + \frac{1}{4}(193.448)}{891.051} = 2.330$ agreeing with the value given in Table XXVIII, which has been constructed, on the supposition of the annuity being payable half-yearly, according to the latter part of the above Rule.

Example XVII.

What annual premium should be required of a person aged 30, for an annuity of £1 to commence at the age of 60, estimating the rate of mortality by the experience of the Equitable, and the improvement of money at $3\frac{1}{2}$ per cent; also supposing the premium payable at the beginning of each year, until the assigned life shall attain the age of 60, (provided it does not fail sooner,) and the annuity to be made payable half-yearly?

Conceiving 891.051 persons of the proposed age

to join into a society to contribute for similar annuities, it follows, from what was stated in the last Example, that a fund sufficient to provide Deferred Annuities of £1 each, for such of them as may survive the age of 60, must, in present money, be $2027.573+\frac{1}{4}(193.448) = £2075.935$. Also, by what was stated in Example XIV it is manifest, that a contribution of £1 per annum from each, payable at the beginning of the year, until they shall have attained the age of 60, found by deducting the number in column N opposite the age of 59, from the number in the same column opposite the age of 29, is in present money $= 16886.896-2221.021 = £14665.875$; hence, by the Rule, given in page 69, we divide the former sum by the latter, and find $\frac{2075.935}{14665.875} = .1416 =$ 2s 10d for the required premium, agreeing with that given in Table XXIX, which has been constructed upon the principles here explained.

From what is here stated, it is obvious that

The annual premium for a Deferred Annuity of £1 *on any assigned age, is found by dividing the number in column* N, *opposite the age at which the annuity is to commence (increased by* $\frac{1}{4}$ *or* $\frac{3}{8}$ *of the number in column* D, *at the same age, if the annuity be made payable half-yearly or quarterly) by the difference between the numbers in column* N, *opposite ages respectively* 1 *year younger than the present and advanced ages.*

Example xviii.

What consideration should be required, either by single or annual payment, for a Deferred Annuity of £60, to a person now 30; the annuity to commence at the age of 50, and payable yearly during the remainder of life; estimating the rate of mortality by the Northampton Table, and the improvement of money at 3 per cent.?

Referring to Table xxxv. we find, by the Rule given in page 277, that the single payment for £1 annuity is $\frac{8104.5434}{1806.5618} = 4.4862$; for £60 annuity the single premium is therefore $= 4.4862 \times 60 =$ £269 3s 5d.

And, by the Rule in last page, the annual premium for £1 annuity is $\frac{8104.543}{32376.61 - 8756.25} = \frac{8104.543}{23620.36} = .34312$: for £60 annuity, the annual premium is, therefore, $.34312 \times 60 =$ £20 11s 9d.

Example xix.

Find, by the assistance of Tables xxviii & xxix, what consideration should be required, either by single or annual payments, for a Deferred Annuity of £100, to a person now 35, the annuity to commence at the age of 50, and to be payable half-yearly during the remainder of life?

Multiplying the numbers taken from the Tables by the amount of annuity proposed, we find
The single payment $= 6.589 \times 100 =$ £658 18s 0d
The annual payment $= .5977 \times 100 =$ £ 59 15s 5d

By reference to the "Rates of Government Annuities on Single Lives, deferred for given periods," shown by Table II, opposite the age of 35, and under 15 years, it will be found that however low the interest of money may be, Government would grant the annuity here proposed for $4.898 \times 100 =$ £489 16s. If therefore the rate of mortality be correctly expressed by the experience of the Equitable, it follows that Government Deferred Annuities now form an eligible mode of investment for individuals, but a most losing concern to the nation.

Example xx.

What sum paid down is equivalent to an annuity of £200, on the Joint Lives of two persons of the respective ages of 20 & 40; the annuity to be deferred for 20 years, and the interest of money to be computed at 3 per cent.?

Estimating the rate of mortality by the experience of the Equitable, and proceeding as directed in page 277, we find by Table xxii, difference of age 20 years, that the required present value becomes $=$
$\frac{5506055}{1854173} \times 200 = 2.9696 \times 200 =$ £593 18s 5d.

But if the rate of mortality be estimated by the Northampton Table, the present value of the proposed annuity, found in like manner by the Table given in page 231, is $\frac{10090788}{5718763} \times 200 = 1.7645 \times 200$ = £352 18s.

On comparing these results, the reader will perceive the importance of procuring the best possible information concerning the rate of human mortality: for if either of the foregoing sums be correct, the other must be egregiously erroneous, though both are calculated at the same rate of interest. Nor will he be at a loss to account for the difference sometimes occurring in the valuations made by different persons, of the same property.

EXAMPLE XXI.

What sum might be given for an annuity of £200, on the life of a person in his 40th year, allowing the purchaser a net interest of 5 per cent., after securing the repayment of his principal at the annuitant's death, by means of an assurance effected at the rate of premiums charged by the Equitable Society?

Referring to Table XXV, opposite the age of 40, and under 5 per cent., the required value in years' purchase is found to be 11.256; and this multiplied by £200, the annuity proposed, gives 11.256×200 = £2251 4s, the answer.

For a further elucidation of this subject the reader is referred back to section 5, page 247.

Example xxii.

What annuity should be required of a person in his 50th year, that is tenant for life to an entailed estate, for an advance of £5000, allowing the lender a net interest of 5 per cent., beside the premium necessary to secure the repayment of the principal by a life assurance, at the rates charged by the Equitable?

By Table xxvi, opposite the age of 50, and under 5 per cent., the annuity to be required for each £100 is found to be 10.244; and this multiplied by 50, the number of £100 in the proposed loan, gives $10.244 \times 50 =$ £512 4s 0d, the answer.

In comparing the rates of annuities to be granted according to the Equitable, $3\frac{1}{2}$ per cent., given in page 261, with the rates to be required for advancing money on annuities, as exhibited by Table xxvi, it must not be concluded that the difference is profit; for when a Public Company grants a life annuity, the rate to be allowed ought to be governed by the interest afforded by the public funds, where the greater part of its receipts must necessarily be invested: nor is it likely to derive any advantage from either delay or forgetfulness on the part of the annuitant to apply for his payments as they become due: but in advancing money on annuity, the payments are often long delayed, and sometimes with difficulty enforced; and a public Company, like

a private individual, generally secures the repayment of the principal advanced, by a life assurance effected with another Office, and is therefore subject to have an extra premium to pay in the event of the borrower's going abroad, and in danger of having the insurance altogether forfeited by the non-compliance with the conditions of the policy: and, what is still worse, the precarious nature of the annuity laws renders most transactions of this kind subject to litigation, and open to the controul of the Court of Chancery.

EXAMPLE XXIII.

Suppose *A*, aged 50, to come into possession of £70 per annum for life, as a bequest under the will of his uncle, it is required to determine the amount of legacy duty chargeable thereon?

By Table XXXVI, 4 per cent. column, the legal value of the proposed legacy is $11.2649 \times 70 =$ £788.543; three per cent. on this amount being taken, agreeably to the schedule in page 254, gives $788.543 \times .03 = 23.65629 =$ £23 13s 2d, the answer.

If the annuity were made payable during the joint lives of *A*, aged 50, and his wife *B*, aged 45, and the life of the survivor of them, the value of the legacy, found by Tables XXXVI and XXXVII, as directed in the Rule given in page 260, would be £1053.15, on which the amount of duty, estimated at 3 per cent. as before, would be £31 11s 11d.

Example xxiv.

Suppose a number of persons, of the age of 60, to join into a society for the purpose of raising a fund, sufficient to pay £1 to the representatives of each member at the end of the year in which that member may happen to die; it is required to find what present sum each ought to contribute, estimating the rate of mortality among the whole mass by the experience of the Equitable, and the improvement of money at 4 per cent.?

Referring to Table xxi, folio 4 per cent., and dividing the number in column M, opposite the given age, by the corresponding number in column D, as directed in page 246, we find the required contribution $= \frac{83.2896}{144.871} = .57492 =$ 11s 6d.

In this manner Table xxxi has been constructed, representing, at 4, 5, and 6 per cent. the average value of £1, to be received at the end of the year in which an assigned life may fail. By this Table the present value of any reversionary sum, receivable on the extinction of a single life, may be determined.

Thus the present value of £700, to be received at the end of the year in which a life of 60 may fail, is

At 4 per cent. $= .57492 \times 700 =$ £402 8s 10d
— 5 per cent. $= .51060 \times 700 =$ £357 8s 5d
— 6 per cent. $= 45668 \times 700 =$ £319 13s 6d

EXAMPLE XXV.

What is the present value of an *Advowson*, or perpetual right to all future Presentations to a Church benefice; estimating the rate of mortality by the experience of the Equitable, and the improvement of money at 5 per cent.; the annual produce of the living being £350, and the age of the Incumbent 64 years?

Taking the Curate's salary,* and other outgoings for taxes, repairs of the Parsonage house, &c. at £150 per annum, there remains a clear annuity of £200, to be entered upon on the demise of 64, and thence to be continued in perpetuity. Hence by Tables VIII and XXIV, 5 per cent. column,

From the present value of a perpetual an. = 20.0000
deduct the present value of an annuity on a life of 64 - - - - - - - - = 8.3056
and there remains - - - - - - - - 11.6944

which multiplied by the £200 clear annuity, gives $11.6944 \times 200 =$ £2338 17s 7d, the answer.

* Curates' salaries are in some measure regulated by legislative enactments. By the 53 Geo. III. c. 149. it is provided that a Curate's stipend shall in no case be less than £80 per annum; nor less than £100 per annum if the population under his care exceed 300 persons, nor less than £120 per annum if the population exceed 500 persons, nor less than £150 per annum if the population exceed 1000 persons, except, however, in each case, the whole produce of the living should be under these respective amounts: but if the living produce more than £400 per annum, the Curate's salary may exceed these respective allowances by £20 per annum in the first case, or by £50 per annum in the others.

Example xxvi.

What is the value of the next Presentation to a Church living producing an annual income of £250, estimating the rate of mortality by the experience of the Equitable, and the improvement of money at 5 per cent., also supposing the age of the Incumbent to be 64 years?

This, being a successive annuity, dependent on the life of a person to be nominated at the decease of the present Incumbent, differs materially from a survivorship annuity, because the party entitled to the presentation, though he may have designed it for any particular person, will have the further privilege of substituting another, in the event of that person dying before the vacancy may occur. The value of the Presentation to the Clerk appointed must, however, depend on his age at the time he obtains possession of the living; but as that age is now unknown, we have no other means of calculating the value of the Presentation than by assuming the worth of it on coming into possession, at some certain number of years' purchase, to be estimated on the net annual produce of the living, after deducting the Curate's salary, and all other outgoings for taxes, &c.

Assuming these outgoings at £100 per annum, there remains an annuity of only £150, during the life of the Clerk to be presented, as a subject of investment: and if we further assume the age of the

person to be appointed at 30 years on coming into possession, (perhaps not an unfair average) by reference to Table XXIV, under 5 per cent., the then value of the living is found to be 14.6019, or say $14\frac{1}{2}$ years' purchase. Hence, by Table XXXI, the present value of this sum, found as in Example XXIV, is $.55688 \times 14\frac{1}{2} = 8.07476$ years' purchase, which multiplied by the £150 clear annuity, gives $8.07476 \times 150 =$ £1211, the answer.

These Presentations are often referred to calculators for valuation; but, independent of the uncertainties already stated, their values are materially affected by the law of *Simony*,* by which it is rendered illegal to give, directly or indirectly, a valuable consideration for any Ecclesiastical preferment; and every person on being presented to a living is required to make an affidavit to the effect that he has made no simoniacal contract. In the opinion of competent judges no valuable consideration proceeding from the presentee can safely form the basis of presenting a man to a living, whoever may be the contracting parties; though in the case of a parent purchasing a right to present his son, the son being no party to the transaction, greater latitude, in the construction of the law of Simony,

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* So termed from Simon the sorcerer, mentioned in the viiith chapter of the Acts of the Apostles.

See the statutes William and Mary, S. 1. c. 16.; 12 Anne, c. 12. and 31 Elizabeth, c. 6; and the case of the Bishop of London and Feitch, in Bythewood on Conveyancing, Vol. III. page 362.

seems to be allowed as respects the security to be taken for vacating the incumbency.

An Advowson being considered as temporal property, and subject to conveyance like any other lay property, is not liable to the difficulties here stated; and, notwithstanding the laws here referred to, it is notorious that Presentations also are not less currently bought and sold.

Example xxvii.

Suppose a Lease granted on the last survivor of two lives, of which one has dropped, and the other is now 65 years of age; it is required to determine the premium which ought to be paid for adding another life of 20 to it, estimating the rate of mortality by the experience of the Equitable, and the interest of money at 4 per cent.; also computing the *improved rent** of the property at £120 per annum?

This question is no other than a proposition to barter the present lease and a sum of money, for a new lease of increased value; the two leases being valued as in the foregoing Examples, the difference must therefore constitute the premium required.

Thus, by the method pointed out in page 260, it will be found that the present value of the renewed lease, considered as an annuity on the last survivor

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* By the phrase Improved Rent is here understood, the excess of the annual value of the property, above what is, by the lease, conditioned to be paid in the shape of reserved, quit, or ground rent, and a due allowance for repairs, &c.

of 65 & 20 is in years' purchase - - - 18.7375
ditto of old lease, at £1 per an. on a life of 65 = 8.6351
difference - - - - - - - - - - - - 10.1024
and this multiplied by the £120 improved rent, gives 10.1024 × 120 = £1212 5s 9d, the answer.

Otherwise the advantage accruing to the lessee from the proposed transaction may be considered as a survivorship annuity, during what may remain of the life of 20, after the extinction of the life of 65. Hence, by the method laid down in page 264,
From the value of £1 per an. on a life of 20 = 18.2424
deduct ditto on the joint lives of 65 & 20 = 8.1400
difference - - - - - - - - - - - - 10.1024
which multiplied by the £120, gives £1212 5s 9d, the same as before.

If, instead of adding a life to the existing lease, the tenant were now desirous to purchase the right of nominating one or more lives at the extinction of the present lease, the amount of fine or consideration may be determined (by assuming the age or ages to be nominated) as in Example XXVI, page 286.

EXAMPLE XXVIII.

Suppose a number of persons, of the age of 30, to contribute each £50 per annum for life, to a common fund for the benefit of their respective survivors; it is required to determine what average sum the survivors of each of them will be entitled to, assum-

ing the rate of mortality among the whole mass to be correctly expressed by the Northampton Table, the money to be improved at 3 per cent., and the different claims to be discharged at the end of the several years in which the contributors may happen to die?

Referring to Table XXXV, folio headed 3 per cent. and dividing the number in column N opposite the given age, by the corresponding number in column M, as directed in page 241, we find the improved amount of £1 per annum on a life of 30 = 30570.053 ÷ 863.554 = 35.4003; and this multiplied by £50, the amount of annual contribution, gives 35.4003 × 50 = £1770 0s 4d, the answer.

But if the contributions be made payable at the beginning of each year, the required amount found by dividing the number in column N at 29, by that in column M at 30, (agreeably to what was stated in page 242,) and multiplying the quotient by the amount of contribution is $\frac{32376.61}{863.554} \times 50 =$ 37.4923 × 50 = £1874 12s 4d.

And if the rate of mortality be estimated by the experience of the Equitable, the money to be improved at $3\frac{1}{2}$ per cent, and the contributions still made payable at the beginning of the year, the required amount, found in like manner by Table XXI, is $\frac{16886.896}{319.997} \times 50 = 52.772 \times 50 =$ £2638 12s.

8. *Miscellaneous Questions.*

The various Examples given in the last Section having been worked out in numbers, and the Rules therein deduced expressed in words at length, it is presumed that the reader who has made himself acquainted with what is therein explained, though unacquainted with the general nature of mathematical investigations, will find no great difficulty in following the reasoning here proposed to be employed by the adoption of symbols, or in transferring the results to numbers at pleasure.

Let the value of £1 annuity on a Single Life $A = \mathrm{A}$
ditto on the Joint Lives of A & B - - $= \mathrm{AB}$
ditto on the Joint Lives of A, B & C - $= \mathrm{ABC}$
ditto on the Last Survivor of A & B - $= \overline{\mathrm{AB}}$
ditto on the Last Survivors of A, B & C $= \overline{\mathrm{ABC}}$
ditto to cease with the 2nd death amongst three persons, A, B & C - - - - $= {}^{2}\overline{\mathrm{ABC}}$
ditto on the joint existence of A and the last survivor of B & C - - - - - $= \mathrm{A}\,\overline{\mathrm{BC}}$
ditto on the life of A after the death of $B = {}_{\mathrm{B}}\mathrm{A}$
ditto on the Joint Lives of A & B after the death of C - - - - - - - - - $= {}_{\mathrm{C}}\mathrm{AB}$
ditto on the Last Survivor of A & B after the death of C - - - - - - - - - $= {}_{\mathrm{C}}\overline{\mathrm{AB}}$
ditto on the Life of A after the joint lives of B & C - - - - - - - - - - $= {}_{\mathrm{BC}}\mathrm{A}$
ditto on ditto after the last survivor of B & $C = {}_{\overline{\mathrm{BC}}}\mathrm{A}$

Then, as examples of the advantage to be derived from the employment of symbols, we may remark that the propositions deduced in pages 260 and 264, become $\overline{AB} = A + B - AB$, and ${}_BA = A - AB$. respectively.

Question I.

An annuity of £1, dependent on the last survivor of *A* & *B*, is to be equally divided between them, while they are both living, but at the death of either the whole reverts to the survivor during the remainder of his life; required the present value of *A*'s interest therein?

Here *A*'s interest may be considered as made up of two parts; first, half of the proposed annuity during the joint lives of himself and *B*, and, secondly the whole of it during what may remain of his life after *B*'s death; hence it follows that

The present value of the first part - - - $= \frac{1}{2}AB$

that of the second part, by Example IX, $= A - AB$

total interest of *A* - - - - - - - $= A - \frac{1}{2}AB$

In this and the following questions, the annuity proposed is £1, but the value of any other annuity may be obtained by multiplying the result by the amount of annual payment. We may likewise remark that the result thus deduced is a general Formula, equally applicable to different ages, different tables of mortality, or different rates of interest; and

which being translated to common language, according to the meaning affixed to the symbols, and the ordinary signification of the arithmetical signs by which they are connected, forms a general rule for the solution of all similar questions.

Question II.

Suppose an annuity of £1 to be equally divided between *A* & *B* during their joint lives, and to be reduced to one half of that amount during the life of the survivor; it is required to determine the present value of *A*'s interest therein?

By the question *A* is entitled to a half of the proposed annuity during the joint lives of himself and *B*, and to the like proportion of it during what may remain of his life after *B*'s death, making together a constant income for his whole life, the present value of which is therefore $= \frac{1}{2}A$, the answer.

In like manner it may be shown that *B*'s interest in the proposed annuity is $\frac{1}{2}B$: the present value of the whole annuity, or the collective shares of *A* & *B* is therefore $= \frac{1}{2}A + \frac{1}{2}B$, or $= \frac{1}{2}(A + B)$.

Question III.

P, being in possession of two separate annuities of £1 each, one on the joint lives of *A* & *B*, and the other on the joint lives of *A* & *C*, disposes of £1 per annum during the joint lives of *A*, *B* & *C*; it is re-

quired to determine the present value of what *P* still reserves to himself?

Here it is manifest that *P*'s annuities may be considered as depending upon the following contingencies, viz. the one on the joint existence of *A* and the joint lives of *B* & *C*, or in other words on the joint existence of *A*, *B* & *C*, and the other on the joint existence of *A* and the last survivor of *B* & *C*, which in symbols we have denoted by $A\overline{BC}$; the first of these, *P* has, by the question, disposed of, and the second he reserved for himself: hence, by deducting the present value of the annuity sold from the aggregate of the two proposed annuities, we have $A\overline{BC} = AB + AC - ABC$, the answer.

By the same kind of reasoning it may be shown that

$$\overline{AB}C = (A + B - AB)C = AC + BC - ABC.$$

Question iv.

P, being entitled to three separate annuities of £1 each, the first payable during the joint lives of *A* & *B*, the second during the joint lives of *A* & *C*, and the third during the joint lives of *B* & *C*, disposes of £2 per annum during the joint lives of *A*, *B* & *C*; it is required to determine the present value of what he still reserves for himself?

Had *P* not disposed of any part of his income, it is manifest that he (or his heirs) would be in receipt

of £3 per annum until the first death among *A*, *B* & *C*, at which period two of the annuities must necessarily cease, leaving him (or his heirs) in receipt of only £1 per annum from that event until the second death, when that also must terminate; but having disposed of £2 per annum, to cease with the first death, the part he reserves to himself simply constitutes an annuity of £1 from the present time until the second death happens among *A*, *B* & *C*, the present value of which we have denoted by $\overset{2}{\overline{ABC}}$: hence, from the present value of the annuities proposed, deducting that of the quantity sold, we have $\overset{2}{\overline{ABC}} = AB + AC + BC - 2ABC$, the answer.

Corollary.—If to each of these equals ABC be added, we have

$$ABC + \overset{2}{\overline{ABC}} = AB + AC + BC - ABC.$$

Question v.

What is the present value of £1 annuity on the last survivor of three lives *A*, *B* & *C*?

Since it is manifest that £1 annuity to cease with the first of the proposed lives, together with similar annuities to cease respectively with the second and third, are identical with the sum of three separate annuities of £1 each on the lives proposed, we have

$ABC+\overset{2}{A}\overline{BC}+A\overline{BC} = A+B+C$, and by last Quest.

$ABC+\overset{2}{A}\overline{BC} = AB+AC+BC-ABC$

difference $\overline{ABC} = A+B+C-AB-AC-BC+ABC$, the present value required.

Otherwise, by substituting a single life D for the last survivor of B & C, the proposed annuity may be considered as dependent on the last survivor of A & D; and its present value, found as in Example IX of last section, becomes $= A+D-AD$: then restoring the value of D, this expression becomes transformed into $A+\overline{BC}-A\overline{BC}$, which, by substituting $B+C-BC$ for its equivalent $\overline{BC}$, and the value of $A\overline{BC}$ as determined in Question III, becomes further transformed into

$$A+B+C-BC-(AB+AC-ABC)$$
$$=A+B+C-BC-AB-AC+ABC,$$

the answer as before.

Question VI.

What is the present value of a Survivorship Annuity of £1 on the life of A, after the extinction of the joint lives of B & C?

Here it is manifest that £1 annuity during the time A may live jointly with B & C, and a similar annuity during what may remain of A's life after the extinction of the joint lives of B & C, together

constitute the like annuity on *A*'s life for the whole duration, or in symbols
$ABC + {}_{BC}A = A$, from which, by transposition, we have ${}_{BC}A = A - ABC$, the answer.

Question VII.

What is the present value of a Survivorship Annuity of £1 on the life of *A*, after the extinction of the last survivor of *B* & *C*?

Reasoning as in the last Question, it is evident that £1 per annum during the time *A* may live jointly with the last survivor of *B* & *C*, and a like annuity for what may remain of *A*'s life, after the extinction of such survivor, together constitute an immediate annuity on *A*'s life, or in symbols
$A\overline{BC} + {}_{\overline{BC}}A = A$ from which we find ${}_{\overline{BC}}A = A - A\overline{BC}$, which, by substituting the value of $A\overline{BC}$, as found in Question III, gives ${}_{\overline{BC}}A = A - AB - AC + ABC$, the answer.

Question VIII.

What is the present value of a Survivorship Annuity of £1 on the joint lives of *A* & *B*, after the extinction of another life *C*?

In this case it is manifest that £1 annuity during the time *A* & *B* may jointly live within the lifetime of *C*, and a like annuity for what may remain of the joint lives of *A* & *B* after the demise of *C*,

together constitute £1 per annum on the joint lives of *A* & *B*, without any restriction, or $ABC + {}_cAB = AB$, whence it follows that ${}_cAB = AB - ABC$, the answer.

Question ix.

What is the present value of a Survivorship Annuity of £1, on the last survivor of *A* & *B*, after the extinction of another life *C*?

Reasoning as in the foregoing questions it may be shown that $\overline{AB}C + {}_c\overline{AB} = \overline{AB}$ from which we obtain - ${}_c\overline{AB} = \overline{AB} - \overline{AB}C$.

But - - $\overline{AB} = A + B - AB$, by Ex. vi, p. 260

And - - $\overline{AB}C = AC + BC - ABC$, by Quest. iii

difference ${}_c\overline{AB} = A + B - AB - AC - BC + ABC$ the answer.

Question x.

Suppose an annuity of £1, dependent on the last survivor of *A*, *B* & *C*, to be enjoyed by *A* during life, and after his decease to become the property of *B* during the remainder of his life, and finally to revert to *C* during what may then remain of his life; it is required to determine the respective shares of *A*, *B* & *C*?

As *A* is here supposed to have possession of the whole annuity for life, his share is evidently = A. And as *B* divides no part of the annuity with *C*, his share is simply a survivorship annuity after the

death of *A*; and its present value is therefore, by Example IX, page 264, $= {}_{A}B = B - AB$.

Nor is it less evident that *C*'s share is nothing more than a survivorship annuity after the extinction of the last survivor of *A* & *B*; the present value of his interest is therefore, by Question VII,

$$\overline{{}_{AB}}C = C - AC - BC + ABC.$$

QUESTION XI.

Suppose an annuity of £1, dependent on the last survivor of *A*, *B* & *C*, to be equally divided among them while they are all living, but after the decease of any one of them, to be equally divided between the two survivors until the second life ceases, and after that event to revert entirely to the remaining survivor; it is required to determine the respective interests of *A*, *B* & *C*?

In this case *A*'s interest may be considered as made up of four distinct parts, the present values of which, found as in the foregoing Questions, will be as under, viz.

1st, his share during the joint lives of *A*, *B* & *C* - - - - - - - - - $= \frac{1}{3}ABC$

2nd, ditto with *B* after the death of *C* - - $= \frac{1}{2} \cdot {}_{C}AB = \frac{1}{2}AB - \frac{1}{2}ABC$

3rd. ditto with *C* after the death of *B* - - $= \frac{1}{2} \cdot {}_{B}AC = \frac{1}{2}AC - \frac{1}{2}ABC$

4th, ditto alone after deaths of *B* & *C* $= \overline{{}_{BC}}A = A - AB - AC + ABC$

Total interest of *A* $= A - \frac{1}{2}AB - \frac{1}{2}AC + \frac{1}{3}ABC$

And since the shares of *B* & *C* depend, respectively, upon similar contingencies to those pointed out in the case of *A*, by interchanging B and C in succession for A, we find by the same formula that

B's interest $= B - \frac{1}{2}AB - \frac{1}{2}BC + \frac{1}{3}ABC$

C's ditto $= C - \frac{1}{2}BC - \frac{1}{2}AC + \frac{1}{3}ABC$

Question xii.

Suppose an annuity of £1, dependent on the last survivor of *A*, *B* & *C*, to be equally divided between *A* & *B* while they are both living, but after the decease of either of them to be equally divided between *C* and the survivor during their joint lives, and after the decease of either of these to revert wholly to the last survivor; it is required to determine the respective interests of *A*, *B* & *C*?

The interest of *A* in this annuity may be comprehended in the two following parts; first what he may receive during the time he may live jointly with *B* & *C* or the survivor of them, and secondly what he may receive after they are both dead, the present value of the first part is therefore $= \frac{1}{2}A\overline{BC}$, and that of the second $= {}_{\overline{BC}}A$.

But by Question iii, $\frac{1}{2}.A\overline{BC} = \frac{1}{2}AB + \frac{1}{2}AC - \frac{1}{2}ABC$

and by Question vii, ${}_{\overline{BC}}A = A - AB - AC + ABC$

Total interest of *A* - $= A - \frac{1}{2}AB - \frac{1}{2}AC + \frac{1}{2}ABC$

Then interchanging B for A we find by the same formula that *B*'s interet $= B - \frac{1}{2}AB - \frac{1}{2}BC + \frac{1}{2}ABC$.

But, C having no share in the proposed annuity until the death of either A or B, it is obvious that his interest is different, and consists of the following parts, viz.

1st, his share with A after the death of B - - $\Big\} = \frac{1}{2}\cdot{}_{B}AC = \frac{1}{2}AC - \frac{1}{2}ABC$

2nd, ditto with B after the death of A - - $\Big\} = \frac{1}{2}\cdot{}_{A}BC = \frac{1}{2}BC - \frac{1}{2}ABC$

3rd, ditto alone after deaths of A & B $\Big\} = {}_{\overline{AB}}C = C - AC - BC + ABC$

Total interest of C - - $= C - \frac{1}{2}AC - \frac{1}{2}BC$

Question XIII.

Suppose an annuity of £1, dependent on the last survivor of A, B & C, to be equally divided between A & B while they are both living, and if A dies first the annuity to be equally divided between B & C until the second death, and afterwards to revert wholly to the survivor; but in the event of B dying first the annuity is to be wholly enjoyed by A during the remainder of his life, and after his death to revert to C; it is required to determine the respective interests of A, B & C?

As A, by the question, shares no part of the annuity with C, but enjoys the half of it during the joint lives of himself and B, and the whole of it during what may remain of his life after B's death, his interest will be found already determined in Question I, $= A - \frac{1}{2}AB$.

B's interest, however, depends on different contingencies, all similar to these involved in the last Question, the present value of his share is therefore $= B - \frac{1}{2}AB - \frac{1}{2}BC + \frac{1}{2}ABC$.

And as *C* shares no part of the annuity during *A*'s life, it is manifest that his interest is composed of the two following parts, viz.

1st, his share with *B* after *A*'s death - - - - $= \frac{1}{2}\cdot_{A}BC = \frac{1}{2}BC - \frac{1}{2}ABC$

2nd, ditto alone after deaths of *A* & *B* $= {}_{\overline{AB}}C = C - AC - BC + ABC$

Total interest of *C* - $= C - AC - \frac{1}{2}BC + \frac{1}{2}ABC$

Question xiv.

Suppose an annuity of £1, dependent on the last survivor of *A*, *B* & *C*, to be enjoyed by *A* during his life, and then to be equally divided between *B* & *C* during what may remain of their joint lives, but at the death of either of them to revert wholly to the last survivor; it is required to determine the respective interests of *A*, *B* & *C*?

By the Question, *A* is to enjoy the whole annuity during life, and his share is therefore denoted by A.

But *B*'s interest is manifestly reducible into the two following parts, viz.

1st, his share with *C* after *A*'s death - $= \frac{1}{2}\cdot_{A}BC = \frac{1}{2}BC - \frac{1}{2}ABC$

2nd, ditto alone after the deaths of *A* & *C* $= {}_{\overline{AC}}B = B - AB - BC + ABC$

Total interest of *B* - $= B - AB - \frac{1}{2}BC + \frac{1}{2}ABC$

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And *C*'s interest depending upon similar contingencies to that of *B*, is found by interchanging C for B in the last formula, thereby becoming $= C - AC - \frac{1}{2}BC + \frac{1}{2}ABC$.

If in this Question, or any of the four preceding ones, the shares of *A*, *B* & *C* be added, the result must evidently give the present value of £1 annuity on the last survivor of them, as determined by Question v.

Question xv.

Suppose an annuity of £1, dependent on the last two survivors of three persons *A*, *B* & *C*, (that is an annuity to cease with the second death,) to be equally divided among them while they are all alive, and afterwards to be equally divided between the two survivors during the remainder of their joint lives; it is required to determine the respective interests of *A*, *B* & *C*?

Here it is obvious that *A*'s interest consists of the first three parts stated in Question xi, viz.

1st, his share during the joint lives of *A*, *B* & *C* - - - $= \frac{1}{3}ABC$

2nd, ditto with *B* after death of *C* $= \frac{1}{2}AB - \frac{1}{2}ABC$

3rd, ditto with *C* after death of *B* $= \frac{1}{2}AC - \frac{1}{2}ABC$

Total interest of *A* - - $= \frac{1}{2}AB + \frac{1}{2}AC - \frac{2}{3}ABC$

And since it is also manifest that the shares of *B* & *C* respectively depend on similar contingencies,

by interchanging B and C successively for A, we find, by the same formula, that

B's interest - - - - - $= \frac{1}{2}AB + \frac{1}{2}BC - \frac{2}{3}ABC$

C's ditto - - - - - - $= \frac{1}{2}BC + \frac{1}{2}AC - \frac{2}{3}ABC$

Question xvi.

Suppose an annuity of £1, dependent on the last two survivors of three persons, A, B & C, to be equally divided between A & B during their joint lives, and after the death of either to be divided in like proportions between C and the survivor, during what may remain of their joint lives; it is required to determine the respective interests of A, B & C?

In this case it is manifest that A's interest consists of one half of the proposed annuity, during the time he may live with B & C, or the survivor of them; its present value is therefore by Question iii $= \frac{1}{2}A\overline{BC} = \frac{1}{2}(AB + AC - ABC)$.

B's interest depending on similar contingencies, is also $= \frac{1}{2}B\overline{AC} = \frac{1}{2}(AB + BC - ABC)$.

But C's interest evidently depends on different contingencies, as he enjoys no part of the annuity until either A or B dies: his interest, therefore, consists of the two following parts, viz.

1st, his share with A after the death of B - - - $\Big\}= \frac{1}{2}\cdot{}_{B}AC = \frac{1}{2}AC - \frac{1}{2}ABC$

2nd, ditto with B after the death of A - - $\Big\}= \frac{1}{2}\cdot{}_{A}BC = \frac{1}{2}BC - \frac{1}{2}ABC$

Total interest of C - - - $\frac{1}{2}(AC + BC) - ABC$

Question xvii.

Suppose an annuity of £1, dependent on the last two survivors of three persons, A, B & C, to be divided equally between A & B during their joint lives, and, in the event of B dying first, to revert wholly to A during the remainder of the joint lives of himself and C, but in the event of A dying first to be equally divided between B & C, during the remainder of their joint lives; it is required to determine the respective shares of A, B & C?

Here A's interest consists of the two following parts, viz.

1st. his share during the joint lives of A & B $= \frac{1}{2}AB$

2nd ditto after the death of B $= {}_B AC = AC - ABC$

Total interest of A - - - - $\frac{1}{2}AB + AC - ABC$

But B being entitled to a moiety of the proposed annuity, during the time he may live with A & C, or the survivor of them, his interest is by Question III, $= \frac{1}{2}B\overline{AC} = \frac{1}{2}(AB + BC - ABC.)$.

And with respect to C, he has no interest until the extinction of the joint lives of A & B, and not then unless A happens to die first; his share therefore simply amounts to one half of the proposed annuity during what may remain of the joint lives of himself and B, after the decease of A, the present value of which is, by Question VIII,

$$= \frac{1}{2} \cdot {}_A BC = \frac{1}{2}(BC - ABC).$$

In this Question or either of the two preceding ones, the collective shares of A, B & C are together equal to the present value of £1 annuity, payable until the second death.

Question xviii.

What is the present value of £1 annuity to be entered upon at the extinction of the last two survivors of three lives A, B & C, and to be payable from that period until the extinction of the third life?

If to the proposed annuity we add £1 per annum until the extinction of the second life, the aggregate evidently constitutes an annuity of £1 on the last survivor of the three given lives; hence it follows that the present value of the proposed annuity is the difference between the present value of £1 annuity on the last survivor of A, B & C, and that of a like annuity to cease at the second death, which in symbols $= \overline{ABC} - \overset{2}{\overline{ABC}}$, but by Questions v and iv, we have shown

that $\overline{ABC} = A + B + C - AB - AC - BC + ABC$

and $\overset{2}{\overline{ABC}} = \qquad\qquad AB + AC + BC - 2ABC$

difference $= A + B + C - 2AB - 2AC - 2BC + 3ABC$ the answer.

Question xix.

What is the present value of £1 annuity to be entered upon at the extinction of the first of three lives *A*, *B* & *C*, and to be continued until the extinction of the last survivor of them?

If to the proposed annuity we add £1 per annum during the three joint lives, the aggregate evidently constitutes an annuity of £1 per annum on the last survivor of the proposed lives; the present value of the required annuity is therefore the difference between the present value of £1 annuity on the last survivor of the assigned lives, and that of a similar annuity on their joint existence, which in symbols is $\overline{ABC} - ABC$.

But $\overline{ABC} = A + B + C - AB - AC - BC + ABC$

and $ABC = \qquad\qquad\qquad\qquad\qquad\qquad ABC$

difference $- A + B + C - AB - AC - BC$

the answer.

Question xx.

What is the present value of £1 annuity to be entered upon at the extinction of the first of three lives, *A*, *B* & *C*, and to be continued until the extinction of the second life?

Reasoning as in the foregoing Questions, it may be shown that the present value of the proposed annuity is equal to the difference between the value of a similar annuity on the last two survivors of the

given lives, and that of a like annuity on their joint existence, which in symbols is $\overset{2}{\overline{ABC}} - ABC$.

But by Quest. IV $\overset{2}{\overline{ABC}} = AB + AC + BC - 2ABC$

and by notation $ABC = \qquad\qquad ABC$

difference - - - $= AB + AC + BC - 3ABC$

the answer.

Question XXI.

What annual premium, payable at the beginning of each year during the joint lives of *A*, *B* & *C*, should be required for a survivorship annuity of £1 on the life of *A*, after the extinction of the joint lives of *B* & *C*?

By Question VI the present value of the proposed annuity in a single payment is shown to be $= A - ABC$; and the present value of £1 per annum payable at the beginning of each year during the joint lives of *A*, *B* & *C*, is evidently denoted by $1 + ABC$; hence, by the Rule given in page 69, we have $\frac{A - ABC}{1 + ABC} = \frac{1 + A}{1 + ABC} - 1$ for the annual premium required.

Question XXII.

What annual premium, payable at the beginning of each year until the determination of the claim, is equivalent to a survivorship annuity of £1 on the life of *A*, after the extinction of the last survivor of *B* & *C*?

The present value of the proposed annuity in a single sum is, by Question VII, shown to be = $A - AB - AC + ABC$.

And the claim is manifestly determined by the failure of the joint existence of A and the last survivor of B & C; for if A should happen to die before such survivor, the proposed annuity cannot take effect, and no interest would remain to pay the premium any longer; but if A should happen to outlive both B & C, the annuity must commence at the extinction of the survivor of them, and the annual premium in consequence cease. The present value of £1 per annum payable at the end of the year until the claim be determined, is therefore expressed by $A\overline{BC}$, and that of a similar annuity payable at the beginning of the year, is $1 + A\overline{BC}$, which, by Question III, $= 1 + AB + AC - ABC$: hence, by the Rule in page 69, the required annual premium is

$$\frac{A - AB - AC + ABC}{1 + AB + AC - ABC} = \frac{A - A\overline{BC}}{1 + A\overline{BC}} = \frac{1 + A}{1 + A\overline{BC}} - 1$$

Question XXIII.

What annual premium, payable at the beginning of each year, until the determination of the claim, is equivalent to a survivorship annuity of £1 on the joint lives of A & B, after the extinction of a third life C?

Reasoning as in the last Question, it may be

shown, that the claim in this case must be determined by the first death, it therefore follows that £1 per annum payable at the beginning of the year until that event, is expressed by $1+ABC$: and it has been shown, in Question VIII, that the present value of the proposed annuity, in a single sum, is $=AB-ABC$. Hence by the Rule quoted in the last two Questions, we have

$$\frac{AB-ABC}{1+ABC} = \frac{1+AB}{1+ABC} - 1$$

for the annual premium which was to be found.

Question XXIV.

What annual premium, payable at the beginning of each year, until the determination of the claim, is equivalent to a survivorship annuity of £1 on the last survivor of *A* & *B*, after the extinction of a third life *C*?

Reasoning as in the foregoing Questions, it may be shown that the annual premium here ceases with the extinction of the joint existence of *C* and the last survivor of *A* & *B*, and that the present value of £1 per annum, payable at the beginning of each year until that event, is $1+\overline{AB}\,C$. Hence, by Question IX, and the Rule in page 69, we have

$$\frac{{}_c\overline{AB}}{1+\overline{AB}\,C} = \frac{\overline{AB}-\overline{AB}\,C}{1+\overline{AB}\,C} = \frac{1+\overline{AB}}{1+\overline{AB}\,C} - 1 = \frac{1+A+B-AB}{1+AC+BC-ABC} - 1$$

for the annual premium required.

9. *On Life Annuities payable by Instalments.*

As a preliminary step to this branch of the subject, it will be found convenient first to generalize the reasoning employed in the second section of this chapter, by putting i, r and v as in page 70, and adopting the symbols explained in pages 129 and 291. By these means, the reader will easily perceive that the present value of a fund sufficient to provide £1 annuity for each of a lives now proposed, is denoted by the series ${}^{1}av + {}^{2}av^2 + {}^{3}av^3 + {}^{4}av^4$ &c.; the several terms of which respectively denote the present values of the payments becoming due at the end of the 1st, 2nd, 3rd, &c. year to the extremity of life. But by the notation in page 291, the present value of each of these annuities is represented by A, that of the whole fund must therefore be denoted by A.a,

Consequently $\mathrm{A}.a = {}^{1}av + {}^{2}av^2 + {}^{3}av^3 + {}^{4}av^4$&c. I.

similarly $\mathrm{AB}.ab = {}^{1}(ab)v + {}^{2}(ab)v^2 + {}^{3}(ab)v^3$&c. II.

and $\mathrm{ABC}.abc = {}^{1}(abc)v + {}^{2}(abc)v^2 + {}^{3}(abc)v^3$&c. III.

Hence, by adding a to each side of the first equation

— — ab — — second —

— — abc — — third, —

we further deduce the following relations, viz.

$(1 + \mathrm{A})a = a + {}^{1}av + {}^{2}av^2 + {}^{3}av^3 + {}^{4}av^4$&c. - - - IV.

$(1 + \mathrm{AB})ab = ab + {}^{1}(ab)v + {}^{2}(ab)v^2 + {}^{3}(ab)v^3$&c. - V.

$(1 + \mathrm{ABC})abc = abc + {}^{1}(abc)v + {}^{2}(abc)v^2 + {}^{3}(abc)v^3$&c. VI.

Moreover, if we denote the number living in any Table of Mortality, opposite an age one year younger than A, by $a_{,}$; that opposite 1 year younger than B, by $b_{,}$&c.; and the value of £1 annuity on a life one year younger than A, by $A_{,}$; that on a life one year younger than B, by $B_{,}$; that on two joint lives respectively one year younger than A & B, by $(AB)_{,}$; and that on two joint lives, whereof the A only is one year younger than the A proposed, by $A_{,}B$, that on three joint lives whereof the A only is a year younger than the A proposed by $A_{,}BC$, &c. &c., and refer Equations I, II and III to such younger ages, we find that

$A_{,}.a_{,}=av+{}^{1}av^{2}+{}^{2}av^{3}+{}^{3}av^{4}+{}^{4}av^{5}$&c. - - VII.

$(AB)_{,}.(ab)_{,}=(ab)v+{}^{1}(ab)v^{2}+{}^{2}(ab)v^{3}$&c. - - VIII.

$A_{,}B.a_{,}b={}^{1}(a_{,}b)v+{}^{2}(a_{,}b)v^{2}+{}^{3}(a_{,}b)v^{3}$&c. - - IX.

$AB_{,}.ab_{,}={}^{1}(ab_{,})v+{}^{2}(ab_{,})v^{2}+{}^{3}(ab_{,})v^{3}$&c. - - X.

$(ABC)_{,}.(abc)_{,}=(abc)v+{}^{1}(abc)v^{2}+{}^{2}(abc)v^{3}$&c. XI.

$A_{,}B_{,}C.a_{,}b_{,}c={}^{1}(a_{,}b_{,}c)v+{}^{2}(a_{,}b_{,}c)v^{2}+{}^{3}(a_{,}b_{,}c)v^{3}$&c. XII.

$A_{,}BC_{,}.a_{,}bc_{,}={}^{1}(a_{,}bc_{,})v+{}^{2}(a_{,}bc_{,})v^{2}+{}^{3}(a_{,}bc_{,})v^{3}$&c. XIII.

$AB_{,}C_{,}.ab_{,}c_{,}={}^{1}(ab_{,}c_{,})v+{}^{2}(ab_{,}c_{,})v^{2}+{}^{3}(ab_{,}c_{,})v^{3}$&c. XIV.

$A_{,}BC.a_{,}bc={}^{1}(a_{,}bc)v+{}^{2}(a_{,}bc)v^{2}+{}^{3}(a_{,}bc)v^{3}$&c. - XV.

$AB_{,}C.ab_{,}c={}^{1}(ab_{,}c)v+{}^{2}(ab_{,}c)v^{2}+{}^{3}(ab_{,}c)v^{3}$&c. - XVI.

$ABC_{,}.abc_{,}={}^{1}(abc_{,})v+{}^{2}(abc_{,})v^{2}+{}^{3}(abc_{,})v^{3}$&c. - XVII.

Again, by multiplying each of the Equations marked IV, V and VI, by v, and comparing the results with those numbered VII, VIII and XI, we find that

$A_{,}.a_{,}=(1+A)av$ or $A_{,}=\frac{av}{a_{,}}.(1+A)$ - - - XVIII.

$(AB)_{,}=\frac{abv}{(ab)_{,}}(1+AB)$ - - - - - - - - - XIX.

$(ABC)_{,}=\frac{abcv}{(abc)_{,}}(1+ABC)$ - - - - - - - - XX.

As a further contraction of the notation here employed, we may call to our aid the symbol $\int.$ to denote the sum of the several terms arising by substituting 1, 2, 3, &c. in succession for the variable quantity, involved in the general term to which it is prefixed; for instance, considering t variable, we have

$\int.{}^{t}av^{t}={}^{1}av+{}^{2}av^{2}+{}^{3}av^{3}+{}^{4}av^{4}+{}^{5}av^{5}$&c.

$\int.{}^{t}(ab)v^{t}={}^{1}(ab)v+{}^{2}(ab)v^{2}+{}^{3}(ab)v^{3}$&c.

$\int.{}^{t}(abc)v^{t}={}^{1}(abc)v+{}^{2}(abc)v^{2}+{}^{3}(abc)v^{3}$&c.

$\int.{}^{t}a_{,}v^{t-1}=a+{}^{1}av+{}^{2}av^{2}+{}^{3}av^{3}+{}^{4}av^{4}$&c.

$\int.{}^{t}(ab)_{,}v^{t-1}=ab+{}^{1}(ab)v+{}^{2}(ab)v^{2}+{}^{3}(ab)v^{3}$&c.

$\int.{}^{t}(abc)_{,}v^{t-1}=abc+{}^{1}(abc)v+{}^{2}(abc)v^{2}+{}^{3}(abc)v^{3}$&c.

and comparing these series with those given in the Equations numbered I, II, III, IV, we obtain

$A.a=\int.{}^{t}av^{t}$, $AB.ab=\int.{}^{t}(ab)v^{t}$, $ABC.abc=\int.{}^{t}(abc)v^{t}$

$(1+A)a=\int.{}^{t}a_{,}v^{t-1}$

$(1+AB).ab=\int.{}^{t}(ab)_{,}v^{t-1}$

and $(1+ABC)abc=\int.{}^{t}(abc)_{,}v^{t-1}$

By the same process it may be shown that

$A_{,}B.a_{,}b=\int.{}^{t}(a_{,}b)v^{t}$, $A_{,}BC.a_{,}bc=\int.{}^{t}(a_{,}bc)v$, &c. &c

Problem 1.

To find the present value of £1 annuity on a single life A, payable by m equal instalments in each year, but to cease with the last instalment becoming due prior to the extinction of the life proposed; supposing the interest to be convertible into principal only once a year.

Putting the required value $= A^M$, and supposing a fund formed sufficient to provide similar annuities to a persons of the same age, we have the present value of such fund $= a.A^M$. Also referring to page 142, we find the number of surviving claimants at the end of the $\frac{x}{m}$th part of the tth year, $= \frac{1}{m}\{(m-x).{}^t a_{,} + x.{}^t a\}$; and, on the supposition of yearly conversions, the present value of the $£\frac{1}{m}$ to which each of these claimants will then be entitled is

$$\frac{1}{m}.v^{t-1} \times \frac{1}{1+\frac{xi}{m}} = \frac{v^{t-1}}{m+xi} = \frac{rv^t}{m+xi};$$

that of all the payments then due is therefore

$$= \frac{rv^t}{m+xi} \times \frac{1}{m}\{(m-x).{}^t a_{,} + x.{}^t a\}$$

$$= \frac{1}{m}\left\{\frac{m-x}{m+xi}.{}^t a_{,} v^{t-1} + \frac{rx}{m+xi}.{}^t a v^t\right\}$$

$$\text{or } = \frac{1}{m(m+xi)}\{(m-x).{}^t a_{,} v^{t-1} + rx.{}^t a v^t\}$$

And if in either of these expressions we make x successively $= 1, 2, 3$ &c. to m. and add the results, the aggregate gives the present value of all the pay-

ments becoming due in the tth year; then making t successively = 1, 2, 3 &c. to the extremity of life, we deduce the total present value of the whole fund, which being made equal to the expression already deduced, enables us to determine the present value of the annuity proposed.

These operations expressed by symbols give

$$a.\mathrm{A}^{\mathrm{M}} = \int.\frac{1}{m}\left\{\int.\left(\frac{m-x}{m+xi}\right)^{t}a_{t}v^{t-1} + \int.\left(\frac{rx}{m+xi}\right)^{t}av^{t}\right\}$$

$$\text{or } a.\mathrm{A}^{\mathrm{M}} = \int.\frac{1}{m(m+xi)}\left\{(m-x).\int.{}^{t}a_{t}v^{t-1} + rx.\int.{}^{t}av^{t}\right\}$$

from the latter of these expressions, by substituting the values of $\int.{}^{t}a_{t}v^{t-1}$ and $\int.{}^{t}av^{t}$ we find that

$$a.\mathrm{A}^{\mathrm{M}} = \int.\frac{1}{m(m+xi)}\left\{(m-x)(1+\mathrm{A}).a + rx\mathrm{A}.a\right\}$$

$$\text{or } \mathrm{A}^{\mathrm{M}} = \int.\frac{1}{m(m+xi)}\left\{(m+xi)\mathrm{A} + m - x\right\}$$

$$\text{or } \mathrm{A}^{\mathrm{M}} = \int.\left\{\frac{1}{m}\mathrm{A} + \frac{m-x}{m(m+xi)}\right\}$$

$$\text{or } \mathrm{A}^{\mathrm{M}} = \mathrm{A} + \int.\frac{m-x}{m(m+xi)}$$

the present value required.

Corollary I.—If the annuity be payable yearly, m and x each = 1 and $\mathrm{A}^{\mathrm{M}} = \mathrm{A}^{\mathrm{I}} = \mathrm{A}$, as it ought.

Corollary II.—If the annuity be payable half-yearly, $m = 2$, x = 1 & 2 successively, and

$$\mathrm{A}^{\mathrm{M}} = \mathrm{A}^{\mathrm{II}} = \mathrm{A} + \frac{1}{2(2+i)} = \mathrm{A} + \frac{1}{4+2i}$$

COROLLARY III.—If the annuity be payable quarterly $m=4$, $x = 1, 2, 3$ & 4 successively, and

$$A^{M}=A^{IV}=A+\frac{3}{4(4+i)}+\frac{2}{4(4+2i)}+\frac{1}{4(4+3i)}$$

COROLLARY IV.—If the annuity be payable momently $A^{M}=A+\int.\frac{m-x}{m(m+xi)}$, which, by expanding the last term into a series, becomes transformed into

$$A+\int.\left(\frac{1}{m}-\frac{xi}{m^2}+\frac{x^2i^2}{m^3}-\frac{x^3i^3}{m^4}\&c.\right)-\int.\left(\frac{x}{m^2}-\frac{x^2i}{m^3}+\frac{x^3i^2}{m^4}\&c.\right)$$

and if the operation indicated by $\int.$ be performed according to the Note in page 143, on the supposition that m is indefinitely great, and $x = 1, 2, 3,$ &c. to m, this expression further becomes

$$=A+(1-\tfrac{1}{2}i+\tfrac{1}{3}i^2-\tfrac{1}{4}i^3\&c.)-(\tfrac{1}{2}-\tfrac{1}{3}i+\tfrac{1}{4}i^2-\tfrac{1}{5}i^3\&c.)$$

$$=A+\frac{1}{i}(i-\tfrac{1}{2}i^2+\tfrac{1}{3}i^3\&c.)+\frac{1}{i^2}(i-\tfrac{1}{2}i^2+\tfrac{1}{3}i^3\&c)-\frac{1}{i}$$

$$=A+\frac{r}{i^2}(i-\tfrac{1}{2}i^2+\tfrac{1}{3}i^3\&c.)-\frac{1}{i}$$

$$=A+\frac{r.(h.\ log.\ r)}{i^2}-\frac{1}{i}=A+\frac{r.(h.\ log.\ r)-i}{i^2}\ *$$

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* Hyp. Log. 1.01 = .0099503308 Hyp. Log. 1.06 = .0582689081
— — 1.02 = .0198026273 — — 1.07 = .0676586485
— — 1.03 = .0295588022 — — 1.08 = .0769610411
— — 1.04 = .0392207131 — — 1.09 = .0861776962
— — 1.05 = .0487901642 — — 1.10 = .0953101798

COROLLARY V.—Taking the value of the quantity $\int.\frac{m-x}{m(m+xi)}$ as determined in the foregoing Corollaries, the correct increments referred to in the Note, page 262, are as under.

When the Interest is	And the Annuity is made payable.		
	Half Yearly.	Quarterly.	Momently.
per Cent.			
2	.2475	.3719	.4967
3	.2463	.3704	.4951
4	.2451	.3689	.4935
5	.2439	.3674	.4919
6	.2427	.3659	.4903
7	.2415	.3644	.4887
8	.2404	.3630	.4872

COROLLARY VI.—Moreover, from the expression $\int.\left\{\frac{1}{m}\mathrm{A}+\frac{m-x}{m(m+xi)}\right\}$ by making x constant, it appears that the present value of a periodical payment of $£\frac{1}{m}$ receivable at the end of the $\frac{x}{m}$th part of each year, during an assigned life A, is $\frac{1}{m}\mathrm{A}+\frac{m-x}{m(m+xi)}$. The present value of £1 annuity payable at the like intervals is therefore $=\mathrm{A}+\frac{m-x}{m+xi}$

Thus, the value of £1 annuity payable at the middle of each year during A's life, is - - $=\mathrm{A}+\frac{1}{2+i}$

ditto of ditto payable at the end of the first quarter in each year . $=\mathrm{A}+\frac{3}{4+i}$

ditto of ditto payable at the end of the third quarter - - - - - $=\mathrm{A}+\frac{1}{4+3i}$

Problem II.

To find the present value of £1 annuity payable by m equal instalments in each year, during the joint existence of two or more lives A, B, C, &c. supposing the interest to be convertible into principal yearly, and the annuity to cease with the last instalment becoming due prior to the extinction of the joint existence of the lives proposed.

Let the required present value be denoted by $(ABC\&c.)^{M}$, and conceive the proposed lives to form one combination, out of abc&c. similar ones, all entitled to like annuities, made payable from a common fund.

Then supposing the combinations which fail during the tth year to fail in equal quantities within equal portions of that year, we have the number of combinations remaining entire at the end of the $(t-1)$th year, or at the beginning of the tth year, $= {}'(abc\&c)_{,}$ and the number remaining entire at the end of the tth year $= {}'(abc\&c)$; the number failing within the tth year is therefore $= {}'(abc\&c)_{,} - {}'(abc\&c)$, hence, by the supposition here made, the number of combinations remaining entire at the end of the $\frac{x}{m}$th part of the tth year is

$$'(abc\&c)_{,} - \frac{x}{m}\left\{'(abc\&c.)_{,} - '(abc\&c.)\right\}$$

$= \frac{1}{m}\left\{(m-x).^{t}(abc\&c.)_{t} + x.^{t}(abc\&c.)\right\}$ and this being multiplied by $\frac{v^{t-1}}{m+xi}$ or by $\frac{rv^{t}}{m+xi}$, the present value of the $£\frac{1}{m}$ to which each of these combinations will then be entitled, gives

$$\frac{1}{m(m+xi)}\left\{(m-x).^{t}(abc\&c.)_{t}.v^{t-1} + rx.^{t}(abc\&c.)v^{t}\right\}$$

for the present value of all the payments becoming due at the end of the $\frac{x}{m}$th part of the tth year. Then proceeding in every respect as in the last Problem, we find

$$(ABC\&c.)^{M} = \int.\left\{\frac{1}{m}ABC\&c. + \frac{m-x}{m(m+xi)}\right\}$$

or $(ABC\&c)^{M} = ABC\&c. + \int.\frac{m-x}{m(m+xi)}$

for the present value of the annuity proposed.

Hence, the several Corollaries deduced in the last Problem are equally applicable to this, on the supposition upon which the above solution is founded.

But if, instead of the *combinations* failing in equal proportions during equal intervals of the same year, we suppose the *individuals*, of which those combinations are formed, to be subject to equal decrements in equal portions of the tth year, the solution, though somewhat more correct, is far more intricate.

For, referring our investigation to two joint lives, on the latter supposition it is manifest, by what has been stated in page 144, that out of ab combinations the number remaining entire at the end of the $\frac{x}{m}$th part of the tth year is

$$\frac{1}{m}\Big\{(m-x).{}^{t}a_{,}+x.{}^{t}a\Big\}\times\frac{1}{m}\Big\{(m-x).{}^{t}b_{,}+x.{}^{t}b\Big\}$$

$$=\frac{1}{m^2}\Big\{(m-x)^2.{}^{t}(ab)_{,}+x^2.{}^{t}(ab)+x(m-x)[{}^{t}(a_{,}b)+{}^{t}(ab_{,})]\Big\}$$

which being multiplied by $\frac{v^{t-1}}{m+xi}$, or $\frac{rv^{t}}{m+xi}$ gives

$$\frac{(m-x)^2}{m^2(m+xi)}.{}^{t}(ab)_{,}.v^{t-1}+\frac{rx^2}{m^2(m+xi)}.{}^{t}(ab)v^{t}$$

$$+\frac{rx(m-x)}{m^2(m+xi)}\Big\{{}^{t}(a_{,}b)v^{t}+{}^{t}(ab_{,})v^{t}\Big\}$$

for the present value of a fund sufficient to provide for the payment of $£\frac{1}{m}$ to each combination remaining entire at the end of the $\frac{x}{m}$th part of the tth year.

Hence, making x successively $= 1, 2, 3,$ &c. to m, and putting

$$\int.\frac{(m-x)^2}{m^2(m+xi)} \quad \text{or} \quad \frac{1}{m^2}.\int.\frac{(m-x)^2}{m+xi} = P$$

$$\int.\frac{rx^2}{m^2(m+xi)} \quad \text{or} \quad \frac{r}{m^2}.\int.\frac{x^2}{m+xi} = Q$$

$$\int.\frac{rx(m-x)}{m^2(m+xi)} \quad \text{or} \quad \frac{r}{m^2}.\int.\frac{x(m-x)}{m+xi} = R$$

we find the present value of all the instalments becoming due within the tth year,

$$= P.{}^{t}(ab)_{,}v^{t-1} + Q.{}^{t}(ab)v^{t} + R\{{}^{t}(a_{,}b)v^{t} + {}^{t}(ab_{,})v^{t}\};$$

then making t successively $= 1, 2, 3$ &c. and adding the results by the Equations in pages 312, 313, we obtain the aggregate present values of all the annuities supposed,

$$= P(1 + AB).ab + Q.AB.ab + R(A_{,}B.a_{,}b + AB_{,}.ab_{,}):$$

and this divided by ab, the number of such annuities, gives for the average value of each

$$(AB)^{M} = P(1 + AB) + Q.AB + R\left(\frac{A_{,}B}{a_{,}} + \frac{AB_{,}}{b_{,}}\right)$$

$$\text{or } (AB)^{M} = (P + Q)AB + R\left(\frac{A_{,}B}{a_{,}} + \frac{AB_{,}}{b_{,}}\right) + P.$$

Again, extending our investigation to Three Joint Lives, it may be shown that, out of abc combinations, the number remaining entire at the end of the $\frac{r}{m}$th part of the tth year, is $\frac{1}{m}\{(m-x).{}^{t}a_{,} + x.{}^{t}a\}$ $\times \frac{1}{m}\{(m-x).{}^{t}b_{,} + x.{}^{t}b\}$ $\times \frac{1}{m}\{(m-x).{}^{t}c_{,} + x.{}^{t}c\}$

Hence, multiplying this number by $\frac{rv^{t}}{m+xi}$, the present value of the $£\frac{1}{m}$ to which each of these combinations will then be entitled, and putting

$$\frac{1}{m^{3}}.\int.\frac{(m-x)^{3}}{m+xi} = S, \quad \frac{r}{m^{3}}.\int.\frac{x^{3}}{m+xi} = T,$$

$$\frac{r}{m^{3}}.\int.\frac{x(m-x)^{2}}{m+xi} = U, \quad \frac{r}{m^{3}}.\int.\frac{x^{2}(m-x)}{m+xi} = V,$$

the present value of all the payments becoming due within the tth year, is expressed by

$$s.^{t}(abc)_{,}v^{t-1} + r.^{t}(abc)v^{t}$$
$$+ u\left\{{}^{t}(a_{,}b_{,}c)v^{t} + {}^{t}(a_{,}bc_{,})v^{t} + {}^{t}(ab_{,}c_{,})v^{t}\right\}$$
$$+ v\left\{{}^{t}(a_{,}bc)v^{t} + {}^{t}(ab_{,}c)v^{t} + {}^{t}(abc_{,})v^{t}\right\}$$

Then making t successively $= 1, 2, 3$ &c, adding the results, and dividing their sum by abc, the number of combinations from which the average is taken, we obtain

$$(\mathrm{ABC})^{\mathrm{M}} = s + (s+r)\mathrm{ABC}$$
$$+ u\left\{\frac{\mathrm{A}_{,}\mathrm{B}_{,}\mathrm{C}}{(ab)_{,}} + \frac{\mathrm{A}_{,}\mathrm{BC}_{,}}{(ac)_{,}} + \frac{\mathrm{AB}_{,}\mathrm{C}_{,}}{(bc)_{,}}\right\}$$
$$+ v\left\{\frac{\mathrm{A}_{,}\mathrm{BC}}{a_{,}} + \frac{\mathrm{AB}_{,}\mathrm{C}}{b_{,}} + \frac{\mathrm{ABC}_{,}}{c_{,}}\right\}$$

for the present value of the annuity proposed.

The value of an annuity payable by instalments, during the existence of a single life, or the joint existence of two or three lives, being determined by these two Problems, that of a similar annuity, payable on any of the contingencies proposed in the last section, may be determined by substituting A^{M} for A, $(\mathrm{AB})^{\mathrm{M}}$ for AB, and so on.

For the purpose of pointing out the application of the formula

$$(\mathrm{AB})^{\mathrm{M}} = (p+q)\mathrm{AB} + r\left(\frac{\mathrm{A}_{,}\mathrm{B}}{a_{,}} + \frac{\mathrm{AB}_{,}}{b_{,}}\right) + p$$

to determine the value of an annuity on Two

Joint Lives, the following Corollaries have been added.

COROLLARY I.—If the annuity be payable yearly, m & x each $=1$; wherefore, $P=0$, $Q=\frac{r}{1+i}=1$, $R=0$, and $(AB)^M=AB$, as it ought.

COROLLARY II.—If the annuity be payable half-yearly, $m=2$ and $x=1$ & 2 in succession; wherefore

$$P=\tfrac{1}{4}\left(\frac{1}{2+i}\right),\ Q=\frac{r}{4}\left(\frac{1}{2+i}+\frac{4}{2+2i}\right),\text{ and }R=\frac{r}{4}\left(\frac{1}{2+i}\right)$$

from which the value of the annuity may be determined.

COROLLARY III.—If the annuity be payable quarterly $m=4$, and $x=1$, 2, 3 & 4, in succession; wherefore

$$P=\frac{1}{16}\left(\frac{9}{4+i}+\frac{4}{4+2i}+\frac{1}{4+3i}\right)$$

$$Q=\frac{r}{16}\left(\frac{1}{4+i}+\frac{4}{4+2i}+\frac{9}{4+3i}+\frac{16}{4+4i}\right)$$

$$R=\frac{r}{16}\left(\frac{3}{4+i}+\frac{4}{4+2i}+\frac{3}{4+3i}\right)$$

from which the value of the annuity may be determined.

COROLLARY IV.—If the annuity be payable momently $P=\frac{1}{m^2}\int\cdot\frac{(m-x)^2}{m+xi}$, which being expanded into a series becomes

$$= \int.\left(\frac{1}{m} - \frac{xi}{m^2} + \frac{x^2 i^2}{m^3} \&c.\right) - 2.\int.\left(\frac{x}{m^2} - \frac{x^2 i}{m^3} + \frac{x^3 i^2}{m^4} \&c.\right)$$
$$+ \int.\left(\frac{x^2}{m^3} - \frac{x^3 i}{m^4} + \frac{x^4 i^2}{m^5} \&c.\right)$$

then making $x = 1, 2, 3$ &c. to m, adding the results, and considering m indefinitely great, this expression becomes further transformed into

$$\left(1 - \frac{i^2}{2} + \frac{i^3}{3} \&c.\right) - 2\left(\frac{1}{2} - \frac{i}{3} + \frac{i^2}{4} \&c.\right) + \left(\frac{1}{3} - \frac{i}{4} + \frac{i^2}{5} \&c\right)$$

$$= \frac{1}{i}\left(i - \frac{i^2}{2} + \frac{i^3}{3} \&c.\right) + \frac{2}{i^2}\left(i - \frac{i^2}{2} + \frac{i^3}{3} \&c.\right)$$

$$+ \frac{1}{i^3}\left(i - \frac{i^2}{2} + \frac{i^3}{3} \&c.\right) - \frac{2}{i} - \frac{1}{i^2} + \frac{1}{2i}$$

$$= \frac{1}{i}\left\{\text{h. log. } (1+i)\right\} + \frac{2}{i^2}\left\{\text{h. log. } (1+i)\right\}$$

$$+ \frac{1}{i^3}\left\{\text{h. log. } (1+i)\right\} - \frac{3i+2}{2i^2}$$

$$= \left(\frac{1}{i} + \frac{2}{i^2} + \frac{1}{i^3}\right) \times \left\{\text{h. log. } (1+i)\right\} - \frac{3i+2}{2i^2}$$

$$= \frac{r^2}{i^3}\left(\text{h. log. } r\right) - \frac{3i+2}{2i^2}.$$

$$Q = \frac{r}{m^2}.\int.\frac{x^2}{m+xi} = r.\int.\left(\frac{x^2}{m^3} - \frac{x^3 i}{m^4} + \frac{x^4 i^2}{m^5} \&c.\right)$$

$$= r\left(\frac{1}{3} - \frac{i}{4} + \frac{i^2}{5} \&c.\right) = \frac{r}{i^3}\left(i - \frac{i^2}{2} + \frac{i^3}{3} \&c.\right) - \frac{r}{i^2}\left(1 - \frac{i}{2}\right)$$

$$= \frac{r}{i^3}\left(\text{h. log. } r\right) - \frac{r(2-i)}{2i^2}.$$

$R = \frac{r}{m^2} \cdot \int \frac{x(m-x)}{m+xi}$, which being expanded as before

$= r \cdot \int \left(\frac{x}{m^2} - \frac{x^2 i}{m^3} + \frac{x^3 i^2}{m^4} \&c. \right) - r \cdot \int \left(\frac{x^2}{m^3} - \frac{x^3 i}{m^4} + \frac{x^4 i^2}{m^5} \&c \right)$

$= r \left(\frac{1}{2} - \frac{i}{3} + \frac{i^2}{4} \&c. \right) - r \left(\frac{1}{3} - \frac{i}{4} + \frac{i^2}{5} \&c. \right)$

$= \frac{r}{i} + \frac{r}{i^2} - \frac{r}{2i} - \frac{r}{i^2} \left(i - \frac{i^2}{2} + \frac{i^3}{3} \&c. \right) - \frac{r}{i^3} \left(i - \frac{i^2}{2} + \frac{i^3}{3} \&c. \right)$

$= \frac{r(2+i)}{2i^2} - \frac{r}{i^2} \left(\text{h. log. } r \right) - \frac{r}{i^3} \left(\text{h. log. } r \right)$

$= \frac{r(2+i)}{2i^2} - \frac{r^2}{i^3} \left(\text{h. log. } r \right)$.

EXAMPLE.—Suppose it were required to find the present value of £1 annuity, payable half yearly, quarterly, or momently, during the joint lives of *A* & *B*, aged 20 & 40, estimating the rate of mortality by the Northampton Table, and the improvement of money at 3 per cent.

By Table XXXVII $AB = 12.0963$ see also page 231

— — $A_{,}B = 12.1563$ — — 232

— — $AB_{,} = 12.2356$ — — 230

and by Table XXXIV $\frac{1}{a_{,}} = 1.01305$

— — $\frac{1}{b_{,}} = 1.02063$

Whence, $\frac{A_{,}B}{a_{,}} = 12.1563 \times 1.01305 = 12.3150$

$\frac{AB_{,}}{b_{,}} = 12.2356 \times 1.02063 = 12.4880$

$\frac{A_{,}B}{a_{,}} + \frac{AB_{,}}{b_{,}} = \text{- - - - - - } = 24.8030$

Hence, if the annuity be payable half-yearly

$$P = \frac{1}{4}\left(\frac{1}{2+i}\right) = .123153$$

$$Q = \frac{r}{4}\left(\frac{1}{2+i}+\frac{4}{2+2i}\right) = .626847$$

$$R = \frac{r}{4}\left(\frac{1}{2+i}\right) = .126848$$

$$\text{And } (AB)^{II} = (P+Q)AB + R\left(\frac{A_{,}B}{a_{,}}+\frac{AB_{,}}{b_{,}}\right)+P$$

$$= .75 \times 12.0963 + .126848 \times 24.803 + .1232 = 12.3416$$

But, if the annuity be payable quarterly

$$P = \frac{1}{16}\left(\frac{9}{4+i}+\frac{4}{4+2i}+\frac{1}{4+3i}\right) = .216436$$

$$Q = \frac{r}{16}\left(\frac{1}{4+i}+\frac{4}{4+2i}+\frac{9}{4+3i}+\frac{16}{4+4i}\right) = .471054$$

$$R = \frac{r}{16}\left(\frac{3}{4+i}+\frac{4}{4+2i}+\frac{3}{4+3i}\right) = .158564$$

$$\text{And } (AB)^{IV} = (P+Q)AB + R\left(\frac{A_{,}B}{a_{,}}+\frac{AB_{,}}{b_{,}}\right)+P$$

$$= .68749 \times 12.0963 + .158564 \times 24.803 + .2164 = 12.4652$$

And if the annuity be payable momently

$$P = \frac{r^2}{i^3}\left(\text{h. log. } r\right) - \frac{2+3i}{2i^2} = .33086$$

$$Q = \frac{r}{i^3}\left(\text{h. log. } r\right) - \frac{r(2-i)}{2i^2} = .33580$$

$$R = \frac{r(2+i)}{2i^2} - \frac{r^2}{i^3}\left(\text{h. log. } r\right) = .16914$$

Whence $(AB)^{M} = (P+Q)AB + R\left(\frac{A,B}{a,} + \frac{AB,}{b,}\right) + P$

$= .66666 \times 12.0963 + .16914 \times 24.803 + .3309 = 12.5901$

Comparing these results with the value of a like annuity on the same lives payable once a year, we find the correct increment on account of the annuity being payable

Half-yearly = 12.3416 − 12.0963 = .2453

Quarterly = 12.4652 − 12.0963 = .3689

Momently = 12.5901 − 12.0963 = .4938

But if we refer to the formula

$$(AB)^{M} = AB + \int \frac{m-x}{m(m+xi)}$$

the corresponding increments, as given in page 317, are as under

For Half-yearly payments .2463

Quarterly — .3704

Momently — .4951

Hence, it appears that the results deduced by the formula $(ABC\&c.)^{M} = ABC\&c. + \int \frac{m-x}{m(m+xi)}$ are sufficiently near for practical application.

Supposing the annuity to be continued during the life of the survivor, its present value, if payable

Yearly, is $A + B - AB$ = 21.3898

Half-yearly $A^{II} + B^{II} - (AB)^{II}$ = 21.6371

Quarterly $A^{IV} + B^{IV} - (AB)^{IV}$ = 21.7617

Momently $A^{M} + B^{M} - (AB)^{M}$ = 21.8862

Problem III.

To find the present value of £1 annuity payable by m equal instalments in each year, during the existence of a single life A; supposing the interest convertible into principal at the like intervals, and the annuity to cease with the last instalment becoming due prior to the extinction of the life proposed.

Supposing a similar annuities to be made payable from a common fund, and reasoning as in the foregoing Problems, we find the number of surviving claimants at the end of the $\frac{x}{m}$th part of the tth year $= \frac{1}{m}\left\{(m-x).{}^{t}a_{t} + x.{}^{t}a\right\}$. Also, supposing the interest convertible into principal m times a year, at the nominal rate denoted by i, the amount of £1 in 1 year, is (by Chap II. Sec. 16.) $= \left(1+\frac{i}{m}\right)^{m}$, and the present value of £1 due one year hence is $\frac{1}{1+\frac{i}{m})^{m}}$ which put $= u$, then will the present value of a like sum, due $t-1$ years hence $= u^{t-1}$. Hence the present value of the £$\frac{1}{m}$ to which each surviving claimant will be entitled at the end of the $\frac{x}{m}$th part of the tth year is $u^{t-1} \times \frac{1}{m} \times \frac{1}{(1+\frac{i}{m})^{x}} = \frac{u^{t-1}.m^{x-1}}{(m+i)^{x}}$: that of all the payments then due is therefore

$$= \frac{(m-x)m^{x-2}}{(m+i)^{x}}.\ {}^{t}a_{t}u^{t-1} + \frac{xm^{x-2}}{u(m+i)^{x}}.\ {}^{t}au^{t}.$$

Then making x successively = 1, 2, 3 &c. to m, and putting $\int.\frac{(m-x)m^{x-2}}{(m+i)^x} = P$, and $\int.\frac{xm^{x-2}}{u(m+i)^x} = Q$, we have the present value of all the payments becoming due within the tth year $= P.{}^{t}a_{,}u^{t-1} + Q.{}^{t}au^{t}$; from which, by making $t = 1, 2, 3$ &c., and adding the results, as in the foregoing Problems, we deduce the present value of the aggregate fund necessary to provide for all the annuities supposed $= P.(1+\mathrm{A})a + Q.\mathrm{A}a$. Hence, dividing this result by a, the number of lives from which the average is taken, we have

$$\mathrm{A}^{\mathrm{M}} = P(1+\mathrm{A}) + Q.\mathrm{A} = (P+Q)\mathrm{A} + P$$

for the present value required. A being the value of £1 annuity payable yearly, calculated at a rate of interest making the amount of £1 in 1 year $= (1+\frac{i}{m})^m$.

Corollary I.—If the annuity be payable half-yearly $P = \frac{1}{4+2i}$, and $Q = \frac{6+i}{8}$, from which the value of the annuity may be determined, provided a Table be formed giving the value of A at the rate abovementioned, of which that in page 332 is a specimen.

Corollary II.—If the annuity be payable quarterly

$$P = \frac{96+32i+3i^2}{4(4+i)^3}, \text{ and } Q = \frac{640+160i+20i^2+i^3}{1024}$$

from which the value of the annuity may be deter-

mined by the assistance of the Table already mentioned.

COROLLARY III.—If the annuity be payable momently

$$P=\int.\frac{(m-x)m^{x-2}}{(m+i)^x}=\int.\frac{(m-x)m^x}{m^2(m+i)^x}=\int.\frac{m-x}{m^2}\left\{1+\frac{i}{m}\right\}^{-x}$$

which by expanding the last factor in terms of x, and putting hyp. log. $(1+\frac{i}{m})=K$ becomes transformed into

$$\int.\frac{m-x}{m^2}\left\{1-Kx+\frac{K^2x^2}{2}-\frac{K^3x^3}{2\cdot3}+\frac{K^4x^4}{2\cdot3\cdot4}\&c.\right\}$$

But we have shown, in page 72, that if m be infinite $i=$ hyp. log. $(1+\frac{i}{m})^m=m\{$hyp. log. $(1+\frac{i}{m})\}=mK$ consequently $K=\frac{i}{m}$, and by substituting this value of K in the above expression, we have

$$P=\int.\frac{m-x}{m^2}\left\{1-\frac{xi}{m}+\frac{x^2i^2}{2m^2}-\frac{x^3i^3}{2\cdot3m^3}+\frac{x^4i^4}{2\cdot3\cdot4m^4}\&c.\right\}$$

$$=\int.\left\{\frac{1}{m}-\frac{xi}{m^2}+\frac{x^2i^2}{2m^3}-\frac{x^3i^3}{2\cdot3m^4}\&c.\right\}$$

$$-\int.\left\{\frac{x}{m^2}-\frac{x^2i}{m^3}+\frac{x^3i^2}{2m^4}-\frac{x^4i^3}{2\cdot3m^5}\&c.\right\}$$

Then making $x=1, 2, 3$ &c. to m, and adding the results, according to the principles stated in the note, page 143,

$$P=\left\{1-\frac{i}{2}+\frac{i^2}{2\cdot3}-\frac{i^3}{2\cdot3\cdot4}\&c.\right\}-\left\{\frac{1}{2}-\frac{2i}{2\cdot3}+\frac{3i^2}{2\cdot3\cdot4}-\frac{4i^3}{2\cdot3\cdot4\cdot5}\&c.\right\}$$

$$=\frac{1}{2}-\frac{i}{2\cdot3}+\frac{i^2}{2\cdot3\cdot4}-\frac{i^3}{2\cdot3\cdot4\cdot5}\&c.$$

$$Q = \int \cdot \frac{xm^{x-2}}{u(m+i)^x} = \int \cdot \frac{xm^x}{m^2 u(m+i)^x} = \frac{1}{u} \cdot \int \cdot \frac{x}{m^2} \left\{ 1 + \frac{i}{m} \right\}^{-x}$$

which being expanded and summed as before, gives

$$Q = \frac{1}{u} \left\{ \frac{1}{2} - \frac{2i}{2 \cdot 3} + \frac{3i^2}{2 \cdot 3 \cdot 4} - \frac{4i^3}{2 \cdot 3 \cdot 4 \cdot 5} \&c. \right\}$$

But $\frac{1}{u} = \left\{ 1 + \frac{i}{m} \right\}^m = 1 + i + \frac{i^2}{2} + \frac{i^3}{2 \cdot 3} + \frac{i^4}{2 \cdot 3 \cdot 4}$ &c, and

$$Q = \left\{ 1 + i + \frac{i^2}{2} + \frac{i^3}{2 \cdot 3} \&c. \right\} \times \left\{ \tfrac{1}{2} - \frac{2i}{2 \cdot 3} + \frac{3i^2}{2 \cdot 3 \cdot 4} - \frac{4i^3}{2 \cdot 3 \cdot 4 \cdot 5} \&c. \right\}$$

$$= \frac{1}{2} + \frac{i}{2 \cdot 3} + \frac{i^2}{2 \cdot 3 \cdot 4} + \frac{i^3}{2 \cdot 3 \cdot 4 \cdot 5} \&c.$$

Hence it appears that $P + Q = 1 + \frac{i^2}{3 \cdot 4} + \frac{i^4}{3 \cdot 4 \cdot 5 \cdot 6} \&c.$

from which $A^M = (P + Q)A + P$ may be easily determined by the assistance of a Table of the value of A, calculated at a rate of interest giving the amount of £1 in 1 year $= (1 + \frac{i}{m})^m$

Corollary iv.—If the annuity be made payable by m equal instalments in each year, during the joint existence of two or more lives A, B &c, and out of ab&c. similar combinations, those that fail within the tth year, be supposed to fail in equal proportions in equal intervals of that year, it may be shown by a process similar to the above, that

$$(AB\&c.)^M = (P + Q)AB\&c. + P,$$

AB&c. being the present value of £1 annuity payable yearly at a rate of interest giving the amount of £1 in 1 year $= \left(1 + \frac{i}{m} \right)^m$.

A TABLE showing the present value of £1 Annuity on a Single Life payable yearly at the nominal rate of 4 per Cent, upon the supposition of the Interest being convertible into Principal half-yearly, or quarterly, according to the rate of mortality among the members of the Equitable.

Age.	Half-yearly Conversions.	Quarterly Conversions.	Age	Half-yearly Conversions.	Quarterly Conversions.
10	19.5133	19.4459	54	11.5571	11.5347
11	19.3805	19.3140	55	11.3067	11.2853
12	19.2420	19.1766	56	11.0537	11.0332
13	19.1048	19.0405	57	10.7984	10.7789
14	18.9619	18.8986	58	10.5407	10.5221
15	18.8201	18.7577	59	10.2809	10.2632
16	18.6794	18.6180	60	10.0191	10.0023
17	18.5397	18.4794	61	9.7471	9.7323
18	18.4014	18.3419	62	9.4679	9.4528
19	18.2644	18.2059	63	9.1847	9.1705
20	18.1287	18.0712	64	8.8988	8.8854
21	17.9875	17.9309	65	8.6105	8.5980
22	17.8404	17.7847	66	8.3202	8.3084
23	17.6941	17.6397	67	8.0282	8.0173
24	17.5419	17.4883	68	7.7353	7.7250
25	17.3905	17.3379	69	7.4419	7.4325
26	17.2326	17.1812	70	7.1493	7.1405
27	17.0754	17.0249	71	6.8506	6.8424
28	16.9186	16.8692	72	6.5539	6.5464
29	16.7624	16.7140	73	6.2527	6.2458
30	16.6070	16.5596	74	5.9562	5.9499
31	16.4453	16.3989	75	5.6583	5.6526
32	16.2842	16.2387	76	5.3514	5.3462
33	16.1236	16.0792	77	5.0446	5.0399
34	15.9567	15.9134	78	4.7304	4.7262
35	15.7832	15.7409	79	4.4191	4.4154
36	15.6098	15.5684	80	4.1149	4.1116
37	15.4294	15.3891	81	3.8238	3.8209
38	15.2489	15.2096	82	3.5436	3.5410
39	15.0611	15.0229	83	3.2840	3.2818
40	14.8658	14.8287	84	3.0471	3.0450
41	14.6625	14.6265	85	2.8353	2.8335
42	14.4508	14.4159	86	2.6507	2.6491
43	14.2302	14.1964	87	2.4906	2.4892
44	14.0074	13.9746	88	2.3382	2.3370
45	13.7822	13.7505	89	2.1768	2.1758
46	13.5475	13.5169	90	1.9616	1.9608
47	13.3028	13.2733	91	1.7073	1.7066
48	13.0546	13.0261	92	1.4177	1.4172
49	12.8026	12.7752	93	1.1239	1.1236
50	12.5467	12.5204	94	.8271	.8269
51	12.2938	12.2685	95	.5298	.5297
52	12.0442	12.0199	96	.2403	.2403
53	11.7984	11.7752	97	0	0

EXAMPLE I.—What is the present value of £50 annuity on a life of 40, payable by half-yearly instalments, estimating the interest of money at 2 per Cent. half-yearly, and the rate of mortality by the experience of the Equitable?

Here A, found by the foregoing Table, is 14.8658

$P = \frac{1}{4+2i} = \frac{1}{4.08} = .245098$, and $Q = \frac{6+i}{8} = \frac{6.04}{8} = .755$

wherefore $(P+Q)A + P = 1.000098 \times 14.8658 + .2451 = 15.1123$, the present value of £1 annuity, from which that of £50 annuity is found to be $50 \times 15.1123 = 755.615$ £755 12s 4d the answer.

EXAMPLE II.—Required the present value of £60 annuity payable by quarterly instalments, during the remainder of a life of 35, estimating the interest of money at 1 per Cent. quarterly, and the rate of mortality by the experience of the Equitable?

By the foregoing Table A is found $= 15.7409$ and by Corollary II. $P = \frac{96+32i+3i^2}{4(4+i)^3} = \frac{97.2848}{263.757} = .368843$

and $Q = \frac{640+160i+20i^2+i^3}{1024} = \frac{646.432064}{1024} = .631281$

wherefore $(P+Q)A + P = 1.000124 \times 15.7409 + .3688 = 16.1116$, the present value of £1 annuity, from which that of £60 annuity is found to be worth $60 \times 16.1116 = 966.696 =$ £966 13s 11d the answer.

Problem iv.

To find the present value of £1 annuity payable by m equal instalments in each year, during an assigned life A; supposing the interest to be convertible into principal at the like intervals, and the annuity (instead of ceasing with the last instalment becoming due within the life-time of the party) to be continued up to the moment of death.

Putting P, Q, & u, as in pages 328, 329, and supposing a similar annuities to be made payable from a common fund, it is manifest, by what has been stated in the last Problem, that the present value of all the instalments becoming due within the tth year is $P.{}^{t}a_{,}u^{t-1} + Q.{}^{t}au^{t}$.

Also conceiving the xth interval of the tth year to be subdivided into n instants, the number of deaths in each will be denoted by $\frac{1}{n}\left(\frac{{}^{t}a_{,} - {}^{t}a}{m}\right)$, and the fraction payable at each death in the yth instant is $\frac{y}{n} \times £\frac{1}{m} = \frac{y}{mn}$, the present value of which is

$$\frac{y}{mn} \times \frac{1}{1+\frac{yi}{mn}} \times \frac{1}{(1+\frac{i}{m})^{x-1}} \times u^{t-1} = \frac{y}{mn+yi} \times \frac{m^{x-1}}{(m+i)^{x-1}} \times u^{t-1};$$

that of all the fractions then due is therefore

$$= \frac{y}{n(mn+yi)} \times \frac{m^{x-2}}{(m+i)^{x-1}}\left({}^{t}a_{,}u^{t-1} - \frac{1}{u}.{}^{t}au^{t}\right)$$

Hence, making $y = 1, 2, 3$ &c. to n, we have

$$\int.\frac{y}{n(mn+yi)} = \int.\left\{\frac{y}{mn^2}-\frac{y^2 i}{m^2 n^3}+\frac{y^3 i^2}{m^3 n^4}\&c.\right\}$$

$$=\frac{1}{2m}-\frac{i}{3m^2}+\frac{i^2}{4m^3}\&c. = \frac{1}{i}-\frac{m}{i^2}\left\{\frac{i}{m}-\frac{1}{2}\left(\frac{i}{m}\right)^2+\frac{1}{3}\left(\frac{i}{m}\right)^3\&c.\right\}$$

$$=\frac{1}{i}-\frac{m}{i^2}\left\{\text{h. log.}\left(1+\frac{i}{m}\right)\right\}=\frac{1}{i^2}\left\{i\text{-h. log.}\left(1+\frac{i}{m}\right)^m\right\}$$

which put $= L$,* then will the present value of all the fractions, becoming due within the xth interval of the tth year, be denoted by

$$\frac{Lm^{x-2}}{(m+i)^{x-1}}\left\{{}^t a_{,} u^{t-1}-\frac{1}{u}.\,{}^t a u^t\right\}$$

Again, making $x = 1, 2, 3$ &c. to m, and putting $\int.\frac{m^{x-2}}{(m+i)^{x-1}} = g$, we obtain $Lg({}^t a_{,} u^{t-1}-\frac{1}{u}.{}^t a u^t)$, for

---

* If i =.03, and m = 1, hyp. log. $\left(1+\frac{i}{m}\right)^m$ = .029558802 and L = .49022
.03, — — = 2, — — = .029777223 — = .24753
.03, — — = 4, — — = .029888058 — = .12430
.03, — — inf. — — = .030000000 — = .00000

If i =.04, and m = 1, hyp. log. $\left(1+\frac{i}{m}\right)^m$ = .039220713 and L = .48705
.04, — — = 2, — — = .039605255 — = .24672
.04, — — = 4, — — = .039801322 — = .12417
.04, — — inf. — — = .040000000 — = .00000

If i =.05, and m = 1, hyp. log. $\left(1+\frac{i}{m}\right)^m$ = .048790164 and L = .48393
.05, — — = 2, — — = .049385224 — = .24591
.05, — — = 4, — — = .049690081 — = .12397
.05, — — inf. — — = .050000000 — = .00000

If i =.06, and m = 1, hyp. log. $\left(1+\frac{i}{m}\right)^m$ = .058268908 and L = .48086
.06, — — = 2, — — = .059117603 — = .24511
.06, — — = 4, — — = .059554449 — = .12376
.06, — — inf. — — = .060000000 — = .00000

the aggregate present value of all the fractions becoming due within the tth year: this again being added to the present value of all the instalments becoming due within the same period, gives

$$P.{}'a_{\overline{t}}v^{t-1} + Q.{}'av^t + Lg({}'a_{\overline{t}}v^{t-1} - \tfrac{1}{u}.{}'av^t)$$

for the total present value of all the payments becoming due from the supposed fund within the tth year.

Whence making $t = 1, 2, 3$ &c, adding the results, and dividing their sum by a, as in the foregoing Problems, we ultimately deduce

$$P(1+\mathrm{A}) + Q.\mathrm{A} + Lg\{(1+\mathrm{A}) - \tfrac{1}{u}.\mathrm{A}\}$$

for the present value required. But we have already shown by the last Problem, that the sum of the first two of these three terms constitutes the present value of a similar annuity, ceasing with the last instalment becoming due prior to the extinction of the proposed life, which in symbols we have denoted by A^{M}. And the third term, by reduction, becomes $= Lg\{1 - (\tfrac{1}{u} - 1)\mathrm{A}\}$; hence by substitution we have

$$\mathrm{A}^{\mathrm{M}} + Lg\{1 - (\tfrac{1}{u} - 1)\mathrm{A}\}$$

as a correct expression for the present value required.

Corollary 1.—If the annuity be payable yearly,

$$m \text{ \& } x \text{ each } = 1,\ \tfrac{1}{u} - 1 = r - 1 = i,$$

$$g = \int \frac{m^{x-2}}{(m+i)^{x-1}} = \frac{1^{-1}}{(1+i)^0} = 1,$$

A^{M} & A each $= \mathrm{A}$, and consequently the present value of the annuity is $\mathrm{A} + L(1 - i\mathrm{A})$

COROLLARY II.—If the annuity be payable half-yearly, $m=2$, $x = 1$ & 2 in succession,

$$\frac{1}{u}-1=\left(1+\frac{i}{2}\right)^{2}-1=\frac{i(4+i)}{4}, \text{ \& } g=\frac{m^{x-2}}{(m+i)^{x-1}}=\frac{4+i}{4+2i},$$

from which the value of the annuity, denoted by $A^{M}+Lg\{1-(\frac{1}{u}-1)\mathbb{A}\}$, may be determined, provided A^{M} and $\mathbb{A}$ be found as directed in last Problem.

COROLLARY III.—If the annuity be payable quarterly, $m=4$, $x=1$, 2, 3 & 4 in succession,

$$\frac{1}{u}-1=\left(1+\frac{i}{4}\right)^{4}-1, \text{ \& } g=\frac{m^{x-2}}{(m+i)^{x-1}}=\frac{256+96i+16i^{2}+i^{3}}{4(4+i)^{3}},$$

from which the value of the annuity may be found as before directed.

COROLLARY IV.—If the annuity be payable momently, $L = 0$, and the required present value $A^{M}+Lg\{1-(\frac{1}{u}-1)\mathbb{A}\}$ becomes $= A^{M}$, as already determined in Corollary III, pages 330, 331.

COROLLARY V.—If the annuity be made payable during the joint existence of two or more lives, *A*, *B*&c. it may be shown, in like manner, that its present value, on the supposition of the combinations failing in equal proportions during equal intervals of the same year, is

$$(AB\&c.)^{M}+Lg\{1-(\tfrac{1}{u}-1)\mathbb{AB}\&c.\}$$

the value of $(AB\&c.)^{M}$, being determined as in page 319 or 331, and that of $\mathbb{AB}$&c. calculated as before directed.

EXAMPLE I.—What is the present value of £120 annuity payable yearly during a life of 50, and continued up to the moment of death, estimating interest at 4 per Cent. and the rate of mortality by the experience of the Equitable?

By Table XXIV $A = 12.5986$, and by the Note in page 335, $L = .48705$; hence, $A + L(1 - iA) = 12.5986 + .48705\,(1 - .04 \times 12.5986) = 12.5986 + .2416 = 12.8402$, the present value of £1 annuity on the proposed life: that of £120 is therefore $= 120 \times 12.8402 = 1540.824$ = £1540 16s 6d, the answer.

EXAMPLE II.—What is the present value of £50 annuity payable half-yearly during a life of 40, and continued up to the moment of death, estimating the interest of money at 2 per Cent. half-yearly, and the rate of mortality by the experience of the Equitable?

By Example I, page 333, $A^{(2)} = 15.1123$, and $A = 14.8658$. Also by Corollary II, in last page,

$$\frac{1}{u} - 1 = \frac{i(4+i)}{4} = .0404, \text{ and } g = \frac{4+i}{4+2i} = .990196.$$

And, by the Note in page 335, $L = .24672$; whence $A^{(2)} + Lg\{1 - (\frac{1}{u} - 1)A\} = 15{\cdot}1123 + {\cdot}24672 \times .990196 \times (1 - {\cdot}0404 \times 14{\cdot}8658) = 15{\cdot}2099$, the present value of £1 annuity, from which that of £50 annuity is found to be $= 50 \times 15{\cdot}2099 = 760{\cdot}495 =$ £760 9s 11d the answer.

EXAMPLE III.—Required the present value of £60 annuity payable by quarterly instalments during a life of 35, and continued up to the moment of death, estimating the interest of money at 1 per Cent. quarterly, and the rate of mortality by the experience of the Equitable?

By Example II, page 333, $A^{IV} = 16{\cdot}1116$, and $\mathbb{A} = 15.7409$. Also, by Corollary III, page 337,

$$\frac{1}{u} - 1 = \left(1 + \frac{i}{4}\right)^4 - 1, \text{ which by Table IV} = .040604;$$

$$g = \frac{256 + 96i + 16i^2 + i^3}{4(4+i)^3} = \frac{259.86566}{263{\cdot}757} = .985247,$$

and by the Note in page 335, $L = .12417$; whence $A^{IV} + Lg\{1 - (\frac{1}{u} - 1)\mathbb{A}\} = 16{\cdot}1557 + {\cdot}12417 \times {\cdot}985247 \times (1 - {\cdot}040604 \times 15{\cdot}7409) = 16{\cdot}1557$, the present value of £1 annuity, from which that of £60 annuity is found to be $= 60 \times 16{\cdot}1557 = 969.342 =$ £969 6s 10d the answer.

SCHOLIUM.—If the interest be convertible into principal yearly, the present value of £1 annuity on a single life A, payable by m equal instalments in each year, and continued up to the moment of death, may be conveniently approximated by the formula

$$A + \frac{m - 1 + P}{2m}$$

where P represents the present value of £1 to be received at the end of the year in which the assigned life may fail, which may be found as directed in Example XXIV, page 284, or already calculated in Tables XXXI and XXXVIII.

Problem v.

To find the present value of £1 annuity on a single life A, deferred for t years, and payable by m equal instalments in each year for the remainder of life.

Let the required value be denoted by ${}_t\mathrm{A}^{\mathrm{M}}$, and that of a similar annuity, for the whole period, on a life t years older than the given life, by ${}'\mathrm{A}^{\mathrm{M}}$. Also put D & N, for the numbers, in columns D & N, opposite the given age in Table XXI or XXXV, and 'D & 'N, those opposite an age t years older than that given.

Then, conceiving a persons of the proposed age, by joint contributions, to raise a fund sufficient to provide similar annuities for such of them as may survive the given term, we have the present value of the whole fund $= a \times {}_t\mathrm{A}^{\mathrm{M}}$. Also the number of survivors (to enter upon their annuities) at the end of the tth year $= {}'a$, each then entitled to an immediate annuity worth ${}'\mathrm{A}^{\mathrm{M}}$, and the then value of the whole fund $= {}'a.{}'\mathrm{A}^{\mathrm{M}}$; its present value, when discounted for t years, is therefore $= {}'av^t.{}'\mathrm{A}^{\mathrm{M}}$. But we have already shown that the present value of the supposed fund is $= a \times {}_t\mathrm{A}^{\mathrm{M}}$; whence it follows that $a \times {}_t\mathrm{A}^{\mathrm{M}} = {}'av^t.{}'\mathrm{A}^{\mathrm{M}}$, and consequently

$$ {}_t\mathrm{A}^{\mathrm{M}} = \frac{{}'av^t}{a}.{}'\mathrm{A}^{\mathrm{M}} $$

a correct expression for the present value of the annuity proposed: observing, however, to find the

value of $'A^{M}$; by Problem I, if the interest be convertible into principal yearly; by Problem III, if the interest be convertible into principal at the like intervals with the instalments of the annuity; or, by Problem IV, if the annuity be continued up to the moment of death; and not forgetting, in the last two cases, that $v^{t} = \{1 + \frac{i}{m}\}^{-mt} = u^{t}$.

COROLLARY I.—If the annuity be payable yearly, and cease with the last payment becoming due in the life-time of the party,

$$_{t}A = \frac{'av^{t}}{a} .'A, \quad \text{or} = \frac{'N}{D} \text{ as shown in page 277.}$$

COROLLARY II.—If the annuity be payable at the beginning of each year, and cease with the last payment becoming due prior to the death of the party, its value at the end of the tth year must be increased by the payment then due; hence, substituting $1 + 'A$ for $'A$ in the last Corollary, we have its present value

$$= \frac{'av^{t}}{a}(1 + 'A) \quad \text{or} = \frac{'N}{D} \quad 'N, \text{ being } = {}'^{-1}N.$$

COROLLARY III.—If the annuity depend on the joint existence of two or more lives A, B &c. its value, found by the same process, is denoted by

$$_{t}(AB\&c.)^{M} = \frac{'(ab\&c.)v^{t}}{ab\&c.} .'(AB\&c.)^{M}$$

A general formula, to which the above Corollaries are both applicable.

EXAMPLE I.—What is the present value of £70 annuity on a life of 40, deferred for 15 years, estimating the interest of money at 4 per Cent. per annum, and the rate of mortality by the experience of the Equitable?

By Table X, $a = 2236$, & $'a = 1744$; also by Tables VI, and XXIV, $v' = .555265$ & $'A = 11.3487$; wherefore $\frac{'av'}{a} = \frac{1744 \times .555265}{2236} = .433087$ and ${}_{,}A = \frac{'av'}{a} . 'A = .433087 \times 11.3487 = 4.9149$, the present value of £1 annuity, from which that of £70 annuity is found to be $= 4.9149 \times 70 =$ £344 0s 10d, the answer.

Again, supposing the annuity to be payable by half-yearly instalments, we find, by Corollary v, page 317, that $\frac{'av'}{a} . 'A^{II} = .433087(11.3487 + .2451) = 5.0211$, and the required value is therefore $= 5.0211 \times 70 =$ £351 9s 6d.

And if the annuity be payable quarterly, we have ${}_{,}A^{IV} = \frac{'av'}{a} . 'A^{IV} = .433087(11.3487 + .3689) = 5.0746$, from which the required value is found to be $5.0746 \times 70 =$ £355 4s 5d.

Finally, supposing the annuity payable momently, ${}_{,}A^{M} = \frac{'av'}{a} . 'A^{M} = .433087(11.3487 + .4935) = 5.1287$, from which the required value is found to be $= 5.1287 \times 70 =$ £359 0s 2d.

Otherwise, supposing the interest convertible into principal yearly, the approximated value of a deferred annuity of £1 on a single life, or on the joint existence of two or more lives, and payable by m equal instalments in each year, may be conveniently expressed by

$$\frac{{}'\mathrm{N} + \frac{m-1}{2m}.{}'\mathrm{D}}{\mathrm{D}}$$

Thus, the approximated value of the annuity proposed in the foregoing Example, if payable half-yearly, is by Table XXI,

$$\frac{2289.074+\frac{1}{4}(201.703)}{465.733} \times 70 = 5.0233 \times 70 = £351\ 12s\ 7d.$$

EXAMPLE II.—Suppose an annuity of £70, dependent on a single life of 40, and deferred for 15 years, to be payable by half-yearly instalments, and continued up to the moment of death; it is required to find its present value, estimating the interest of money at 2 per Cent. half-yearly, and the rate of mortality by the experience of the Equitable?

Here ${}'\mathrm{A}^{\mathrm{M}}$, the present value of an immediate annuity of £1, on a life of 55, payable by half-yearly instalments, and continued up to the moment of death, found by Problem IV, is 11.6856.

and $v' = \left\{1 + \frac{i}{m}\right\}^{-mt} = (1.02)^{-30}$, by Table VI $= .552071$

hence ${}_{t}\mathrm{A}^{\mathrm{M}} = \frac{{}'av'}{a}.{}'\mathrm{A}^{\mathrm{M}} = \frac{1744 \times .552071}{2236} \times 11.6856 =$

5.0317; and $5.0317 \times 70 = £352\ 4s\ 5d$ the answer.

Problem VI.

To find the present value of a Temporary Annuity of £1 on a single life A, payable by m equal instalments in each year for the next t years.

Let the required present value $= {}_{\overline{t}|}\mathrm{A}^{\mathrm{M}}$, that of a similar annuity on the same life for the whole duration $= \mathrm{A}^{\mathrm{M}}$, and the remaining symbols as in last Problem.

Then it is manifest that a Temporary annuity of £1 for the next t years, and a similar annuity Deferred for the same period, together constitute a continued annuity on the whole life, or in symbols ${}_{\overline{t}|}\mathrm{A}^{\mathrm{M}} + {}_{t}\mathrm{A}^{\mathrm{M}} = \mathrm{A}^{\mathrm{M}}$, from which it follows that ${}_{\overline{t}|}\mathrm{A}^{\mathrm{M}} = \mathrm{A}^{\mathrm{M}} - {}_{t}\mathrm{A}^{\mathrm{M}}$, or by substituting the value of ${}_{t}\mathrm{A}^{\mathrm{M}}$ as determined by the last Problem, we have

$$ {}_{\overline{t}|}\mathrm{A}^{\mathrm{M}} = \mathrm{A}^{\mathrm{M}} - \frac{{}^{t}av^{t}}{a} \cdot {}^{t}\mathrm{A}^{\mathrm{M}} $$

for the present value of the annuity proposed; observing, however, to find the values of A^{M} and ${}^{t}\mathrm{A}^{\mathrm{M}}$, by Problems I, III, or IV, according as the interest is convertible into principal yearly, or at the same periods as the instalments of the annuity, or the annuity continued to the moment of death; and that v^{t} in the last two cases is $= \left\{1 + \frac{i}{m}\right\}^{-mt} = w^{t}$.

Corollary I.—If the annuity be payable yearly, and cease with the last payment becoming due in the life-time of the party $\mathrm{A}^{\mathrm{M}} = \mathrm{A}$, ${}^{t}\mathrm{A}^{\mathrm{M}} = {}^{t}\mathrm{A}$, and ${}_{\overline{t}|}\mathrm{A} = \mathrm{A} - \frac{{}^{t}av^{t}}{a} \cdot {}^{t}\mathrm{A}$, or $= \frac{\mathrm{N} - {}^{t}\mathrm{N}}{\mathrm{D}}$ as shown in page 274.

COROLLARY II.—If the annuity be payable at the beginning of the year, we have only to substitute $1+A$ for A^{M}, and $1+{}'A$ for ${}'A^{M}$, and the required present value (then an annuity for $t-1$ years beside the first payment) becomes

$$1+{}_{\overline{t-1}|}A = 1+A-\frac{{}'av'}{a}(1+{}'A) \text{ or } = \frac{N_{,}-{}'N_{,}}{D}$$ as shown in page 274.

COROLLARY III.—If the annuity depend on the joint existence of two or more lives, A, B &c. its value, found by the same process, is denoted by

$${}_{\overline{n}|}(AB\&c.)^{M} = (AB\&c.)^{M} - \frac{{}'(ab\&c.)v'}{ab\&c.}\cdot{}'(AB\&c.)^{M}$$

a general formula, to which the foregoing Corollaries are also applicable.

EXAMPLE I.—What is the present value of a Temporary annuity of £70, for the next 15 years on a life of 40, estimating the interest of money at 4 per Cent. per annum, and the rate of mortality by the experience of the Equitable?

By Table XXIV, opposite the given age, $A = 14{\cdot}9390$, and by Example I, page 342, $\frac{{}'av'}{a}\cdot{}'A$ is $= 4{\cdot}9149$.

Wherefore ${}_{\overline{n}|}A = A - \frac{{}'av'}{a}\cdot{}'A = 14{\cdot}9390 - 4{\cdot}9149$ $= 10{\cdot}0241$, the present value of £1 annuity, and $10{\cdot}0241 \times 70 = 701.687$ = £701 13s 9d the answer.

Again, supposing the annuity to be payable by half-yearly instalments, and the interest to be still convertible into principal yearly,

$$\overline{\ }A^{II} = A^{II} - \frac{{}^{t}av^{t}}{a} \cdot {}^{t}A^{II},$$

which by Corollary v, page 317, and Example I, page 342, becomes $= 14{\cdot}9390 + {\cdot}2451 - 5{\cdot}0211 = 10{\cdot}1630$, from which the required value is found to be $= 10{\cdot}1630 \times 70 =$ £711 8s 2d.

Or if the annuity be payable quarterly, we have

$$\overline{\ }A^{IV} = A^{IV} - \frac{{}^{t}av^{t}}{a} \cdot {}^{t}A^{IV} = 14{\cdot}9390 + {\cdot}3689 - 5{\cdot}0746 = 10{\cdot}2333,$$

from which the required value becomes $= 10{\cdot}2333 \times 70$ $716{\cdot}331 =$ £716 6s 8d.

Moreover, supposing the annuity to be payable momently, $\overline{\ }A^{M} = A^{M} - \frac{{}^{t}av^{t}}{a} \cdot {}^{t}A^{M} = 14{\cdot}9390 + {\cdot}4935 - 5{\cdot}1287 = 10{\cdot}3038$, from which the required value is $10{\cdot}3038 \times 70 =$ £721 5s 4d.

Otherwise, supposing the interest convertible into principal yearly, the approximated value of a Temporary annuity, payable by m equal instalments in each year, may be determined by the formula

$$\frac{N - {}^{t}N + \frac{m-1}{2m}(D - {}^{t}D)}{D}$$

Thus the approximated value of the annuity proposed in the foregoing Example, if payable half-yearly, is

$$\frac{6957{\cdot}623 - 2289{\cdot}074 + \frac{1}{4}(465{\cdot}733 - 201{\cdot}703)}{465{\cdot}733} \times 70 =$$

£711 12s 2d. which is nearly the same as before.

EXAMPLE II.—Suppose an annuity of £70, payable by half-yearly instalments for the next 15 years, provided a life now 40 should survive that period, or to be continued up to the moment of death in the event of the life failing within such period; it is required to find its present value, estimating the interest of money at 2 per Cent. half-yearly, and the rate of mortality by the experience of the Equitable?

Here A^M, found in page 338, - - = 15·2099

and $\frac{'av'}{a}.'A^M$, found in page 343, - = 5·0317

wherefore $_{\overline{n}|}A^M = A^M - \frac{'av'}{a}.'A^M = 15{\cdot}2099 - 5{\cdot}0317$

$= 10{\cdot}1782$, the present value of each £1 of the proposed annuity, from which that of £70 per Annum is found to be $= 10{\cdot}1782 \times 70 =$ £712 9s 6d. the answer.

SCHOLIUM.

By connecting what has been advanced in this Section with the mode of reasoning employed in the last, it is manifest that the solutions given to the first twenty of the foregoing Miscellaneous Questions, may be made applicable to cases where the annuities are payable by instalments, and restricted in point of time, so as to be rendered Temporary or Deferred.

10. *Of Increasing and Decreasing Annuities.*

Problem 1.

To find the present value of a Temporary Annuity for t years, increasing in the following manner, viz.

$£m$	at the end of the	. . .	1st year
$m+n$	—	. . .	2nd
$m+2n$	—	. . .	3rd
$m+(x-1)n$	—	. . .	xth

Referring to any Table formed as directed in the second section of this Chapter; putting D, N, & S, for the respective numbers, in the columns so marked, opposite the proposed age; ^{x}D, ^{x}N, & ^{x}S, for those, in the like columns, opposite an age x years older than that proposed; and supposing D similar annuities made payable from a common fund; it is manifest, from the construction of the Table, that the present value of all the payments becoming due at the end of the xth year, is denoted by $\{m+(x-1)n\}{}^{x}\mathrm{D} = (m-n).{}^{x}\mathrm{D} + nx.{}^{x}\mathrm{D}$.

Hence, making x successively $= 1, 2, 3,$ &c. to t, and adding the results, we find the present value of the whole fund necessary to provide for the payment of all the annuities supposed

$$= (m-n)({}^{1}\mathrm{D} + {}^{2}\mathrm{D} + {}^{3}\mathrm{D} \ldots\ldots\ldots {}^{t}\mathrm{D})$$
$$+ n({}^{1}\mathrm{D} + 2.{}^{2}\mathrm{D} + 3.{}^{3}\mathrm{D} \ldots\ldots\ldots t.{}^{t}\mathrm{D})$$

But by the construction of the Table here referred to, it is manifest that

$$\begin{aligned} {}^{1}D + {}^{2}D + {}^{3}D \ldots . {}^{t}D &= N - {}^{t}N \\ {}^{2}D + {}^{3}D \ldots . {}^{t}D &= {}^{1}N - {}^{t}N \\ {}^{3}D \ldots . {}^{t}D &= {}^{2}N - {}^{t}N \end{aligned}$$

hence $${}^{1}D + 2.{}^{2}D + 3.{}^{3}D \ldots t.{}^{t}D = N + {}^{1}N + {}^{2}N \ldots {}^{t}N_{,} - t.{}^{t}N$$

and, by substitution, the present value of the supposed fund is transformed into

$$(m-n)(N - {}^{t}N) + n(N + {}^{1}N + {}^{2}N \ldots \ldots {}^{t}N_{,} - t.{}^{t}N)$$

Again, referring to the construction of column S, we find in like manner that

$$N + {}^{1}N + {}^{2}N \ldots {}^{t}N_{,} + {}^{t}N + {}^{t+1}N \text{\&c.} = S$$

and $${}^{t}N + {}^{t+1}N \text{\&c.} = {}^{t}S$$

diff. $$N + {}^{1}N + {}^{2}N \ldots {}^{t}N_{,} = S - {}^{t}S$$

from which, by further substitution, the present value of the supposed fund becomes transformed into

$$(m-n)(N - {}^{t}N) + n(S - {}^{t}S - t.{}^{t}N)$$

and this expression, divided by D, the number of annuities supposed, gives as an average

$$\frac{(m-n)(N - {}^{t}N) + n(S - {}^{t}S - t.{}^{t}N)}{D}$$

for the present value of the annuity proposed.

Corollary 1.—If t be not less than the greatest tabular duration of the life proposed, ${}^{t}N$ & ${}^{t}S$ each $= 0$, and the present value of an annuity commenc-

ing at $£m$, and increasing $£n$ annually to the end of life, is

$$\frac{(m-n)\mathrm{N}+n\mathrm{S}}{\mathrm{D}}$$

COROLLARY II.—Supposing t as in the last Corollary, and $n = m$, the present value of an annuity commencing at $£n$, and increasing $£n$ annually to the end of life, is

$$\frac{n\mathrm{S}}{\mathrm{D}}$$

COROLLARY III.—If $n = 0$, the present value of a constant annuity of $£m$, for the next t years, is

$$\frac{m(\mathrm{N}-{}'\mathrm{N})}{\mathrm{D}}$$

agreeing with the rule given for finding the present value of a temporary annuity in page 274.

COROLLARY IV.—If n be taken negatively, the present value of an annuity commencing at $£m$, and *decreasing* by $£n$ annually, is

$$\frac{(m+n)(\mathrm{N}-{}'\mathrm{N})-n(\mathrm{S}-{}'\mathrm{S}-t.{}'\mathrm{N})}{\mathrm{D}}$$

but in this case n must not exceed $\frac{m}{t-1}$, otherwise the annuity, towards the latter part of the proposed term, must become negative.

COROLLARY V.—If in the last expression t be not less than the greatest tabular duration of the proposed life, ${}'\mathrm{N}$ & ${}'\mathrm{S}$ each $= 0$, and the present

value of an annuity commencing at £m, and *decreasing* by £n annually to the end of life, is

$$\frac{(m+n)\mathrm{N}-n\mathrm{S}}{\mathrm{D}}$$

EXAMPLE 1.—What is the present value of an annuity commencing at £50, and increasing £10 annually during the remaining part of a life of 40, estimating the interest of money at 4 per cent., and the rate of mortality by the experience of the Equitable?

By Corollary 1 and Table XXI, the required value is

$$\frac{(m-n)\mathrm{N}+n\mathrm{S}}{\mathrm{D}}$$

$$=\frac{(50-10)\times 6957{\cdot}623+10\times 89063{\cdot}8}{465{\cdot}733}=2509{\cdot}9$$

= £2509 18s 0d the answer.

EXAMPLE II.—What is the present value of a temporary annuity for 10 years on a life of 50, the annuity commencing at £20, and increasing £5 annually; estimating the improvement of money at 3 per Cent., and the rate of mortality by the Northampton Table?

Here the required value by the general formula is

$$\frac{(m-n)(\mathrm{N}-{}'\mathrm{N})+n(\mathrm{S}-{}'\mathrm{S}-t.{}'\mathrm{N})}{\mathrm{D}}$$

By Table XXXV $\mathrm{N}-{}'\mathrm{N}=8104{\cdot}543-3382{\cdot}152=4722{\cdot}391$

$\mathrm{S}-{}'\mathrm{S}-t.{}'\mathrm{N}=85391{\cdot}56-28057{\cdot}34-33821{\cdot}52=23512{\cdot}70$

Hence $\dfrac{15\times 4722{\cdot}391+5\times 23512{\cdot}70}{651{\cdot}702}=289.088=$

£289 1s 9d, the answer.

EXAMPLE III.—What is the present value of a decreasing annuity on a life of 65, the annuity commencing at £200, and diminishing £5 annually; estimating interest at 4 per Cent., and the rate of mortality by the experience of the Equitable?

By Corollary V, the required value is denoted by

$$\frac{(m+n)\mathrm{N}-n\mathrm{S}}{\mathrm{D}}$$

Hence by Table XXI $\dfrac{205\times 868.997-5\times 6466.39}{100.635}=$ 1448.923 = £1448 18s 6d the answer.

EXAMPLE IV.—Required the present value of a Temporary annuity for 14 years on a life of 40, the annuity to commence at £150, and to decrease £10 annually to the end of the term; reckoning the interest of money at 3 per Cent. and estimating the rate of mortality by the Northampton Table?

By Corollary IV the required value is denoted by

$$\frac{(m+n)(\mathrm{N}-{}'\mathrm{N})-n(\mathrm{S}-{}'\mathrm{S}-t.{}'\mathrm{N})}{\mathrm{D}}$$

and by Table XXXV we find that $(m+n)(\mathrm{N}-{}'\mathrm{N})$

$$=160\times(16545.194-5852.506)=1710830$$

$$n(\mathrm{S}-{}'\mathrm{S}-t.{}'\mathrm{N})$$

$$=10(209130.1-56521.5-14\times 5852.506)=706735$$

Hence $\dfrac{1710830-706735}{1114.334}=901.072=$ £901 1s 5d, the answer.

Problem II.

To find the present value of a life annuity payable at the beginning of the year, and increasing in the following manner, viz.

$£m$ at the beginning of the - - - 1st year
$m+n$ — — - - - 2nd —
$m+2n$ — — - - - 3rd —

and so on for t payments.

Let the symbols remain as in last Problem, and put ${}^{x-1}\mathrm{D} = {}^{x}\mathrm{D}_{,}$, ${}^{x-1}\mathrm{N} = {}^{x}\mathrm{N}_{,}$ &c.

Then supposing D similar annuities, made payable from a common fund, it is manifest that the number of survivors, at the beginning of the xth year, is identical with the number of survivors at the end of the $(x-1)$th year, and the present value of £1 for each survivor, is, by the construction of the Table $= {}^{x}\mathrm{D}_{,}$; that of all the payments due at the beginning of the xth year, is therefore

$$= \{m+(x-1)n\}.{}^{x}\mathrm{D}_{,} = (m-n).{}^{x}\mathrm{D}_{,}+nx.{}^{x}\mathrm{D}_{,}$$

Hence, making $x = 1, 2, 3$ &c, to t, and proceeding as in the last Problem, we find the present value of the fund necessary to provide for the supposed annuities

$$= (m-n)(\mathrm{D}+{}^{1}\mathrm{D}+{}^{2}\mathrm{D} \ldots\ldots {}^{t}\mathrm{D}_{,})$$
$$+n(\mathrm{D}+2.{}^{1}\mathrm{D}+3.{}^{2}\mathrm{D} \ldots\ldots t.{}^{t}\mathrm{D}_{,})$$
$$= (m-n)(\mathrm{N}_{,}-{}^{t}\mathrm{N}_{,})+n(\mathrm{S}_{,}-{}^{t}\mathrm{S}_{,}-t.{}^{t}\mathrm{N}_{,})$$

and the average value of each annuity is therefore

$$= \frac{(m-n)(\mathrm{N}_{,}-{}^{t}\mathrm{N}_{,})+n(\mathrm{S}_{,}-{}^{t}\mathrm{S}_{,}-t.{}^{t}\mathrm{N}_{,})}{\mathrm{D}}$$

COROLLARY I.—If n be taken negatively, the present value of an annuity payable at the beginning of each year for t payments, commencing at £m, and *decreasing* by £n annually, is

$$\frac{(m+n)(N_{,}-{}'N_{,})-n(S_{,}-{}'S_{,}-t.{}'N_{,})}{D}$$

COROLLARY II.—If in this expression $n=\frac{m}{t}$ the present value of the annuity becomes

$$=\frac{m\{N_{,}-\frac{1}{t}(S-{}'S)\}}{D}$$

SCHOLIUM.—All the Corollaries deduced from the last Problem equally apply to this, by merely substituting $N_{,}$ '$N_{,}$ $S_{,}$ & '$S_{,}$ for N 'N S & 'S, and the formulæ in both Problems are equally applicable to two or more joint lives, provided Joint Life Tables were constructed as directed in the beginning of this Chapter.

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11. *On Contingent Annuities.*

PROBLEM I.

To find the present value of £1 annuity to commence at the extinction of an assigned life A, provided that event happen before the extinction of another life B, and then to be continued payable during the remainder of a third life C; the first payment becoming due at the end of the year in which A's life may happen to fail.

Referring to the new Table, at the age of the expectant *C*, and supposing a number of similar annuities equal to the number in column D, at that age, to be made payable from a common fund, to as many families or combinations of lives, each consisting of three persons of the respective ages of *A*, *B* & *C*, it is manifest that the number of *new* claimants on the supposed fund, at the end of the *t*th year, will be identical with the number of the supposed families in each of which the *A* shall die before the *B* during that year, leaving the *C* to survive it.

If all the expectants that may survive the *t*th year were then to become claimants, that is, then to receive £1 each, and afterwards to continue to receive £1 per annum during the remainders of their respective lives, it is also manifest, from the construction of the Table, that the present fund, necessary to provide for such payments, would be denoted by $'N_{,}$. But by the conditions of the Problem, the number of new claimants at the end of the *t*th year must be less than the number of *C*'s then surviving, in the same proportion that the number of families in which the *A*'s may die before the *B*'s during the *t*th year, is less than the whole number of families supposed; which proportion is found by page 158 to be as $\frac{D('a_{,}-'a)('b_{,}+'b)}{2ab} : D$ or as $('a_{,}-'a)('b_{,}+'b) : 2ab$. Hence by diminishing the fund in the same proportion, we have

$$2ab : ({}^{t}a_{,}-{}^{t}a)({}^{t}b_{,}+{}^{t}b) : : {}^{t}\mathrm{N}_{,} : \frac{({}^{t}a_{,}-{}^{t}a)({}^{t}b_{,}+{}^{t}b).{}^{t}\mathrm{N}_{,}}{2ab} =$$

the present value of all the claims of those who begin to receive their annuities at the end of the tth year. And if in this expression we further put

$${}^{t}\mathrm{K}_{,}=({}^{t}a_{,}-{}^{t}a)({}^{t}b_{,}+{}^{t}b).{}^{t}\mathrm{N}_{,}$$

and make t successively = 1, 2, 3, &c. by adding the results thereby deduced, we obtain

$$\frac{1}{2ab}\left\{\mathrm{K}+{}^{1}\mathrm{K}+{}^{2}\mathrm{K}+{}^{3}\mathrm{K}+{}^{4}\mathrm{K}\&\mathrm{c}\right\}$$

for the present fund necessary to provide for all the annuities supposed, and this sum divided by D, the number of such annuities gives as an average

$$\frac{\mathrm{K}+{}^{1}\mathrm{K}+{}^{2}\mathrm{K}+{}^{3}\mathrm{K}+{}^{4}\mathrm{K}\&\mathrm{c}.}{2ab.\mathrm{D}}$$

for the present value of the annuity proposed.

This Formula, consisting of as many terms as are equal to the number of years contained between the age of the oldest of the given lives and the greatest tabular duration, and each term being composed of three factors, is far from being convenient to determine the present values of contingent annuities; but no simpler formula, that is mathematically correct, has hitherto been deduced for that purpose.

An approximation to the true value may, however, be obtained with less labour by dividing the terms of which the numerator is composed into classes, and supposing the terms in each class to observe some assigned law of progression.

Thus, supposing the terms in each class to be z in number, and to form an arithmetical progression, of which the common difference is denoted by y, we have

$$\mathrm{K}+{}^{1}\mathrm{K}+{}^{2}\mathrm{K}+{}^{3}\mathrm{K}\ \ldots\ \ldots\ {}^{z-1}\mathrm{K}$$
$$=\mathrm{K}+(\mathrm{K}+y)+(\mathrm{K}+2y)+(\mathrm{K}+3y)\ \ldots\ (\mathrm{K}+\overline{z-1}\cdot y)$$

the sum of which is $z\mathrm{K}+\frac{z(z-1)}{2}y$; and by continuing the progression, we find that $\mathrm{K}+zy={}^{z}\mathrm{K}$, from which $y=\frac{{}^{z}\mathrm{K}-\mathrm{K}}{z}$.

Then substituting this value of y in the foregoing expression, we find the sum of the

z terms in the 1st class $=\frac{z+1}{2}\cdot\mathrm{K}+\frac{z-1}{2}\cdot{}^{z}\mathrm{K}$.

z — — — $=\frac{z+1}{2}\cdot{}^{z}\mathrm{K}+\frac{z-1}{2}\cdot{}^{2z}\mathrm{K}$.

z — — — $=\frac{z+1}{2}\cdot{}^{2z}\mathrm{K}+\frac{z-1}{2}\cdot{}^{3z}\mathrm{K}$.

and so on to the extremity of life, the aggregate of which being taken, and substituted for

$$\mathrm{K}+{}^{1}\mathrm{K}+{}^{2}\mathrm{K}+{}^{3}\mathrm{K}+{}^{4}\mathrm{K}\&\mathrm{c.}\ \text{gives}$$

$$\frac{\frac{z+1}{2}\mathrm{K}+z({}^{z}\mathrm{K}+{}^{2z}\mathrm{K}+{}^{3z}\mathrm{K}\&\mathrm{c.})}{2ab.\mathrm{D}}$$

for the approximated value of the annuity proposed.

The labour of calculation may be still further abridged, by taking any given part of each of the terms K, ^{z}K, ^{2z}K&c. and the like part of $2ab$.D

EXAMPLE.—Suppose *A*, aged 35, to be entitled to an estate in the event of his surviving his father *B*, aged 60, and desirous of providing an annuity of £500 to his wife *C*, aged 30, to commence at his death, in the event of his dying before his father; it is required to determine the present value of *C*'s contingent annuity; estimating the interest of money at 4 per Cent., and the rate of mortality by the experience of the Equitable?

By the Formula $\frac{K + {}^1K + {}^2K + {}^3K + {}^4K \&c.}{2ab.D}$ the correct value may be determined by finding each term in the numerator from the expression ${}^tK_{,} = ({}^ta_{,} - {}^ta)({}^tb_{,} + {}^tb).{}^tN_{,}$ observing that the first factor $({}^ta_{,} - {}^ta)$ is given by Table x, in the column of decrements; the second factor $({}^tb_{,} + {}^tb)$ by Table XLV; and the third, ${}^tN_{,}$ by Table XXI.

$$K = \frac{({}^ta_{,} - {}^ta)({}^tb_{,} + {}^tb).{}^tN_{,}}{10000} = 27 \times .3002 \times 12877{\cdot}97 = 104381{\cdot}1$$

${}^1K =$	—	—	$= 27 \times {\cdot}2910 \times 12143{\cdot}64$	$= 95412{\cdot}6$
${}^2K =$	—	—	$= 28 \times {\cdot}2817 \times 11444{\cdot}68$	$= 90270{\cdot}9$
${}^3K =$	—	—	$= 28 \times {\cdot}2722 \times 10779{\cdot}73$	$= 82158{\cdot}7$
${}^4K =$	—	—	$= 28 \times {\cdot}2625 \times 10147{\cdot}20$	$= 74582{\cdot}0$
${}^5K =$	—	—	$= 28 \times {\cdot}2526 \times 9545{\cdot}60$	$= 67514{\cdot}1$
${}^6K =$	—	—	$= 28 \times {\cdot}2425 \times 8973{\cdot}71$	$= 60931{\cdot}5$
${}^7K =$	—	—	$= 28 \times {\cdot}2322 \times 8430{\cdot}14$	$= 54809{\cdot}3$
${}^8K =$	—	—	$= 29 \times {\cdot}2217 \times 7913{\cdot}78$	$= 50880{\cdot}1$
${}^9K =$	—	—	$= 30 \times {\cdot}2110 \times 7423{\cdot}36$	$= 46989{\cdot}9$
${}^{10}K =$	—	—	$= 30 \times {\cdot}2002 \times 6957{\cdot}62$	$= 41787{\cdot}4$
Carried forward			- - - - - -	769717·6

Brought forward			- - - - - -	769717·6
$^{11}K = \frac{(^{t}a_{,} - {}^{t}a)(^{t}b_{,} + {}^{t}b).{}^{t}N_{,}}{10000}$			$= 30 \times \cdot 1893 \times 6515 \cdot 41 =$	37001·0
$^{12}K =$	—	—	$= 31 \times \cdot 1783 \times 6095 \cdot 60 =$	33692·1
$^{13}K =$	—	—	$= 32 \times \cdot 1672 \times 5697 \cdot 11 =$	30481·8
$^{14}K =$	—	—	$= 33 \times \cdot 1560 \times 5319 \cdot 12 =$	27382·8
$^{15}K =$	—	—	$= 35 \times \cdot 1449 \times 4960 \cdot 81 =$	25158·8
$^{16}K =$	—	—	$= 37 \times \cdot 1339 \times 4621 \cdot 21 =$	22894·9
$^{17}K =$	—	—	$= 39 \times \cdot 1230 \times 4299 \cdot 42 =$	20624·2
$^{18}K =$	—	—	$= 41 \times \cdot 1122 \times 3994 \cdot 72 =$	18376·5
$^{19}K =$	—	—	$= 41 \times \cdot 1014 \times 3706 \cdot 43 =$	15409·0
$^{20}K =$	—	—	$= 42 \times \cdot 0906 \times 3433 \cdot 87 =$	13066·5
$^{21}K =$	—	—	$= 43 \times \cdot 0799 \times 3176 \cdot 53 =$	10913·6
$^{22}K =$	—	—	$= 44 \times \cdot 0694 \times 2933 \cdot 90 =$	8958·9
$^{23}K =$	—	—	$= 45 \times \cdot 0592 \times 2705 \cdot 48 =$	7207·3
$^{24}K =$	—	—	$= 46 \times \cdot 0495 \times 2490 \cdot 78 =$	5671·4
$^{25}K =$	—	—	$= 46 \times \cdot 0405 \times 2289 \cdot 07 =$	4264·4
$^{26}K =$	—	—	$= 46 \times \cdot 0324 \times 2099 \cdot 80 =$	3129·4
$^{27}K =$	—	—	$= 47 \times \cdot 0254 \times 1922 \cdot 40 =$	2294·9
$^{28}K =$	—	—	$= 48 \times \cdot 0196 \times 1756 \cdot 35 =$	1652·3
$^{29}K =$	—	—	$= 49 \times \cdot 0150 \times 1601 \cdot 14 =$	1176·8
$^{30}K =$	—	—	$= 50 \times \cdot 0114 \times 1456 \cdot 27 =$	830·0
$^{31}K =$	—	—	$= 51 \times \cdot 0085 \times 1321 \cdot 17 =$	572·7
$^{32}K =$	—	—	$= 52 \times \cdot 0061 \times 1195 \cdot 32 =$	379·1
$^{33}K =$	—	—	$= 53 \times \cdot 0041 \times 1078 \cdot 27 =$	234·2
$^{34}K =$	—	—	$= 54 \times \cdot 0025 \times 969 \cdot 63 =$	130·9
$^{35}K =$	—	—	$= 54 \times \cdot 0013 \times 869 \cdot 00 =$	61·0
$^{36}K =$	—	—	$= 55 \times \cdot 0005 \times 775 \cdot 99 =$	21·3
$^{37}K =$	—	—	$= 55 \times \cdot 0001 \times 690 \cdot 24 =$	3·8
$K + {}^{1}K + {}^{2}K + {}^{3}K$ &c. =				1061307·2

Also $\frac{2ab.D}{10000} = \frac{2 \times 2374 \times 1524 \times 771{\cdot}106}{10000} = 557968{\cdot}6$

Wherefore $\frac{1061307{\cdot}2}{557968{\cdot}6} = 1{\cdot}9021 =$ the value of £1 annuity on the contingency proposed; and $1{\cdot}9021 \times 500 = 951{\cdot}05 =$ £951 1s, the answer.

Otherwise, by the approximating Formula,

$$\frac{\frac{z+1}{2}K + z({}^{1}K + {}^{2}K + {}^{3}K \&c.)}{2ab.D}$$

by making $z = 5$ we find as before

$K = 104381{\cdot}1$ and $\frac{z+1}{2}K = 3K = 313143{\cdot}3$

${}^{5}K = 67514{\cdot}1$

${}^{10}K = 41787{\cdot}4$

${}^{15}K = 25158{\cdot}8$

${}^{20}K = 13066{\cdot}5$

${}^{25}K = 4264{\cdot}4$

${}^{30}K = 830{\cdot}0$

${}^{35}K = 61{\cdot}0$

$152682{\cdot}2 \times 5 = \text{- - - -} \quad 763411{\cdot}0$

$\frac{z+1}{2}K + z({}^{1}K + {}^{2}K + {}^{3}K \&c.) = \text{- - - -} \quad 1076554{\cdot}3$

And this, divided by $\frac{2ab.D}{10000}$, as before determined, gives $\frac{1076554{\cdot}3}{557968{\cdot}6} = 1{\cdot}9294$ for the approximated value of £1 annuity on the proposed contingency: hence, $1{\cdot}9294 \times 500 = 964{\cdot}7 =$ £964 14s, for the approximated value of the given annuity, which exceeds the true value already found by £13 13s, or about $1\frac{1}{2}$ per Cent. on the correct value of the annuity.

Another method of finding an approximated value of the proposed annuity, may be deduced by supposing the numerator of the formula given in page 356, to be divided into classes, each consisting of an odd number of terms, and taking the middle term as an average of all the terms composing that class.

Thus, supposing each class composed of 5 terms, we have ${}^{2}K$ for an average of K, ${}^{1}K$, ${}^{2}K$, ${}^{3}K$, ${}^{4}K$

${}^{7}K$ — — ${}^{5}K$, ${}^{6}K$, ${}^{7}K$, ${}^{8}K$, ${}^{9}K$

${}^{12}K$ — — ${}^{10}K$, ${}^{11}K$, ${}^{12}K$, ${}^{13}K$, ${}^{14}K$

and so on; so that on this supposition the approximated value of the numerator $K+{}^{1}K+{}^{2}K$&c. becomes $= 5({}^{2}K+{}^{7}K+{}^{12}K$&c); and the approximated value of £1 annuity on the proposed contingency

$$= \frac{5({}^{2}K+{}^{7}K+{}^{12}K+{}^{17}K\text{\&c})}{2ab.D}$$

Hence, taking the values of the terms as already determined in the foregoing pages, we have

${}^{2}K$	=	90270·9
${}^{7}K$	=	54809·3
${}^{12}K$	=	33692·1
${}^{17}K$	=	20624·2
${}^{22}K$	=	8958·9
${}^{27}K$	=	2294·9
${}^{32}K$	=	379·1
${}^{37}K$	=	3·8
$5({}^{2}K+{}^{7}K+{}^{12}K$&c)	=	$211033{\cdot}2\times 5 = 1055166{\cdot}0$

$\frac{1055166{\cdot}0}{557968{\cdot}6} = 1{\cdot}8911$ = the approximated value of £1 annuity on the proposed contingency, and $1{\cdot}8911 \times 500 = 945{\cdot}55$ = £945 11s, which is within £5 10s, of the true value of the annuity proposed.

Problem II.

Supposing the payment of the annuity proposed in the last Problem restricted to the next n years, it is required to find its present value.

Reasoning as in the last Problem it may be shown that, if all the C's who may survive the tth year were then to receive £1 each, and the survivors, at the end of every succeeding year, up to the end of the nth year from this time, were also to receive £1 each, the present value of the fund necessary to provide for such payments would be denoted by ${}^{t}\mathrm{N}_{,} - {}^{n}\mathrm{N}$. And by reducing this quantity in the ratio of the whole number of annuities supposed, to the number of new claimants at the end of the tth year, produced by the A's dying before the B's during that year, leaving the C's to survive it, we have

$$\frac{({}^{t}a_{,} - {}^{t}a)({}^{t}b_{,} + {}^{t}b)({}^{t}\mathrm{N}_{,} - {}^{n}\mathrm{N})}{2ab}$$

for the present value of all the claims of those who begin to receive their annuities at the end of the

tth year. Hence, making t successively = 1, 2, 3&c to n, adding the results, and dividing their sum by D, the number of annuities supposed, we obtain

$$\int \frac{({}^{t}a_{,} - {}^{t}a)({}^{t}b_{,} + {}^{t}b)({}^{t}\mathrm{N}_{,} - {}^{n}\mathrm{N})}{2ab.\mathrm{D}}$$

as a correct expression for the present value of the annuity proposed.

Or if we put ${}^{t}\mathrm{K}_{,}$ as in the last Problem, and ${}^{t}\mathrm{P}_{,} = ({}^{t}a_{,} - {}^{t}a)({}^{t}b_{,} + {}^{t}b)$, the above expression, in a more developed form, becomes

$$= \frac{\mathrm{K} + {}^{1}\mathrm{K} + {}^{2}\mathrm{K} \ldots {}^{n}\mathrm{K}_{,} - (\mathrm{P} + {}^{1}\mathrm{P} + {}^{2}\mathrm{P} \ldots {}^{n}\mathrm{P}_{,}).{}^{n}\mathrm{N}}{2ab.\mathrm{D}}$$

Corollary.—If from the present value of the annuity proposed in the last Problem, we deduct that of the temporary contingent annuity herein determined, we have

$$\frac{{}^{n}\mathrm{K} + {}^{n+1}\mathrm{K} + {}^{n+2}\mathrm{K}\&\mathrm{c} + (\mathrm{P} + {}^{1}\mathrm{P} + {}^{2}\mathrm{P} \ldots\ldots {}^{n}\mathrm{P}_{,}).{}^{n}\mathrm{N}}{2ab.\mathrm{D}}$$

for the present value of a contingent annuity, deferred for n years.

Example 1.—The ages of A, B & C, as well as the rates of interest and mortality, remaining as supposed in page 358, it is required to determine the present value of C's contingent annuity, supposing it to cease at the end of the 20th year from this time?

By finding the value of each term of the numerator from the expression

$$\int. \frac{({}^{t}a_{,}-{}^{t}a)({}^{t}b_{,}+{}^{t}b)({}^{t}\mathrm{N}_{,}-{}^{n}\mathrm{N})}{2ab.\mathrm{D}}$$ we have

The	1st term	=	$27 \times \cdot3002 \times 9444\cdot10$	=	76548·2
	2nd —	=	$27 \times \cdot2910 \times 8709\cdot77$	=	68432·6
	3rd —	=	$28 \times \cdot2817 \times 8010\cdot81$	=	63186·0
	4th —	=	$28 \times \cdot2722 \times 7345\cdot86$	=	55987·1
	5th —	=	$28 \times \cdot2625 \times 6713\cdot33$	=	49343·0
	6th —	=	$28 \times \cdot2526 \times 6111\cdot73$	=	43227·0
	7th —	=	$28 \times \cdot2425 \times 5539\cdot84$	=	37615 4
	8th —	=	$28 \times \cdot2322 \times 4996\cdot27$	=	32483·7
	9th —	=	$29 \times \cdot2217 \times 4479\cdot91$	=	28802·6
	10th —	=	$30 \times \cdot2110 \times 3989\cdot49$	=	25253·4
	11th —	=	$30 \times \cdot2002 \times 3523\cdot75$	=	21163·6
	12th —	=	$30 \times \cdot1893 \times 3081\cdot54$	=	17500·0
	13th —	=	$31 \times \cdot1783 \times 2661\cdot73$	=	14712·2
	14th —	=	$32 \times \cdot1672 \times 2263\cdot24$	=	12109·3
	15th —	=	$33 \times \cdot1560 \times 1885\cdot25$	=	9705·2
	16th —	=	$35 \times \cdot1449 \times 1526\cdot94$	=	7743·8
	17th —	=	$37 \times \cdot1339 \times 1187\cdot34$	=	5882·4
	18th —	=	$39 \times \cdot1230 \times 865\cdot55$	=	4152·0
	19th —	=	$41 \times \cdot1122 \times 560\cdot85$	=	2580·0
	20th —	=	$41 \times \cdot1014 \times 272\cdot56$	=	1133·1
					577560·6

Also $\frac{2ab\cdot\mathrm{D}}{10000} = \frac{2 \times 2374 \times 1524 \times 771\cdot106}{10000} = 557968\cdot6$

wherefore $\int. \frac{({}^{t}a_{,}-{}^{t}a)({}^{t}b_{,}+{}^{t}b)({}^{t}\mathrm{N}_{,}-{}^{n}\mathrm{N})}{2ab.\mathrm{D}} = \frac{577560\cdot6}{557968\cdot6} =$ 1·0351 = the present value of £1 annuity on the

proposed contingency, and $1{\cdot}0351 \times 500 = 517{\cdot}55 =$ £517 11s, the answer.

Otherwise, by dividing the numerator into 4 classes of 5 terms each, and taking the middle term in each class as an average, in the manner described in page 361, we have

The	3rd term =	63186·0
	8th — =	32483·7
	13th — =	14712·2
	18th — =	4152·0
their sum . . . =		114533·9

Hence $\frac{5 \times 114533{\cdot}9}{557968{\cdot}6} \times 500 = 1{\cdot}0264 \times 500 = 513{\cdot}2$ = £513 4s, = the approximated value of the annuity proposed, which is only £4 7s deficient of the true value already determined.

EXAMPLE II.—The ages of *A*, *B* & *C*, as well as the rates of interest and mortality remaining as before, it is required to find the present value of *C*'s contingent annuity, supposing it deferred for 20 years?

The present value of *C*'s entire interest were it not restricted to time, found in page 360, - - - - - - - - - -	= 1·9021
Ditto of his interest for the next 20 years, found in the last Example - - -	= 1·0351
their difference - - - - - - - - -	.8670

which multiplied by £500, the amount of annuity proposed, gives $\cdot 867 \times 500 = 433{\cdot}5 =$ £433 10s, the answer.

Referring to the Examples by which the component parts of this solution were deduced, it is, however, manifest that the labour necessary to obtain them is exceedingly great. But if we proceed by approximation from the Formula

$$\frac{{}^{n}\mathrm{K} + {}^{n+1}\mathrm{K} + {}^{n+2}\mathrm{K}\,\&c + (\mathrm{P} + {}^{1}\mathrm{P} + {}^{2}\mathrm{P} \ldots\ldots {}^{n}\mathrm{P}).{}^{n}\mathrm{N}}{2ab.\mathrm{D}}$$

the quantity of labour will be materially reduced.

Thus ${}^{20}\mathrm{K} + {}^{21}\mathrm{K} + {}^{22}\mathrm{K} \ldots\ldots {}^{37}\mathrm{K}$ consists of 18 terms, which may be divided into three classes of 5 terms each, and one class of 3 terms, and $\mathrm{P} + {}^{1}\mathrm{P} + {}^{2}\mathrm{P} \ldots\ldots {}^{19}\mathrm{P}$ may be divided into 4 classes of 5 terms each.

Then, taking the middle term as the average of each class, we have

$${}^{22}\mathrm{K} = 44 \times \cdot0694 \times 2933\cdot90 = 8958\cdot9$$
$${}^{27}\mathrm{K} = 47 \times \cdot0254 \times 1922\cdot40 = 2294\cdot9$$
$${}^{32}\mathrm{K} = 52 \times \cdot0061 \times 1195\cdot32 = \underline{\;379\cdot1}$$
$$11632\cdot9 \times 5 = 58164\cdot5$$
$${}^{36}\mathrm{K} = 55 \times \cdot0005 \times 775\cdot99 = \underline{\;21\cdot3}$$
$$3 \qquad\qquad 63\cdot9$$
$${}^{20}\mathrm{K} + {}^{21}\mathrm{K} + {}^{22}\mathrm{K} + {}^{23}\mathrm{K} \ldots\ldots {}^{37}\mathrm{K} = 58228\cdot4$$
$${}^{2}\mathrm{P} = 28 \times \cdot2817 = 7\cdot8876$$
$${}^{7}\mathrm{P} = 28 \times \cdot2322 = 6\cdot5016$$
$${}^{12}\mathrm{P} = 31 \times \cdot1783 = 5\cdot5273$$
$${}^{17}\mathrm{P} = 39 \times \cdot1230 = \underline{\;4\cdot7970}$$
$$24\cdot7135$$
$$5$$
$$(\mathrm{P} + {}^{1}\mathrm{P} \ldots {}^{19}\mathrm{P}).{}^{20}\mathrm{N} = 123\cdot5675 \times 3433\cdot87 = 424314\cdot6$$
$$\text{approximated value of the numerator} = 482543\cdot0$$

and the denominator $2ab.D \div 10000$, found as in the foregoing Examples, is 557968·6.

Hence, $\frac{482543 \cdot 0}{557968 \cdot 6} \times 500 = \cdot 8648 \times 500 = 432 \cdot 4 =$ £432 8s the approximated value of the annuity proposed, which differs only £1 2s from the correct value already determined.

Problem III.

To find the present value of £1 annuity to commence at the extinction of an assigned life *A*, provided that event happen after the extinction of another life *B*, and then to be continued payable during the remainder of a third life, *C*; the first payment of the annuity becoming due at the end of the year in which *A*'s life may happen to fail.

Were *C* entitled to £1 per annum after the death of *A*, in the event of *A* dying *before* *B*, and also to a like annuity in the event of *A* dying *after* *B*, it is manifest that the two contingent annuities together would constitute a survivorship annuity to *C*, after the death of *A*, the present value of which is, by page 264, $= C - AC$.

Hence it follows, that if from the aggregate present value of the two contingent annuities, we deduct that of the first, denoted by V, and found by Problem I, there remains

$$C - AC - V$$

for the present value of the annuity proposed.

In the same manner it may be shown, that the present value of a similar annuity, to cease after the expiration of n years, is

$$\overline{n|}C - \overline{n|}AC - \overline{n|}V$$

and the present value of a similar annuity deferred for n years is

$${}_nC - {}_nAC - {}_nV$$

observing that, in either case, the value of the third term may be determined by the last Problem.

EXAMPLE—What is the present value of a contingent annuity of £500 to be entered upon at the death of *A*, aged 35, provided he shall die after *B*, aged 60, and then to be continued payable until the death of *C*, now aged 30; estimating the interest of money at 4 per Cent., and the rate of mortality by the experience of the Equitable?

By Table XXIV - - - - - -	C =	16·7007
— XXVII - - - - - - -	AC =	13·2458
their difference - - -	C−AC =	3·4549
and, by page 360, the correct value of V =		1·9021
C−AC−V - - - - - - - - - -	=	1·5528

= the value of £1 annuity on the contingency proposed; hence $1{\cdot}5528 \times 500 = 776.4$ = £776 8s the answer.

12. *On Successive Annuities.*

Problem I.

Suppose an annuity of £1 dependent on an assigned life *A*, to be continued during the life of a successor *B*, who is to be nominated at *A*'s death: it is required to determine the present value of the annuity on the succeeding life, and that of the whole annuity on the two lives in succession.

Let A′ represent the present value of £1 annuity on *A*'s life, and continued up to the day of his death; and B′ the value of a like annuity on *B*'s life, at the time of nomination.

Then, since it is manifest that the interest of £1 receivable during *A*'s life, and the principal to be received at his death, together constitute an equivalent to £1 now paid down, it follows that if we deduct the present value of the former from that of the two together, there remains $1-i\mathrm{A}'$, for the present value of £1 to be received at *A*'s death: and the present value of £B′, to be received at the same period, is therefore

$$=(1-i\mathrm{A}')\mathrm{B}'$$

= the present value of £1 annuity on the succeeding life.

And if to this expression we add the present value of £1 annuity during *A*'s life, we obtain

$$A' + (1 - iA')B'$$

for the present value of the whole annuity on the two lives in succession.

COROLLARY I.—If instead of being made payable during the succeeding life *B*, the annuity were continued from *A*'s death for a term of years certain, the same Formulæ are applicable by merely substituting for B′ the value of £1 annuity for the given term.

COROLLARY II.—If in the expression $(1 - iA')B'$, which is applicable to determine the value of the next Presentation to a Living, we substitute for B′ the value of a perpetual annuity of £1 $(=\frac{1}{i})$ we have

$$\frac{1}{i} - A'$$

for the present value of the *Advowson*, or perpetual right to all future Presentations.

EXAMPLE I.—Suppose *A*, aged 50, to be in possession of a Living producing a net income of £200 per annum, beside providing for the Curate's salary, &c., and to have the right of nominating a successor at his death; it is required to determine the present value of his interest, assuming his successor to be 30 years of age at the time of nomina-

tion, and estimating the improvement of money at 4 per Cent. and the rate of mortality by the experience of the Equitable?

Here A′, found as in Example I, page 338, = 12·84; and B′, found by Table XXIV, is 16·7.*

Hence $A' + (1 - iA')B' = 12{\cdot}84 + (1 - {\cdot}04 \times 12{\cdot}84) \times 16{\cdot}7 = 12{\cdot}84 + 8{\cdot}12 = 20{\cdot}96$ = the present value of £1 annuity on the two succeeding lives; that of £200 annuity is therefore = $20{\cdot}96 \times 200$ = £4192, the answer.

Note.—The value of the next Presentation to the same Living at the like rate of interest is $(1 - iA')B' \times 200 = 8{\cdot}12 \times 200$ = £1624. And the present value of the Advowson = $(\frac{1}{i} - A') \times 200 = (25 - 12{\cdot}84) \times 200 = 12{\cdot}16 \times 200$ = £2432.

EXAMPLE II.—Suppose an estate worth £1000 per annum, held on lease, terminable with the extinction of an assigned life *A*, now 60 years of age, at a pepper corn rent; it is required to determine the Fine which ought now to be paid for the right of renewing the lease, by nominating another life *B*, at *A*'s death; estimating the improvement of money at 5 per cent. and the rate of mortality by the experience of the Equitable?

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* The age of *B* being merely assumed, renders it unnecessary to aim at any greater exactness by increasing the value of B′, on account of the fraction which *B* may happen to survive beyond the last yearly payment which may become due prior to his death.

By Corollary 1, page 336, $A' = A + L(1 - iA)$, and $1 - iA' = 1 - iA - iL(1 - iA) = (1 - iL)(1 - iA)$ $= (1 - \cdot05 \times \cdot48393)(1 - \cdot05 \times 9\cdot2773) = \cdot9758 \times \cdot53614 = \cdot52316 =$ the present value of £1, to be received on *A*'s death. Then assuming the age of *B*, the life to be nominated, at 10 years, by Table XXIV. we have $B' = 16\cdot732$, and consequently $(1 - iA')B' \times 1000 = \cdot52316 \times 16\cdot732 \times 1000 =$ £8754, the answer.

If the interest of money were reckoned at 4 per Cent., the present value of the same lease found in like manner would be £11518.

EXAMPLE III.—Required the present value of a Lease determinable at the expiration of 21 years, after the extinction of an assigned life *A*, now aged 50, estimating the interest of money at 4 per Cent., and the rate of mortality by the experience of the Equitable, also supposing the improved rent to be £60 per annum?

Here A', found as in Example 1, page 338, $= 12\cdot8402$, and B', the present value of £1 per annum for 21 years, found by Table VIII, is $14\cdot02916$. Hence $A' + (1 - iA')B' = 12\cdot8402 + (1 - \cdot04 \times 12\cdot8402) \times 14\cdot02916 = 12\cdot8402 + 6\cdot8236 = 19\cdot6638 =$ the present value of £1 annuity on the proposed contingency; and $19\cdot6638 \times 60 = 1179\cdot828 =$ £1179 16s 7d, the answer.

Problem II.

Suppose an annuity of £1 to be made payable to any number of lives *A*, *B*, *C*, *D*,&c. in succession; it is required to determine the present value of all the payments which may be received by any one life in the series, and that of the whole annuity on all the lives in succession.

Let the values of the respective interests of *A*, *B*, *C*, &c., at the periods when they successively come into possession, be denoted by A′, B′, C′ &c, and put $1-iA' = \mathfrak{A}$, $1-iB' = \mathfrak{B}$, $1-iC' = \mathfrak{C}$ &c.

Then by the last Problem it is manifest that the present value of *B*'s annuity $= (1-iA')B' = \mathfrak{A}\cdot B'$, and at *A*'s death, *C* being the second life in succession, the then value of his annuity will, for the same reason, be denoted by $(1-iB')C' = \mathfrak{B}\cdot C'$, which being further discounted for *A*'s life gives $\mathfrak{A}\cdot\mathfrak{B}\cdot C'$ for its present value. In like manner it may be shown that the present value of *D*'s interest $= \mathfrak{A}\cdot\mathfrak{B}\cdot\mathfrak{C}\cdot D'$, that of *E*, $= \mathfrak{A}\cdot\mathfrak{B}\cdot\mathfrak{C}\cdot\mathfrak{D}\cdot E'$ &c. &c.

From this mode of reasoning it is manifest that

A's interest	=	A′
B's	=	$\mathfrak{A}\cdot B'$
C's	=	$\mathfrak{A}\cdot\mathfrak{B}\cdot C'$
D's	=	$\mathfrak{A}\cdot\mathfrak{B}\cdot\mathfrak{C}\cdot D'$
E's	=	$\mathfrak{A}\cdot\mathfrak{B}\cdot\mathfrak{C}\cdot\mathfrak{D}\cdot E'$
&c.		&c.

and by adding the whole together, it is equally manifest that the present value of the whole annuity on all the lives in succession, is denoted by

$$A' + \mathfrak{A}\cdot B' + \mathfrak{A}\cdot\mathfrak{B}\cdot C' + \mathfrak{A}\cdot\mathfrak{B}\cdot\mathfrak{C}\cdot D' + \mathfrak{A}\cdot\mathfrak{B}\cdot\mathfrak{C}\cdot\mathfrak{D}\cdot E' \text{ &c.}$$

COROLLARY I.—As it is obvious, from the nature of the Problem, that the age of A only can at present be known, no reason can be assigned for assuming B, C, D, &c. to be of different ages; if therefore C, D &c. be all taken of the same age as B, the present value of £1 annuity on any number of successive lives, is more simply expressed by

$$A' + \mathfrak{A}\cdot B' + \mathfrak{A}\cdot\mathfrak{B}\cdot B' + \mathfrak{A}\cdot\mathfrak{B}^2\cdot B' + \mathfrak{A}\cdot\mathfrak{B}^3\cdot B' \text{ &c.}$$
$$= A' + \mathfrak{A}\cdot B'(1 + \mathfrak{B} + \mathfrak{B}^2 + \mathfrak{B}^3 \text{ &c.})$$

and if in this expression the number of lives be taken without limit, the series $1 + \mathfrak{B} + \mathfrak{B}^2 + \mathfrak{B}^3$ &c. $= \frac{1}{1-\mathfrak{B}} = \frac{1}{1-(1-iB')} = \frac{1}{iB'}$ which being substituted for its equivalent in the above expression, gives

$$A' + \mathfrak{A}\cdot B'\left\{\frac{1}{iB'}\right\} = A' + \frac{\mathfrak{A}}{i} = A' + \frac{1-iA'}{i} = A' + \frac{1}{i} - A'$$

$= \frac{1}{i}$ = the present value of a perpetual annuity of £1 as it ought.

COROLLARY II.—If P and Q, and their respective heirs, be entitled to supply all future vacancies in a Church Living, by alternate Presentations, supposing P to have the first Presentation after the decease of the Incumbent A, we find by taking the alternate terms of the first series in the last Corollary that

$$P\text{'s interest} = \mathfrak{A}\cdot B' + \mathfrak{A}\cdot\mathfrak{B}^2\cdot B' + \mathfrak{A}\cdot\mathfrak{B}^4\cdot B'\&c.$$
$$= \frac{\mathfrak{A}\cdot B'}{1-\mathfrak{B}^2}$$
$$Q\text{'s interest} = \mathfrak{A}\cdot\mathfrak{B}\cdot B' + \mathfrak{A}\cdot\mathfrak{B}^3\cdot B' + \mathfrak{A}\cdot\mathfrak{B}^5\cdot B'\&c.$$
$$= \frac{\mathfrak{A}\cdot\mathfrak{B}\cdot B'}{1-\mathfrak{B}^2}$$

EXAMPLE.—Suppose *A*, aged 60, to be in possession of a Living producing a net income of £500 per annum, after deducting all outgoings for Curate's salary, &c., and that *P* and *Q*, and their respective heirs are entitled, by alternate Presentations, to supply all future vacancies in the same; it is required to determine the respective interests of *A*, *P* & *Q*, estimating the improvement of money at 5 per Cent., and the rate of mortality by the experience of the Equitable?

By page 336 the present value of *A*'s interest is denoted in years' purchase by $A + L(1 - iA)$, which by taking the value of L, from the Note in page 335, and that of A from Table XXIV, $= 9{\cdot}2773 + {\cdot}48393(1 - {\cdot}05 \times 9{\cdot}2773) = 9{\cdot}2773 + {\cdot}2594 = 9{\cdot}5367 = A'$: hence $1 - iA' = {\cdot}52316 = \mathfrak{A}$.

Again, assuming the age of the Clerk to be appointed at each presentation to be about 30, so that the annuity of £1 on his life, and continued up to the day of his death, may be taken at 5 per Cent. $= 14{\cdot}6 = B'$ we have $\mathfrak{B} = 1 - iB' = 1 - {\cdot}05 \times 14{\cdot}6 = {\cdot}27$. Hence P's interest $= \frac{\mathfrak{A}\cdot B'}{1-\mathfrak{B}^2} = \frac{{\cdot}52316 \times 14{\cdot}6}{1-{\cdot}27^2} = \frac{7{\cdot}6381}{{\cdot}9271} = 8{\cdot}239$ years' purchase.

And Q's interest $= \frac{\mathfrak{A}\cdot\mathfrak{B}\cdot B'}{1-\mathfrak{B}^2} = \frac{\cdot 52316 \times \cdot 27 \times 14\cdot 6}{1-\cdot 27^2}$

$= 2\cdot 224$ years' purchase.

Then collecting the results, and multiplying the years' purchase by the annual value of the Living, we find that

A's interest	$= 9\cdot 537 \times 500$	$=$	£4768 10s
P's —	$= 8\cdot 239 \times 500$	$=$	4119 10s
Q's —	$= 2\cdot 224 \times 500$	$=$	1112 0s

Problem III.

Suppose a Copyhold Estate dependent on n lives A, B, C, &c. to be always renewable whenever any of these lives may fail, by substituting a new life on the payment of a fine f: it is required to determine the present value of all the fines which may become chargeable henceforth in perpetuity.

Let the life put in at each renewal, (which we suppose to be the best life that can be selected), be designated by P, and put the value of £1 annuity on A's life, and continued up to the day of death $= A'$, that on B's life $= B'$ &c.; then it is manifest by the last Problem, that the present value of £1 receivable at A's death, $= 1 - iA'$ which we designate by $\mathfrak{A}$, that of £1 receivable at B's death $= 1 - iB' = \mathfrak{B}$ &c.

Then referring to the life of A, and those that follow in direct succession, it is manifest that the present value of the first fine is denoted by $f\mathfrak{A}$;

and at A's death the value of the second fine, payable at the extinction of the life P then nominated, is $f\mathfrak{P}$, the present value of which when further discounted for A's life is $f\mathfrak{A}\mathfrak{P}$. In like manner it may be shewn, that at the failure of A's immediate successor the value of the third fine, payable at the extinction of the second P then nominated, will be $f\mathfrak{P}$, which being discounted for the two preceding lives, gives the present value of the third fine $= \mathfrak{A}\mathfrak{P} \times f\mathfrak{P} = f\mathfrak{A}\mathfrak{P}^2$. Similarly it may be shewn, that the present value of the fourth fine is $f\mathfrak{A}\mathfrak{P}^3$; that of the fifth, $f\mathfrak{A}\mathfrak{P}^4$, and so on. Hence the aggregate present value of all the fines payable at the death of A and those that follow in direct succession is denoted by the infinite series

$$f\mathfrak{A}+f\mathfrak{A}\,\mathfrak{P}+f\mathfrak{A}\,\mathfrak{P}^2+f\mathfrak{A}\,\mathfrak{P}^3 \text{ \&c.}$$
$$=f\mathfrak{A}\left\{1+\mathfrak{P}+\mathfrak{P}^2+\mathfrak{P}^3 \text{ \&c.}\right\}=\frac{f\mathfrak{A}}{1-\mathfrak{P}}$$

Proceeding in the same manner with B, C, &c., and their respective successors, it may be shewn that the present value of all the fines chargeable for such renewals as may be effected in direct succession from $B=\frac{f\mathfrak{B}}{1-\mathfrak{P}}$, from $C=\frac{f\mathfrak{C}}{1-\mathfrak{P}}$, and so on.

Having thus determined the present value of all the fines chargeable for such renewals as may be effected in direct succession for each of the proposed lives separately, their sum produces

$$\frac{f\mathfrak{A}}{1-\mathfrak{P}}+\frac{f\mathfrak{B}}{1-\mathfrak{P}}+\frac{f\mathfrak{C}}{1-\mathfrak{P}}\&c. = \frac{f}{1-\mathfrak{P}}(\mathfrak{A}+\mathfrak{B}+\mathfrak{C}\ \&c.)$$

$$= \frac{f}{i\mathrm{P}'}(\mathfrak{A}+\mathfrak{B}+\mathfrak{C}\ \&c.)$$

$$= \frac{f}{i\mathrm{P}'}\{(1-i\mathrm{A}')+(1-i\mathrm{B}') +(1-i\mathrm{C}')\ \&c.\}$$

$$= \frac{f}{\mathrm{P}'}\left\{\frac{n}{i}-(\mathrm{A}'+\mathrm{B}'+\mathrm{C}'\&c.)\right\}$$

Corollary i.—If an estate, subject to the proposed fines, were now purchased for the sum s, its whole value, including the purchase money and the renewal fines, would be $s+\frac{f}{\mathrm{P}'}\left\{\frac{n}{i}-(\mathrm{A}'+\mathrm{B}'+\mathrm{C}'\ \&c.)\right\}$

Corollary ii.—If the whole value of the estate in fee simple be denoted by $\mathfrak{w}$, we have

$$\mathfrak{w} = s+\frac{f}{\mathrm{P}'}\left\{\frac{n}{i}-(\mathrm{A}'+\mathrm{B}'+\mathrm{C}'\ \&c.)\right\}$$

$$s = \mathfrak{w}-\frac{f}{\mathrm{P}'}\left\{\frac{n}{i}-(\mathrm{A}'+\mathrm{B}'+\mathrm{C}'\ \&c.)\right\}$$

$$f = \frac{i\mathrm{P}'(\mathfrak{w}-s)}{n-i(\mathrm{A}'+\mathrm{B}'+\mathrm{C}'\ \&c.)}$$

Corollary iii.—If the present value of the renewal fines be multipled by i, the equivalent annual rent is found to be $\frac{f}{\mathrm{P}'}\{n-i(\mathrm{A}'+\mathrm{B}'+\mathrm{C}'+\&c.)\}$

Corollary iv.—If $n=1$, the present value of all the fines necessary to keep in force a lease on one life

$$=\frac{f}{\mathrm{P}'}\left\{\frac{1}{i}-\mathrm{A}'\right\} \text{ or } =\frac{f\mathfrak{A}}{1-\mathfrak{P}}$$

COROLLARY V.—If an estate were held on a lease for the last survivor of two or more lives, subject to be renewed on the payment of a fine f whenever the last of those lives becomes extinct, the present value of such fines may be determined by the formula

$$\frac{f}{P'}\left\{\frac{1}{i} - A'\right\}$$

by substituting for A′ the present value of £1 annuity on the last survivor of the lives now existing, and for P′ the present value of a similar annuity on the last survivor of the lives to be nominated at each renewal.

EXAMPLE I.—Suppose a copyhold estate dependent on three lives, *A*, *B*, and *C*, of the respective ages of 33, 40, and 50, to be renewable on the payment of £1000 whenever any one of the lives may happen to fail. It is required to find the present value of all the renewal fines chargeable henceforth in perpetuity, estimating the improvement of money at 5 per Cent., and the rate of mortality by the experience of the Equitable.

Here $n = 3$.

$A' = 14{\cdot}6019 + {\cdot}48393(1 - {\cdot}05 \times 14{\cdot}6019) =$	14·7324
$B' = 13{\cdot}2747 + {\cdot}48393(1 - {\cdot}05 \times 13{\cdot}2747) =$	13·4374
$C' = 11{\cdot}4017 + {\cdot}48393(1 - {\cdot}05 \times 11{\cdot}4017) =$	11·6097
$A' + B' + C' =$ - - - - - - - - - -	39·7795
$\frac{n}{i} = \frac{3}{{\cdot}05}$ - - - - -	60·0000
$\frac{n}{i} - (A' + B' + C') =$ - - - - - - -	20·2205

Then assuming the age of the life nominated at each renewal to be about 10 years, we have $P' = 16{\cdot}7$; and consequently

$$\frac{f}{P'}\left\{\frac{n}{i} - (A' + B' + C')\right\} = \frac{1000 \times 20{\cdot}2205}{16{\cdot}7} = £1210,$$

the present value of the fines proposed.

Note.—The annual rent equivalent to this fine is $1210 \times {\cdot}05 = 60{\cdot}5 = £60\ 10s.$

Example II.—Suppose a copyhold estate dependent on a single life A, aged 30, and renewable by the substitution of another single life P, on the payment of £1000 whenever A or any of his successors may happen to fail. It is required to determine the present value of all the fines chargeable henceforth in perpetuity, estimating the improvement of money at 5 per Cent., and the rate of mortality by the experience of the Equitable.

Here $A' = 14{\cdot}7324$; $f = 1000$; $i = {\cdot}05$; and P', taken as in the last Example, $= 16{\cdot}7$. Wherefore $\frac{f}{P'}\left\{\frac{1}{i} - A'\right\} = \frac{1000}{16{\cdot}7}(20 - 14{\cdot}7324) = 315{\cdot}4 =$ £315 8s, the answer.

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TABLES.

TABLE I.

*Showing the relative Prices of different Funds, according to the Annual Dividends paid thereon, so as to produce the same Rate of Interest.**

Price of Land in Years purchase.	Price of 3 per Cent. Stock.	Price of 3½ per Cent. Stock.	Price of 4 per Cent. Stock.	Price of 5 per Cent. Stock.	Price of 6 per Cent. Stock.	Price of Bank Stock, or 8 per Cents.	Price of India Stock, or 10 per Cents.	Annual Interest afforded.		
	£	£	£	£	£	£	£	£	*s.*	*d.*
20	60	70	80	100	120	160	200	5	0	0
20⅓	61	71	81⅓	101⅔	122	162⅔	203⅓	4	18	9
20⅔	62	72⅓	82⅔	103⅓	124	165⅓	206⅔	3	16	9
21	63	73½	84	105	126	168	210	4	15	3
21⅓	64	74⅔	85⅓	106⅔	128	170⅔	213⅓	4	13	9
21⅔	65	76	86⅔	108⅓	130	173⅓	216⅔	4	12	3
22	66	77	88	110	132	176	220	4	10	11
22⅓	67	78	89⅓	111⅔	134	178⅔	223⅓	4	9	7
22⅔	68	79⅓	90⅔	113⅓	136	181⅓	226⅔	4	8	3
23	69	80½	92	115	138	184	230	4	7	0
23⅓	70	81⅔	93⅓	116⅔	140	186⅔	233⅓	4	5	9
23⅔	71	83	94⅔	118⅓	142	189⅓	236⅔	4	4	6
24	72	84	96	120	144	192	240	4	3	4
24⅓	73	85	97⅓	121⅔	146	194⅔	243⅓	4	2	2
24⅔	74	86⅓	98⅔	123⅓	148	197⅓	246⅔	4	1	1
25	75	87½	100	125	150	200	250	4	0	0
25⅓	76	88⅔	101⅓	126⅔	152	202⅔	253⅓	3	18	11
25⅔	77	90	102⅔	128⅓	154	205⅓	256⅔	3	17	11
26	78	91	104	130	156	208	260	3	16	11
26⅓	79	92	105⅓	131⅔	158	210⅔	263⅓	3	15	11
26⅔	80	93⅓	106⅔	133⅓	160	213⅓	266⅔	3	15	0
27	81	94½	108	135	162	216	270	3	14	1
27⅓	82	95⅔	109⅓	136⅔	164	218⅔	273⅓	3	13	2
27⅔	83	97	110⅔	138⅓	166	221⅓	276	3	12	4
28	84	98	112	140	168	224	280	3	11	5
28⅓	85	99	113⅓	141⅔	170	226⅔	283⅓	3	10	7
28⅔	86	100⅓	114⅔	143⅓	172	229⅓	286⅔	3	9	9
29	87	101½	116	145	174	232	290	3	9	0
29⅓	88	102⅔	117⅓	146⅔	176	234⅔	293⅓	3	8	2
29⅔	89	104	118⅔	148⅓	178	237⅓	296⅔	3	7	5
30	90	105	120	150	180	240	300	3	6	8
30⅓	91	106	121⅓	151⅔	182	242⅔	303⅓	3	5	11
30⅔	92	107⅓	122⅔	153⅓	184	245⅓	306⅔	3	5	3
31	93	108½	124	155	186	248	310	3	4	6
31⅓	94	109⅔	125⅓	156⅔	188	250⅔	313⅓	3	3	10
31⅔	95	111	126⅔	158⅓	190	253⅓	316⅔	3	3	2
32	96	112	128	160	192	256	320	3	2	6
32⅓	97	113	129⅓	161⅔	194	258⅔	323⅓	3	1	10
32⅔	98	114⅓	130⅔	163⅓	196	261⅓	326⅔	3	1	3
33	99	115½	132	165	198	264	330	3	0	7
33⅓	100	116⅔	133⅓	166⅔	200	266⅔	333⅓	3	0	0

* For the various causes which produce a difference in these relative prices, see pages 30 and 31.

TABLE II.

RATES OF GOVERNMENT ANNUITIES:

ON SINGLE LIVES,

DEFERRED FOR GIVEN PERIODS.†

Present Age.	*Money* Value of £1 Annuity to commence at the end of				Present Age.	*Money* Value of £1 Annuity to commence at the end of			
	5 Years.	10 Years.	15 Years.	20 Years.		5 Years.	10 Years.	15 Years.	20 Years.
21	12 497	9.136	6.575	4.638	46	8.552	5.479	3.332	1.880
22	12.377	9.022	6.470	4.545	47	8.355	5.300	3.187	1.768
23	12 254	8.912	6.357	4.447	48	8.147	5.125	3.032	1.646
24	12.130	8.785	6.255	4.343	49	7.939	4.945	2,884	1.532
25	12.009	8.668	6.139	4.249	50	7.735	4.760	2.737	1,423
26	11.867	8.538	6 023	4.145	51	7.532	4.576	2.587	1.305
27	11.735	8.416	5.908	4.039	52	7.321	4.393	2.441	1.198
28	11.602	8.282	5.786	3.934	53	7.117	4.215	2.288	1.092
29	11.453	8.147	5.667	3.833	54	6.895	4.027	2.143	.990
30	11.314	8.014	5.544	3.721	55	6.685	3.837	1.994	.884
21	11.161	7.877	5.419	3.613	56	6.463	3.651	1.847	- -
32	11.016	7.740	5.294	3.506	57	6.240	3.461	1.702	- -
33	10.858	7.594	5.160	3.388	58	6.019	3.274	1.559	- -
34	10.707	7.446	5.031	3.277	59	5.786	3.075	1.422	- -
35	10.542	7.290	4.898	3.166	60	5.550	2.886	1.286	- -
36	10.377	7.139	4.761	3.051	61	5.320	2.689	- -	- -
37	10.217	6.988	4.619	2.931	62	5.079	2.503	- -	- -
38	10.040	6.826	4 484	2.818	63	4.838	2.306	- -	- -
39	9.861	6.662	4.336	2.701	64	4.589	2.117	- -	- -
40	9,681	6.492	4.201	2.580	65	4.336	1.930	- -	- -
41	9.498	6.326	4.055	2.466	66	4.084	- -	- -	- -
42	9.312	6.165	3.909	2 348	67	3.833	- -	- -	- -
43	9.133	6.000	3.765	2.232	68	3.581	- -	- -	- -
44	8.941	5.826	3.626	2.114	69	3.329	- -	- -	- -
45	8.748	5.653	3.478	2.000	70	3.081	- -	- -	- -

† These Annuities do not vary with the price of Stock, and are granted only to those who have previously purchased an Annuity of £5 or upwards on the whole duration of Life.

TABLE II.

RATES OF GOVERNMENT ANNUITIES:

ON SINGLE LIVES,

FOR THE WHOLE DURATION.

Age.	Annuity granted for every £100 Money, when the price of Three per Cent. Stock is										The Price charged for a given Annuity in Years purchase, when the price of Three per Cent. Stock is					Age.
	80			85			90			95			100*		80	85	90	95	100	
	£	s.	d.	£	s.	d.	£	s.	d.	£	s.	d.	£	s.						
21	5	17	6	5	10	7	5	4	5	4	18	11	4	14	17.02	18.09	19.15	20.22	21.28	21
22	5	17	6	5	10	7	5	4	5	4	18	11	4	14	17.02	18.09	19.15	20.22	21.28	22
23	5	18	9	5	11	9	5	5	7	5	0	0	4	15	16.84	17.89	18.95	20.00	21.01	23
24	6	0	0	5	12	11	5	6	8	5	1	1	4	16	16.67	17.71	18.75	19.79	20.83	24
25	6	1	3	5	14	1	5	7	9	5	2	1	4	17	16.49	17.53	18.56	19.59	20.62	25
26	6	1	3	5	14	1	5	7	9	5	2	1	4	17	16.49	17.53	18.56	19.59	20.62	26
27	6	2	6	5	15	4	5	8	11	5	3	2	4	18	16.33	17.35	18.37	19.39	20.41	27
28	6	3	9	5	16	6	5	10	0	5	4	3	4	19	16.16	17.17	18.18	19.19	20.20	28
29	6	5	0	5	17	8	5	11	1	5	5	3	5	0	16.00	17.00	18.00	19.00	20.00	29
30	6	6	3	5	18	10	5	12	3	5	6	4	5	1	15.84	16.83	17.82	18.81	19.80	30
31	6	7	6	6	0	0	5	13	4	5	7	4	5	2	15.68	16.67	17.65	18.63	19.61	31
32	6	8	9	6	1	2	5	14	5	5	8	5	5	3	15.53	16.51	17.48	18.45	19.42	32
33	6	10	0	6	2	4	5	15	7	5	9	6	5	4	15.38	16.35	17.31	18.27	19.23	33
34	6	11	3	6	3	7	5	16	8	5	10	6	5	5	15.24	16.19	17.14	18.09	19.05	34
35	6	12	6	6	4	8	5	17	9	5	11	7	5	6	15.09	16.04	16.98	17.92	18.87	35
36	6	15	0	6	7	1	6	0	0	5	13	8	5	8	14.81	15.74	16.67	17.59	18.52	36
37	6	16	3	6	8	3	6	1	1	5	14	9	5	9	14.68	15.60	16.51	17.43	18.35	37
38	6	17	6	6	9	5	6	2	3	5	15	9	5	10	14.55	15.46	16.36	17.27	18.18	38
39	7	0	0	6	11	9	6	4	5	5	17	11	5	12	14.29	15.18	16.07	16.96	17.86	39
40	7	1	7	6	12	11	6	5	7	5	18	11	5	13	14.16	15.04	15.93	16.81	17.70	40
41	7	3	9	6	15	4	6	7	9	6	1	1	5	15	13.91	14.78	15.65	16.52	17.39	41
42	7	6	3	6	17	8	6	10	0	6	3	2	5	17	13.68	14.53	15.38	16.24	17.09	42
43	7	7	6	6	18	10	6	11	1	6	4	3	5	18	13.56	14.41	15.25	16.10	16.95	43
44	7	10	0	7	1	2	6	13	4	6	6	4	6	0	13.33	14.17	15.00	15.83	16.67	44
45	7	12	6	7	3	7	6	15	7	6	8	5	6	2	13.11	13.93	14.75	15.57	16.39	45
46	7	15	0	7	5	10	6	17	9	6	10	6	6	4	12.90	13.71	14.52	15.32	16.13	46
47	7	17	6	7	8	3	7	0	0	6	12	8	6	6	12.70	13.49	14.29	15.08	15.87	47
48	8	0	0	7	10	7	7	2	2	6	14	9	6	8	12.50	13.28	14.07	14.84	15.59	48
49	8	3	9	7	14	1	7	5	7	6	17	11	6	11	12.21	12.98	13.74	14.50	15.27	49
50	8	6	3	7	16	1	7	7	9	7	0	0	6	13	12.03	12.78	13.53	14.27	15.04	50

* This column, from which the others have been deduced, shows the Annuity granted for £100 *Stock*, when its price is 80 or upwards.—*For further explanation, see pages 36 and 37.*

TABLE II.

RATES OF GOVERNMENT ANNUITIES: †

ON SINGLE LIVES,

FOR THE WHOLE DURATION.

Age.	Annuity granted for every £100 Money, when the price of Three per Cent. Stock is										The Price charged for a given Annuity in Years purchase, when the price of Three per Cent. Stock is					Age.
	80			85			90			95	100	80	85	90	95	100	
	£	s.	d.	£	s.	d.	£	s.	d.	£ s. d.	£ s.						
51	8	10	0	8	0	0	7	11	1	7 3 2	6 16	11.78	12.50	13.24	13.97	14.71	51
52	8	12	6	8	2	4	7	13	4	7 5 3	6 18	11.60	12.32	13.04	13.77	14.49	52
53	8	16	3	8	5	10	7	16	8	7 8 5	7 1	11.35	12.06	12.77	13.48	14.19	53
54	9	0	0	8	9	5	8	0	0	7 11 7	7 4	11.11	11.81	12.50	13.19	13.89	54
55	9	3	9	8	12	11	8	3	4	7 14 9	7 7	10.88	11.57	12.25	12.93	13.61	55
56	9	7	6	8	16	6	8	6	8	7 17 11	7 10	10.67	11.33	12.00	12.67	13.33	56
57	9	12	6	9	1	2	8	11	1	8 2 1	7 14	10.39	11.04	11.69	12.34	12.99	57
58	9	17	6	9	5	10	8	15	6	8 6 4	7 18	10.13	10.76	11.39	12.03	1 266	58
59	10	2	6	9	10	7	9	0	0	8 10 6	8 2	9.88	10.49	11.11	11.73	12.35	59
60	10	6	3	9	14	1	9	3	4	8 13 8	8 5	9.69	10.30	10.91	11.52	12.12	60
61	10	11	3	9	18	10	9	7	9	8 17 11	8 9	9.47	10.06	10.65	11.24	11.83	61
62	10	17	6	10	4	8	9	13	4	9 3 2	8 14	9.20	9.77	10.35	10.92	11.49	62
63	11	3	9	10	10	7	9	18	10	9 8 5	8 19	8.94	9.50	10.06	10.61	11.17	63
64	11	11	3	10	17	8	10	5	7	9 14 9	9 5	8.65	9.19	9.73	10.27	10.81	64
65	11	18	9	11	4	8	10	12	3	10 1 0	9 11	8.38	8.90	9.42	9.95	10.47	65
66	12	7	6	11	12	11	11	0	0	10 8 5	9 18	8.08	8.59	9.09	9.60	10.10	66
67	12	16	3	12	1	2	11	7	9	10 15 10	10 5	7.80	8.29	8.78	9.27	9.76	67
68	13	6	3	12	10	7	11	16	8	11 4 3	10 13	7.51	7.98	8.45	8.92	9.39	68
69	13	17	6	13	1	2	12	6	8	11 13 8	11 2	7.21	7.66	8.11	8.56	9.01	69
70	14	8	9	13	11	9	12	16	8	12 3 2	11 11	6.93	7.36	7.79	8.23	8.66	70
71	15	1	3	14	3	6	13	7	9	12 14 3	12 1	6.64	7.05	7.47	7.88	8.30	71
72	15	16	3	14	17	8	14	1	1	13 6 4	12 13	6.33	6.72	7.12	7.51	7.90	72
73	16	11	3	15	11	9	14	14	5	13 18 11	13 5	6.04	6.42	6.79	7.17	7.55	73
74	17	8	9	16	8	3	15	10	0	14 13 8	13 19	5.74	6.09	6.45	6.81	7.17	74
75 or above	18	6	3	17	4	8	16	5	7	15 8 5	14 13	5.46	5.80	6.10	6.48	6.83	75 or above

† All Government Annuities are transferrable, free from stamp duty, or enrolment fees, and payable half-yearly during the life of the nominee (or of the longest liver of two nominees,) and an additional quarter is payable at death.

TABLE II.

RATES OF GOVERNMENT ANNUITIES:

ON TWO JOINT LIVES AND THE LIFE OF THE SURVIVOR.

Younger Age.	Annuity granted for every £100 Three per Cent. Stock* when the Price of that Stock is 80 or upwards, and the difference between the ages is																	
	Under 5 Years.		5 & under 10 Years.		10 & under 15 Years.		15 & under 20 Years.		20 & under 25 Years.		25 & under 30 Years.		30 & under 35 Years.		35 Yrs. & upwards.	
	£	s.	£	s.	£	s.	£	s.	£	s.	£	s.	£	s.	£	s.
21	3	16	3	17	3	19	4	0	4	2	4	3	4	4	4	6
22	3	17	3	18	3	19	4	1	4	2	4	4	4	5	4	7
23	3	17	3	18	4	0	4	1	4	3	4	4	4	6	4	7
24	3	18	3	19	4	1	4	2	4	4	4	5	4	7	4	8
25	3	18	4	0	4	1	4	3	4	4	4	6	4	8	4	9
26	3	19	4	0	4	2	4	4	4	5	4	7	4	8	4	10
27	3	19	4	1	4	3	4	4	4	6	4	8	4	9	4	11
28	4	0	4	2	4	3	4	5	4	7	4	9	4	10	4	12
29	4	1	4	2	4	4	4	6	4	8	4	10	4	11	4	13
30	4	1	4	3	4	5	4	7	4	9	4	11	4	12	4	14
31	4	2	4	4	4	6	4	8	4	10	4	12	4	14	4	15
32	4	3	4	5	4	7	4	9	4	11	4	13	4	15	4	16
33	4	4	4	6	4	8	4	10	4	12	4	14	4	16	4	17
34	4	4	4	7	4	9	4	11	4	13	4	15	4	17	5	0
35	4	5	4	8	4	10	4	12	4	15	4	17	4	19	5	2
36	4	6	4	9	4	11	4	13	4	16	4	18	5	1	5	3
37	4	7	4	10	4	12	4	15	4	17	5	0	5	2	5	4
38	4	8	4	11	4	13	4	16	4	19	5	1	5	4	5	6
39	4	9	4	12	4	14	4	17	5	0	5	3	5	5	5	8
40	4	10	4	13	4	16	4	19	5	2	5	5	5	7	5	9
41	4	11	4	14	4	17	5	0	5	3	5	6	5	9	-	-
42	4	12	4	15	4	19	5	2	5	5	5	8	5	11	-	-
43	4	14	4	17	5	0	5	3	5	7	5	10	5	13	-	-
44	4	15	4	18	5	2	5	5	5	9	5	12	5	15	-	-
45	4	16	5	0	5	3	5	7	5	10	5	14	5	17	-	-
46	4	17	5	1	5	5	5	9	5	12	5	16	-	-	-	-
47	4	19	5	3	5	7	5	11	5	15	5	18	-	-	-	-
48	5	0	5	5	5	9	5	13	5	17	5	19	-	-	-	-
49	5	2	5	6	5	11	5	15	5	19	6	2	-	-	-	-
50	5	4	5	8	5	13	5	18	6	1	6	4	-	-	-	-

* For the mode of finding the Rate allowed for £ 100 money, or of determining the price of a given Annuity, see Pages 36 and 37.

TABLE II.

RATES OF GOVERNMENT ANNUITIES:

ON TWO JOINT LIVES AND THE LIFE OF THE SURVIVOR.

Younger Age.	Annuity granted for every £100 Three per Cent. Stock when the Price of that Stock is 80 or upwards, and the difference between the ages is									
	Under 5 Years.		5 & under 10 Years.		10& under 15 Years.		15& under 20 Years,		20& under 25 Years.	
	£	s.	£	s.	£	s.	£	s.	£	s.
51	5	6	5	10	5	15	6	0	6	3
52	5	7	5	12	5	18	6	3	6	6
53	5	9	5	15	6	0	6	4	6	9
54	5	11	5	17	6	3	6	7	6	12
55	5	14	6	0	6	5	6	10	6	15
56	5	16	6	2	6	8	6	13	-	-
57	5	18	6	5	6	11	6	17	-	-
58	6	1	6	7	6	15	7	1	-	-
59	6	4	6	11	6	18	7	5	-	-
60	6	6	6	14	7	2	7	9	-	-
61	6	9	6	18	7	6	-	-	-	-
62	6	12	7	2	7	11	-	-	-	-
63	6	16	7	6	7	16	-	-	-	-
64	7	0	7	11	8	1	-	-	-	-
65	7	4	7	16	8	6	-	-	-	-
66	7	9	8	2	-	-	-	-	-	-
67	7	14	8	8	-	-	-	-	-	-
68	8	0	8	13	-	-	-	-	-	-
69	8	6	9	1	-	-	-	-	-	-
70	8	11	9	7	-	-	-	-	-	-
71	8	18	-	-	-	-	-	-	-	-
72	9	6	-	-	-	-	-	-	-	-
73	9	15	-	-	-	-	-	-	-	-
74	10	4	-	-	-	-	-	-	-	-
75 or above.	10	14	-	-	-	-	-	-	-	-

TABLE III.

Showing the Interest and Amount of £1 in 12, 9, 6, or 3 Months; also the Present Worth and Discount of £1 due 12, 9 6, or 3 Months hence.—Simple Interest page 35.

Rate of Ann. Interest.	Time.	Interest. *i*	Amount. *r*	Present worth. *v*	Discount. *d*
2 Per Cent.	1 Year	.02	1.02	.980392	.019608
	¾ ——	.015	1.015	.985222	.014778
	½ ——	.01	1.01	.990099	.009901
	¼ ——	.005	1.005	.995025	.004975
2½ Per Cent	1 Year	.025	1.025	.975610	.024390
	¾ ——	.01875	1.01875	.981595	.018405
	½ ——	.0125	1.0125	.987654	.012346
	¼ ——	.00625	1.00625	.993789	.006211
3 Per Cent.	1 Year	.03	1.03	.970874	.029126
	¾ ——	.0225	1.0225	.977995	.022005
	½ ——	.015	1.015	.985222	.014778
	¼ ——	.0075	1.0075	.992556	.007444
3½ Per Cent	1 Year	.035	1.035	.966184	.033816
	¾ ——	.02625	1.02625	.974421	.025579
	½ ——	.0175	1.0175	.982801	.017199
	¼ ——	.00875	1.00875	.991326	.008674
4 Per Cent.	1 Year	.04	1.04	.961538	.038462
	¾ ——	.03	1.03	.970874	.029126
	½ ——	.02	1.02	.980392	.019608
	¼ ——	.01	1.01	.990099	.009901
4½ Per Cent	1 Year	.045	1.045	.956938	.043062
	¾ ——	.03375	1.03375	.967352	.032648
	½ ——	.0225	1.0225	.977995	.022005
	¼ ——	.01125	1.01125	.988875	.011125
5 Per Cent.	1 Year	.05	1.05	.952381	.047619
	¾ ——	.0375	1.0375	.963856	.036144
	½ ——	.025	1.025	.975610	.024390
	¼ ——	.0125	1.0125	.987654	.012346
6 Per Cent.	1 Year	.06	1.06	.943396	.056604
	¾ ——	.045	1.045	.956938	.043062
	½ ——	.03	1.03	.970874	.029126
	¼ ——	.015	1.015	.985222	.014778
7 Per Cent.	1 Year	.07	1.07	.934579	.065421
	¾ ——	.0525	1.0525	.950119	.049881
	½ ——	.035	1.035	.966184	.033816
	¼ ——	.0175	1.0175	.982801	.017199
8 Per Cent.	1 Year	.08	1.08	.925926	.074074
	¾ ——	.06	1.06	.943396	.056604
	½ ——	.04	1.04	.961538	.038462
	¼ ——	.02	1.02	.980392	.019608

TABLE IV.

Showing the Annual Interest produced at a given Nominal Rate, and the Amount of £1 in one Year, when the Interest is convertible into principal Half-yearly, Quarterly, or Momently. Also showing the Yearly Amount of an Annuity of £1 payable by Half-yearly, Quarterly, or Momently Instalments.—See pages 39, 53, and 72

Nominal Rate of Interest.	Payable.	Annual Rate produced. £	s.	d.	Amount of £1 in one Year. r	Logarithm of such Amount λr	Yearly Amount of £1 Annuity.
2 per Cent.	y	2	0	0	1.020000	0.008600172	1 000000
	h	2	0	2½	1.020100	.008642748	1.005000
	q	2	0	3½	1.020150	.008664247	1.007500
	m	2	0	4¾	1.020201	.008685890	1.010000
2½ per Cent.	y	2	10	0	1.025000	0.010723865	1.000000
	h	2	10	3¾	1.025156	.010790064	1.006250
	q	2	10	5½	1.025235	.010823574	1.009375
	m	2	10	7½	1.025315	.010857362	1.012500
3 per Cent.	y	3	0	0	1.030000	0.012837225	1.000000
	h	3	0	5¼	1.030225	.012932084	1.007500
	q	3	0	8	1.030339	.012980219	1.011250
	m	3	0	11	1.030454	.013028834	1.015000
3½ per Cent.	y	3	10	0	1.035000	0.014940350	1.000000
	h	3	10	7¼	1.035306	.015068836	1.008750
	q	3	10	11	1.035462	.015134191	1.013125
	m	3	11	3	1.035620	.015200307	1.017500
4 per Cent.	y	4	0	0	1.040000	0.017033339	1.000000
	h	4	0	9½	1.040400	.017200344	1.010000
	q	4	1	2½	1.040604	.017285495	1.015000
	m	4	1	7½	1.040811	.017371779	1.020000
4½ per Cent.	y	4	10	0	1.045000	0.019116290	1.000000
	h	4	11	0½	1.045506	.019326633	1.011250
	q	4	11	6¼	1.045765	.019434138	1.016875
	m	4	12	0¾	1.046028	.019543252	1.022500
5 per Cent.	y	5	0	0	1.050000	0 021189299	1.000000
	h	5	1	3	1.050625	.021447731	1.012500
	q	5	1	10¾	1.050946	.021580128	1.018750
	m	5	2	6½	1.051271	.021714724	1.025000
6 per Cent.	y	6	0	0	1.060000	0.025305865	1.000000
	h	6	1	9½	1.060900	.025674449	1.015000
	q	6	2	8¾	1.061364	.025864169	1.022500
	m	6	3	8	1.061837	.026057669	1.030000
7 per Cent.	y	7	0	0	1.070000	0.029383778	1.000000
	h	7	2	5¼	1.071225	.029880700	1.017500
	q	7	3	8½	1.071859	.030137672	1.026250
	m	7	5	0¼	1.072508	.030400614	1.035000
8 per Cent.	y	8	0	0	1.080000	0.033423755	1.000000
	h	8	3	2½	1.081600	.034066679	1.020000
	q	8	4	10¼	1.082432	.034400687	1.030000
	m	8	6	7	1.083287	.034743559	1.040000

TABLE V.

Showing the *Amount* of £1 improved at Compound Interest, for any number of years not exceeding 100.—See page 40.

Years.	1 Per Cent.	1½ Per Cent	2 Per Cent.	2½ Per Cent	3 Per Cent	3½ Per Cent
1	1.010000	1.015000	1.020000	1.025000	1.030000	1.035000
2	1.020100	1.030225	1.040400	1.050625	1.060900	1 071225
3	1.030301	1.045678	1.061208	1.076891	1 092727	1.108718
4	1.040604	1.061363	1.082432	1.103813	1.125509	1.147523
5	1.051010	1.077284	1.104081	1.131408	1.159274	1.187686
6	1.061520	1.093444	1.126162	1.159693	1.194052	1.229255
7	1.072135	1.109845	1.148686	1.188686	1.229874	1.272279
8	1.082856	1.126492	1.171659	1.218403	1.266770	1 316809
9	1.093685	1.143389	1.195093	1 248863	1.304773	1.362897
10	1.104622	1.160540	1.218994	1.280085	1.343916	1.410599
11	1.115668	1.177948	1.243374	1.312087	1.384234	1 459970
12	1.126825	1.195616	1.268242	1.344889	1.425761	1 511069
13	1.138093	1.213550	1.293607	1.378511	1.468534	1.563956
14	1.149474	1.231754	1.319479	1.412974	1.512590	1.618695
15	1.160969	1.250231	1.345868	1.448298	1.557967	1.675349
16	1.172579	1.268984	1.372786	1.484506	1.604706	1.733986
17	1.184305	1.288019	1.400241	1.521618	1.652848	1.794676
18	1.196148	1.307339	1.428246	1.559659	1.702433	1 857489
19	1.208109	1.326948	1.456811	1.598650	1.753506	1.922501
20	1.220190	1.346851	1.485947	1.638616	1.806111	1.989789
21	1.232392	1.367055	1.515666	1.679582	1.860295	2.059431
22	1.244716	1.387562	1.545980	1.721571	1.916103	2.131512
23	1.257163	1.408376	1.576899	1.764611	1.973587	2.206114
24	1.269735	1.429502	1.608437	1.808726	2.032794	2.283328
25	1.282432	1.450945	1.640606	1.853944	2.093778	2.363245
26	1.295256	1.472709	1.673418	1.900293	2.156591	2.445959
27	1.308209	1.494800	1.706886	1.947800	2.221289	2.531567
28	1.321291	1.517222	1.741024	1.996495	2.287928	2.620172
29	1.334504	1.539980	1.775845	2.046407	2.356566	2 711878
30	1.347849	1.563080	1.811362	2.097568	2.427262	2.806794
31	1.361327	1.586527	1.847589	2.150007	2.500080	2.905031
32	1.374940	1.610324	1.884541	2.203757	2.575083	3.006708
33	1.388689	1.634479	1.922231	2.258851	2.652335	3.111942
34	1.402576	1.658997	1.960676	2.315322	2.731905	3.220860
35	1.416602	1.683882	1.999890	2.373205	2.813862	3.333590
36	1.430768	1.709141	2.039887	2.432535	2 898278	3.450266
37	1.445076	1.734777	2.080685	2 493349	2.985227	3.571025
38	1 459527	1.760799	2.122299	2.555682	3.074783	3.696011
39	1.474122	1.787211	2.164745	2.619574	3.167027	3.825372
40	1.488863	1.814019	2.208040	2.685064	3.262038	3.959260
41	1.503752	1.841229	2.252200	2.752190	3.359899	4 097834
42	1.518790	1.868847	2.297244	2 820995	3.460696	4.241258
43	1.533978	1.896879	2 343189	2.891520	3.564517	4.389702
44	1.549318	1.925333	2.390053	2.963808	3.671452	4.543342
45	1.564811	1.954212	2.437854	3.037903	3.781596	4.702359
46	1.580459	1.983525	2.486611	3.113851	3.895044	4.866941
47	1.596264	2.013277	2.536344	3.191697	4.011895	5.037284
48	1.612227	2.043477	2.587070	3.271490	4.132252	5.213589
49	1.628349	2.074129	2.638812	3.353277	4.256219	5.396065
50	1.644632	2.105240	2.691588	3.437109	4.383906	5.584927

TABLE V.

Showing the *Amount* of £1 improved at Compound Interest, for any number of years not exceeding 100.

Years.	4 per Cent.	4½ per Cent	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
1	1.040000	1.045000	1.050000	1.060000	1.070000	1.080000
2	1.081600	1.092025	1.102500	1.123600	1.144900	1.166400
3	1.124864	1.141166	1.157625	1.191016	1.225043	1.259712
4	1.169859	1.192519	1.215506	1.262477	1.310796	1.360489
5	1.216653	1.246182	1.276282	1.338226	1.402552	1.469328
6	1.265319	1.302260	1.340096	1.418519	1.500730	1.586874
7	1.315932	1.360862	1.407100	1.503630	1.605781	1.713824
8	1.368569	1.422101	1.477455	1.593848	1.718186	1.850930
9	1.423312	1.486095	1.551328	1.689479	1.838459	1.999005
10	1.480244	1.552969	1.628895	1.790848	1.967151	2.158925
11	1.539454	1.622853	1.710339	1.898299	2.104852	2.331639
12	1.601032	1.695881	1.795856	2.012196	2.252192	2.518170
13	1.665074	1.772196	1.885649	2.132928	2.409845	2 719624
14	1.731676	1.851945	1.979932	2.260904	2.578534	2.937194
15	1.800944	1.935282	2.078928	2.396558	2.759032	3.172169
16	1.872981	2.022370	2.182875	2.540352	2.952164	3.425943
17	1.947901	2.113377	2.292018	2.692773	3.158815	3.700018
18	2.025817	2.208479	2.406619	2.854339	3.379932	3.996020
19	2.106849	2.307860	2.526950	3.025600	3.616528	4.315701
20	2.191123	2.411714	2.653298	3.207135	3.869684	4.660957
21	2.278768	2.520241	2.785963	3.399564	4.140562	5.033834
22	2.369919	2.633652	2.925261	3.603537	4.430402	5.436540
23	2.464716	2.752166	3.071524	3.819750	4.740530	5.871464
24	2.563304	2.876014	3.225100	4.048935	5.072367	6.341181
25	2.665836	3.005434	3.386355	4.291871	5.427433	6.848475
26	2.772470	3.140679	3.555673	4.549383	5.807353	7.396353
27	2.883369	3.282010	3.733456	4 822346	6.213868	7.988061
28	2.998703	3.429700	3.920129	5.111687	6.648838	8.627106
29	3.118651	3.584036	4.116136	5.418388	7.114257	9.317275
30	3.243398	3.745318	4.321942	5.743491	7.612255	10.062657
31	3.373133	3.913857	4.538039	6.088101	8.145113	10.867669
32	3.508059	4.089981	4.764941	6.453387	8.715271	11.737083
33	3.648381	4.274030	5.003189	6.840590	9.325340	12.676050
34	3.794316	4.466362	5.253348	7.251025	9.978114	13.690134
35	3.946089	4.667348	5.516015	7.686087	10.676581	14.785344
36	4.103933	4.877378	5.791816	8.147252	11.423942	15.968172
37	4.268090	5.096860	6.081407	8.636087	12.223618	17.245626
38	4.438813	5.326219	6.385477	9.154252	13.079271	18.625276
39	4.616366	5.565899	6.704751	9.703507	13.994820	20.115298
40	4.801021	5.816365	7.039989	10.285718	14.974458	21.724522
41	4.993061	6.078101	7.391988	10.902861	16.022670	23.462483
42	5.192784	6.351615	7.761588	11 557033	17.144257	25.339482
43	5.400495	6.637438	8.149667	12.250455	18.344355	27.366640
44	5.616515	6.936123	8.557150	12.985482	19.628460	29.555972
45	5.841176	7.248248	8.985008	13.764611	21.002452	31.920449
46	6.074823	7.574420	9.434258	14.590487	22.472623	34.474085
47	6.317816	7.915268	9.905971	15.465917	24.045707	37.232012
48	6.570528	8.271456	10.401270	16 393872	25.728907	40.210573
49	6.833349	8.643671	10.921333	17.377504	27.529930	43.427419
50	7.106683	9.032636	11.467400	18.420154	29.457025	46.901613

TABLE V.

Showing the *Amount* of £1 improved at Compound Interest, for any number of years not exceeding 100.

Years.	1 per Cent.	1½ per Cent.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.
51	1.661078	2.136818	2.745420	3.523036	4.515423	5.780399
52	1.677689	2.168870	2.800328	3.611112	4.650886	5.982713
53	1.694466	2.201404	2.856335	3.701390	4.790412	6.192108
54	1.711411	2.234425	2.913461	3.793925	4.934125	6.408832
55	1.728525	2.267946	2.971731	3.888773	5.082149	6.633141
56	1.745810	2.301964	3.031165	3.985992	5.234613	6.865301
57	1.763268	2.336494	3.091789	4.085642	5.391651	7.105587
58	1.780901	2.371541	3.153624	4.187783	5.553401	7.354282
59	1.798710	2.407114	3.216697	4.292478	5.720003	7.611682
60	1.816697	2.443220	3.281031	4.399790	5.891603	7.878091
61	1.834864	2.479868	3.346651	4.509784	6.068351	8.153824
62	1.853213	2.517067	3.413584	4.622529	6.250402	8.439208
63	1.871745	2.554823	3.481856	4.738092	6.437914	8.734580
64	1.890462	2.593145	3.551493	4.856545	6.631051	9 040291
65	1.909367	2.632042	3.622523	4.977958	6.829983	9.356701
66	1.928461	2.671522	3.694974	5.102407	7.034882	9.684185
67	1.947746	2.711594	3.768873	5.229967	7.245929	10.023132
68	1.967223	2.752267	3.844251	5.360717	7.463307	10.373941
69	1.986895	2.793550	3.921136	5.494734	7.687206	10.737029
70	2.006764	2.835454	3.999558	5.632103	7.917822	11.112825
71	2.026832	2.877986	4.079549	5.772905	8.155357	11.501774
72	2.047100	2.921156	4.161140	5.917228	8.400017	11.904336
73	2.067571	2.964974	4.244363	6.065159	8.652018	12.320988
74	2.088247	3.009449	4.329250	6.216788	8.911578	12.752223
75	2.109129	3.054590	4.415835	6.372207	9.178926	13.198550
76	2.130220	3.100409	4.504152	6.531513	9.454293	13.660500
77	2.151522	3.146913	4.594235	6.694800	9.737922	14.138617
78	2.173037	3.194117	4.686120	6.862170	10.030060	14.633469
79	2.194767	3.242029	4.779842	7.033725	10.330962	15.145640
80	2.216715	3.290659	4.875439	7.209568	10.640891	15.675738
81	2.238882	3.340020	4.972948	7.389807	10.960117	16.224388
82	2.261271	3.390120	5.072407	7.574552	11.288921	16.792242
83	2.283884	3.440971	5.173855	7.763916	11.627588	17.379970
84	2.306723	3.492586	5.277332	7.958014	11.976416	17.988269
85	2.329790	3.544975	5.382879	8.156964	12.335709	18.617859
86	2.353088	3.598150	5.490536	8.360888	12.705780	19.269484
87	2.376619	3.652123	5.600347	8.569911	13.086953	19.943916
88	2.400385	3.706905	5.712354	8.784158	13.479562	20.641953
89	2.424389	3.762509	5.826601	9.003762	13.883949	21.364421
90	2.448633	3.818947	5.943133	9.228856	14.300467	22.112176
91	2.473119	3.876231	6.061996	9.459578	14.729481	22.886102
92	2.497850	3.934374	6.183236	9.696067	15.171366	23.687116
93	2 522828	3.993390	6.306900	9.938469	15.626507	24.516165
94	2.548056	4.053291	6.433038	10.186931	16.095302	25.374230
95	2.573537	4.114090	6.561699	10.441604	16.578161	26.262329
96	2.599272	4.175800	6.692933	10.702644	17.075506	27.181510
97	2.625265	4.238437	6.826792	10.970210	17.587771	28.132863
98	2.651518	4.302013	6.963328	11.244465	18.115404	29.117513
99	2.678033	4.366543	7.102594	11.525577	18.658866	30.136626
100	2.704813	4.432041	7.244646	11.813716	19.218632	31.191408

TABLE V.

Showing the *Amount* of £1 improved at Compound Interest, for any number of years not exceeding 100.

Years.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
51	7.390951	9.439105	12.040770	19.525364	31.519017	50.653742
52	7.686589	9.863865	12.642808	20.696885	33.725348	54.706041
53	7.994052	10.307739	13.274949	21.938698	36.086122	59.082524
54	8.313814	10.771587	13.938696	23.255020	38.612151	63.809126
55	8.646367	11.256308	14.635631	24.650322	41.315001	68.913856
56	8.992222	11.762842	15.367412	26.129341	44.207052	74.426965
57	9.351910	12.292170	16.135783	27.697101	47.301545	80.381122
58	9.725987	12.845318	16.942572	29.358927	50.612653	86.811612
59	10.115026	13.423357	17.789701	31.120463	54.155539	93.756540
60	10.519627	14.027408	18.679186	32.987691	57.946427	101.257064
61	10.940413	14.658641	19.613145	34.966952	62.002677	109.357629
62	11.378029	15.318280	20.593802	37.064969	66.342864	118.106239
63	11.833150	16.007603	21.623493	39.288868	70.986865	127.554738
64	12.306476	16.727945	22.704667	41.646200	75.955945	137.759117
65	12.798735	17.480702	23.839901	44.144972	81.272861	148.779847
66	13.310685	18.267334	25.031896	46.793670	86.961962	160.682234
67	13.843112	19.089364	26.283490	49.601290	93.049299	173.536813
68	14.396836	19.948385	27.597665	52.577368	99.562750	187.419758
69	14.972710	20.846063	28.977548	55.732010	106.532142	202.413339
70	15.571618	21.784136	30.426426	59.075930	113.989392	218.606406
71	16.194483	22.764422	31.947747	62.620486	121.968650	236.094918
72	16.842262	23.788821	33.545134	66.377715	130.506455	254.982512
73	17.515953	24.859318	35.222391	70.360378	139.641907	275.381113
74	18.216591	25.977987	36.983510	74.582001	149.416840	297.411602
75	18.945255	27.146996	38.832686	79.056921	159.876019	321.204530
76	19.703065	28.368611	40.774320	83.800336	171.067341	346.900892
77	20.491187	29.645199	42.813036	88.828356	183.042054	374.652964
78	21.310835	30.979233	44.953688	94.158058	195.854998	404.625201
79	22.163268	32.373298	47.201372	99.807541	209.564848	436.995217
80	23.049799	33.830096	49.561441	105.795993	224.234388	471.954834
81	23.971791	35.352451	52.039513	112.143753	239.930795	509.711221
82	24.930663	36.943311	54.641489	118.872378	256.725950	550.488119
83	25.927889	38.605760	57.373563	126.004721	274.696767	594.527168
84	26.965005	40.343019	60.242241	133.565004	293.925541	642.089342
85	28.043605	42.158455	63.254353	141.578904	314.500328	693.456489
86	29.165349	44.055586	66.417071	150.073639	336.515351	748.933008
87	30.331963	46.038087	69.737925	159.078057	360.071426	808.847649
88	31.545242	48.109801	73.224821	168.622740	385.276426	873.555461
89	32.807051	50.274742	76.886062	178.740105	412.245776	943.439897
90	34.119333	52.537105	80.730365	189.464511	441.102980	1018.91509
91	35.484107	54.901275	84.766883	200.832382	471.980188	1100.42830
92	36.903471	57.371832	89.005227	212.882325	505.018802	1188.46256
93	38.379610	59.953565	93.455489	225.655264	540.370118	1283.53956
94	39.914794	62.651475	98.128263	239.194580	578.196026	1386.22273
95	41.511386	65.470792	103.034676	253.546255	618.669748	1497.12055
96	43.171841	68.416977	108.186410	268.759030	661.976630	1616.89019
97	44.898715	71.495741	113.595731	284.884572	708.314994	1746.24141
98	46.694664	74.713050	119.275517	301.977646	757.897044	1885.94072
99	48.562450	78.075137	125.239293	320.096305	810.949837	2036.81598
100	50.504948	81.588518	131.501258	339.302084	867.716326	2199.76126

TABLE VI.

Showing the *Present Value* of £1 to be received at the end of any number of years not exceeding 100.—See page 45.

Years	1 per Cent.	1½ per Cent.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.
1	.990099	.985222	.980392	.975610	.970874	.966184
2	.980296	.970662	.961169	.951814	.942596	.933511
3	.970590	.956317	.942322	.928599	.915142	.901943
4	.960980	.942184	.923845	.905951	.888487	.871442
5	.951466	.928260	.905731	.883854	.862609	.841973
6	.942045	.914542	.887971	.862297	.837484	.813501
7	.932718	.901027	.870560	.841265	.813092	.785991
8	.923483	.887711	.853490	.820747	.789409	.759412
9	.914340	.874592	.836755	800728	.766417	.733731
10	.905287	.861667	.820348	.781198	.744094	.708919
11	.896324	.848933	.804263	.762145	.722421	.684946
12	.887449	.836387	.788493	.743556	.701380	.661783
13	.878662	.824027	.773033	.725420	.680951	.639404
14	.869963	.811849	.757875	.707727	.661118	.617782
15	.861349	.799852	.743015	.690466	.641862	.596891
16	.852821	.788031	.728446	.673625	.623167	.576706
17	.844377	.776385	.714163	.657195	.605016	.557204
18	.836017	.764912	.700159	.641166	.587395	.538361
19	.827740	.753607	.686431	.625528	.570286	.520156
20	.819544	.742471	.672971	.610271	.553676	.502566
21	.811430	.731498	.659776	.595386	.537549	.485571
22	.803396	.720687	.646839	.580865	.521893	.469151
23	.795442	.710037	.634156	.566697	.506692	.453286
24	.787566	.699544	.621721	.552875	.491934	.437957
25	.779768	.689206	.609531	.539391	.477606	.423147
26	.772048	.679020	.597579	.526235	.463695	.408838
27	.764404	.668986	.585862	.513400	.450189	.395012
28	.756836	.659099	.574375	.500878	.437077	.381654
29	.749342	.649359	.563112	.488661	.424346	.368748
30	.741923	.639762	.552071	.476743	.411987	.356278
31	.734577	.630308	.541246	.465115	.399987	.344230
32	.727304	.620994	.530633	.453771	.388337	.332590
33	.720103	.611816	.520229	.442703	.377026	.321343
34	.712973	.602774	.510028	.431905	.366045	.310476
35	.705914	.593866	.500028	.421371	.355383	.299977
36	.698925	.585090	.490223	.411094	.345032	.289833
37	.692005	.576443	.480611	.401067	.334983	.280032
38	.685153	.567924	.471187	.391285	.325226	.270562
39	.678370	.559531	.461948	.381741	.315754	.261413
40	.671653	.551262	.452890	.372431	.306557	.252572
41	.665003	.543116	.444010	.363347	.297628	.244031
42	.658419	.535089	.435304	.354485	.288959	.235779
43	.651900	.527182	.426769	.345839	.280543	.227806
44	.645445	.519391	.418401	.337404	.272372	.220102
45	.639055	.511715	.410197	.329174	.264439	.212659
46	.632728	.504153	.402154	.321146	.256737	.205468
47	.626463	.496702	.394268	.313313	.249259	.198520
48	.620260	.489362	.386538	.305671	.241999	.191806
49	.614119	.482130	.378958	.298216	.234950	.185320
50	.608039	.475005	.371528	.290942	.228107	.179053

TABLE VI.

Showing the *Present Value* of £1 to be received at the end of any number of years not exceeding 100.

Years.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent	7 per Cent.	8 per Cent.
1	.961538	.956938	.952381	.943396	.934579	.925926
2	.924556	.915730	.907029	.889996	.873439	.857339
3	.888996	.876297	.863838	.839619	.816298	.793832
4	.854804	.838561	.822702	.792094	.762895	.735030
5	.821927	.802451	.783526	.747258	.712986	.680583
6	.790315	.767896	.746215	.704961	.666342	.630170
7	.759918	.734828	.710681	.665057	.622750	.583490
8	.730690	.703185	.676839	.627412	.582009	.540269
9	.702587	.672904	.644609	.591898	.543934	.500249
10	.675564	.643928	.613913	.558395	.508349	.463193
11	.649581	.616199	.584679	.526788	.475093	.428883
12	.624597	.589664	.556837	.496969	.444012	.397114
13	.600574	.564272	.530321	.468839	.414964	.367698
14	.577475	.539973	.505068	.442301	.387817	.340461
15	.555265	.516720	.481017	.417265	.362446	.315242
16	.533908	.494469	.458112	.393646	.338735	.291890
17	.513373	.473176	.436297	.371364	.316574	.270269
18	.493628	.452800	.415521	.350344	.295864	.250249
19	.474642	.433302	.395734	.330513	.276508	.231712
20	.456387	.414643	.376889	.311805	.258419	.214548
21	.438834	.396787	.358942	.294155	.241513	.198656
22	.421955	.379701	.341850	.277505	.225713	.183941
23	.405726	.363350	.325571	.261797	.210947	.170315
24	.390121	.347703	.310068	.246979	.197147	.157699
25	.375117	.332731	295303	.232999	.184249	.146018
26	.360689	.318402	.281241	.219810	.172195	.135202
27	.346817	.304691	.267848	.207368	.160930	.125187
28	.333477	.291571	.255094	.195630	.150402	.115914
29	.320651	.279015	.242946	.184557	.140563	.107328
30	.308319	.267000	.231377	.174110	.131367	.099377
31	.296460	.255502	.220359	.164255	.122773	.092016
32	.285058	.244500	.209866	.154957	.114741	.085200
33	.274094	.233971	.199873	.146186	.107235	.078889
34	.263552	.223896	.190355	.137912	.100219	.073045
35	.253415	.214254	.181290	.130105	.093663	.067635
36	.243669	.205028	.172657	.122741	.087535	.062625
37	.234297	.196199	.164436	.115793	.081809	.057986
38	.225285	.187750	.156605	.109239	.076457	.053690
39	.216621	.179665	.149148	.103056	.071455	.049713
40	.208289	.171929	.142046	.097222	.066780	.046031
41	.200278	.164525	.135282	.091719	.062412	.042621
42	.192575	.157440	.128840	.086527	.058329	.039464
43	.185168	.150661	.122704	.081630	.054513	.036541
44	.178046	.144173	.116861	.077009	.050946	.033834
45	.171198	.137964	.111297	.072650	.047613	.031328
46	.164614	.132023	.105997	.068538	.044499	.029007
47	.158283	.126338	.100949	.064658	.041587	.026859
48	.152195	.120898	.096142	.060998	.038867	.024869
49	.146341	.115692	.091564	.057546	.036324	.023027
50	.140713	.110710	.087204	.054288	.033948	.021321

TABLE VI.

Showing the *Present Value* of £1 to be received at the end of any number of years not exceeding 100.

Years.	1 per Cent.	1½ per Cent.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.
51	.602019	.467985	.364243	.283846	.221463	.172998
52	.596058	.461069	.357101	.276923	.215013	.167148
53	.590156	.454255	.350099	.270169	.208750	.161496
54	.584313	.447542	.343234	.263579	.202670	.156035
55	.578528	.440928	.336504	.257151	.196767	.150758
56	.572800	.434412	.329906	.250879	.191036	.145660
57	.567129	.427992	.323437	.244760	.185472	.140734
58	.561514	.421661	.317095	.238790	.180070	.135975
59	.555954	.415435	.310878	.232966	.174825	.131377
60	.550450	.409296	.304782	.227284	.169733	.126934
61	.545000	.403247	.298806	.221740	.164789	.122642
62	.539604	.397288	.292947	.216332	.159990	.118495
63	.534261	.391417	.287203	.211055	.155330	.114487
64	.528971	.385632	.281572	.205908	.150806	.110616
65	.523734	.379933	.276051	.200886	.146413	.106875
66	.518548	.374318	.270638	.195986	.142149	.103261
67	.513414	.368787	.265331	.191206	.138009	.099769
68	.508331	.363337	.260129	.186542	.133989	.096395
69	.503298	.357967	.255028	.181992	.130086	.093136
70	.498315	.352677	.250028	.177554	.126297	.089986
71	.493381	.347465	.245125	.173223	.122619	.086943
72	.488496	.342330	.240319	.168998	.119047	.084003
73	.483659	.337271	.235607	.164876	.115580	.081162
74	.478871	.332287	.230987	.160855	.112214	.078418
75	.474130	.327376	.226458	.156931	.108945	.075766
76	.469435	.322538	.222017	.153104	.105772	.073204
77	.464787	.317771	.217664	.149370	.102691	.070728
78	.460185	.313075	.213396	.145726	.099700	.068337
79	.455629	.308449	.209212	.142172	.096796	.066026
80	.451118	.303890	.205110	.138705	.093977	.063793
81	.446651	.299399	.201088	.135322	.091240	.061636
82	.442229	.294975	.197145	.132021	.088582	.059551
83	.437851	.290616	.193279	.128801	.086002	.057538
84	.433516	.286321	.189490	.125659	.083497	.055592
85	.429223	.282089	.185774	.122595	.081065	.053712
86	.424973	.277920	.182132	.119605	.078704	.051896
87	.420766	.273813	.178560	.116687	.076412	.050141
88	.416600	.269767	.175059	.113841	.074186	.048445
89	.412475	.265780	.171627	.111065	.072026	.046807
90	.408391	.261852	.168261	.108356	.069928	.045224
91	.404348	.257983	.164962	.105713	.067891	.043695
92	.400344	.254170	.161728	.103135	.065914	.042217
93	.396380	.250414	.158556	.100619	.063994	.040789
94	.392456	.246713	.155448	.098165	.062130	.039410
95	.388570	.243067	.152400	.095771	.060320	.038077
96	.384723	.239475	.149411	.093435	.058563	.036790
97	.380914	.235936	.146482	.091156	.056858	.035546
98	.377142	.232449	.143610	.088933	.055202	.034344
99	.373408	.229014	.140794	.086764	.053594	.033182
100	.369711	.225629	.138033	.084647	.052033	.032060

TABLE VI.

Showing the *Present Value* of £1 to be received at the end of any number of years not exceeding 100.

Years.	4 per Cent	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
51	.135301	.105942	.083051	.051215	.031727	.019742
52	.130097	.101380	.079096	.048316	.029651	.018280
53	.125093	.097014	.075330	.045582	.027711	.016925
54	.120282	.092837	.071743	.043001	.025899	.015672
55	.115656	.088839	.068326	.040567	.024204	.014511
56	.111207	.085013	.065073	.038271	.022621	.013436
57	.106930	.081353	.061974	.036105	.021141	.012441
58	.102817	.077849	.059023	.034061	.019758	.011519
59	.098863	.074497	.056212	.032133	.018465	.010666
60	.095060	.071289	.053536	.030314	.017257	.009876
61	.091404	.068219	.050986	.028598	.016128	.009144
62	.087889	.065281	.048558	.026980	.015073	.008467
63	.084508	.062470	.046246	.025453	.014087	.007840
64	.081258	.059780	.044044	.024012	.013166	.007259
65	.078133	.057206	.041946	.022653	.012304	.006721
66	.075128	.054743	.039949	.021370	.011499	.006223
67	.072238	.052385	.038047	.020161	.010747	.005762
68	.069460	.050129	.036235	.019020	.010044	.005336
69	.066788	.047971	.034509	.017943	.009387	.004940
70	.064219	.045905	.032866	.016927	.008773	.004574
71	.061749	.043928	.031301	.015969	.008199	.004236
72	.059374	.042037	.029811	.015065	.007662	.003922
73	.057091	.040226	.028391	.014213	.007161	.003631
74	.054895	.038494	.027039	.013408	.006693	.003362
75	.052784	.036836	.025752	.012649	.006255	.003113
76	.050754	.035250	.024525	.011933	.005846	.002883
77	.048801	.033732	.023357	.011258	.005463	.002669
78	.046924	.032280	.022245	.010620	.005106	.002471
79	.045120	.030890	.021186	.010019	.004772	.002288
80	.043384	.029559	.020177	.009452	.004460	.002119
81	.041716	.028287	.019216	.008917	.004168	.001962
82	.040111	.027069	.018301	.008412	.003895	.001817
83	.038569	.025903	.017430	.007936	.003640	.001682
84	.037085	.024787	.016600	.007487	.003402	.001557
85	.035659	.023720	.015809	.007063	.003180	.001442
86	.034287	.022699	.015056	.006663	.002972	.001335
87	.032969	.021721	.014339	.006286	.002777	.001236
88	.031701	.020786	.013657	.005930	.002596	.001145
89	.030481	.019891	.013006	.005595	.002426	.001060
90	.029309	.019034	.012387	.005278	.002267	.000981
91	.028182	.018215	.011797	.004979	.002119	.000909
92	.027098	.017430	.011235	.004697	.001980	.000841
93	.026056	.016680	.010700	.004432	.001851	.000779
94	.025053	.015961	.010191	.004181	.001730	.000721
95	.024090	.015274	.009705	.003944	.001616	.000668
96	.023163	.014616	.009243	.003721	.001511	.000618
97	.022272	.013987	.008803	.003510	.001412	.000573
98	.021416	.013385	.008384	.003312	.001319	.000530
99	.020592	.012808	.007985	.003124	.001233	.000491
100	.019800	.012257	.007604	.002947	.001152	.000455

TABLE VII.

Showing the *Amount* of £1 *per Annum* forborn and improved for any number of years not exceeding 100.—See page 48.

Years.	1 per Cent.	1½ per Cent.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.010000	2.015000	2.020000	2.025000	2.030000	2.035000
3	3.030100	3.045225	3.060400	3.075625	3.090900	3.106225
4	4.060401	4.090903	4.121608	4.152516	4.183627	4.214943
5	5.101005	5.152266	5.204040	5.256329	5.309136	5.362466
6	6.152015	6.229550	6.308121	6.387737	6.468410	6.550152
7	7.213535	7.322994	7.434283	7.547430	7.662462	7.779408
8	8.285670	8.432839	8.582969	8.736116	8.892336	9.051687
9	9.368526	9.559331	9.754628	9.954519	10.159106	10.368496
10	10.462211	10.702720	10.949721	11.203382	11.463879	11.731393
11	11.566833	11.863260	12.168715	12.483466	12.807796	13.141992
12	12.682501	13.041208	13.412090	13.795553	14.192030	14.601962
13	13.809326	14.236824	14.680332	15.140442	15.617790	16.113030
14	14.947419	15.450374	15.973938	16.518953	17.086324	17.676986
15	16.096893	16.682128	17.293417	17.931927	18.598914	19.295681
16	17.257862	17.932359	18.639285	19.380225	20.156881	20.971030
17	18.430441	19.201343	20.012071	20.864730	21.761588	22.705016
18	19.614746	20.489362	21.412312	22.386349	23.414435	24.499691
19	20.810894	21.796701	22.840559	23.946007	25.116868	26.357181
20	22.019003	23.123649	24.297370	25.544658	26.870374	28.279682
21	23.239193	24.470506	25.783317	27.183274	28.676486	30.269471
22	24.471585	25.837555	27.298984	28.862856	30.536780	32.328902
23	25.716301	27.225117	28.844963	30.584427	32.452884	34.460414
24	26.973464	28.633493	30.421862	32.349038	34.426470	36.666528
25	28.243199	30.062995	32.030300	34.157764	36.459264	38.949857
26	29.525631	31.513940	33.670906	36.011708	38.553042	41.313102
27	30.820887	32.986649	35.344324	37.912001	40.709634	43.759060
28	32.129096	34.481449	37.051210	39.859801	42.930923	46.290627
29	33.450387	35.998671	38.792235	41.856296	45.218850	48.910799
30	34.784891	37.538651	40.568079	43.902703	47.575416	51.622677
31	36.132740	39.101731	42.379441	46.000271	50.002678	54.429471
32	37.494067	40.688258	44.227030	48.150278	52.502759	57.334502
33	38.869007	42.298582	46.111570	50.354034	55.077841	60.341210
34	40.257696	43.933061	48.033802	52.612885	57.730177	63.453152
35	41.660272	45.592058	49.994478	54.928207	60.462082	66.674013
36	43.076874	47.275940	51.994367	57.301413	63.275944	70.007603
37	44.507642	48.985081	54.034255	59.733948	66.174223	73.457869
38	45.952718	50.719858	56.114940	62.227297	69.159449	77.028895
39	47.412245	52.480657	58.237238	64.782979	72.234233	80.724906
40	48.886367	54.267868	60.401983	67.402554	75.401260	84.550278
41	50.375230	56.081887	62.610023	70.087617	78.663298	88.509537
42	51.878982	57.923116	64.862223	72.839808	82.023196	92.607371
43	53.397772	59.791963	67.159468	75.660803	85.483892	96.848629
44	54.931750	61.688842	69.502657	78.552323	89.048409	101.23833
45	56.481068	63.614175	71.892710	81.516131	92.719861	105.78167
46	58.045879	65.568387	74.330564	84.554034	96.501457	110.48403
47	59.626338	67.551912	76.817176	87.667885	100.39650	115.35097
48	61.222602	69.565189	79.353519	90.859582	104.40840	120.38826
49	62.834829	71.608666	81.940590	94.131072	108.54065	125.60185
50	64.463178	73.682795	84.579401	97.484349	112.79687	130.99791

TABLE VII.

Showing the *Amount* of £1 *per Annum* forborn and improved for any number of years not exceeding 100.

Years.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
1	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
2	2.040000	2.045000	2.050000	2.060000	2.070000	2.080000
3	3.121600	3.137025	3.152500	3.183600	3.214900	3.246400
4	4.246464	4.278191	4.310125	4.374616	4.439943	4.506112
5	5.416323	5.470710	5.525631	5.637093	5.750739	5.866601
6	6.632975	6.716892	6.801913	6.975319	7.153291	7.335929
7	7.898294	8.019152	8.142008	8.393838	8.654021	8.922803
8	9.214226	9.380014	9.549109	9.897468	10.259803	10.636628
9	10.582795	10.802114	11.026564	11.491316	11.977989	12.487558
10	12.006107	12.288209	12.577893	13.180795	13.816448	14.486562
11	13.486351	13.841179	14.206787	14.971643	15.783599	16.645487
12	15.025805	15.464032	15.917127	16.869941	17.888451	18.977126
13	16.626838	17.159913	17.712983	18.882138	20.140643	21.495297
14	18.291911	18.932109	19.598632	21.015066	22.550488	24.214920
15	20.023588	20.784054	21.578564	23.275970	25.129022	27.152114
16	21.824531	22.719337	23.657492	25.672528	27.888054	30.324283
17	23.697512	24.741707	25.840366	28.212880	30.840217	33.750226
18	25.645413	26.855084	28.132385	30.905653	33.999033	37.450244
19	27.671229	29.063562	30.539004	33.759992	37.378965	41.446263
20	29.778079	31.371423	33.065954	36.785591	40.995492	45.761964
21	31.969202	33.783137	35.719252	39.992727	44.865177	50.422921
22	34.247970	36.303378	38.505214	43.392290	49.005739	55.456755
23	36.617889	38.937030	41.430475	46.995828	53.436141	60.893296
24	39.082604	41.689196	44.501999	50.815577	58.176671	66.764759
25	41.645908	44.565210	47.727099	54.864512	63.249038	73.105940
26	44.311745	47.570645	51.113454	59.156383	68.676470	79.954415
27	47.084214	50.711324	54.669126	63.705766	74.483823	87.350768
28	49.967583	53.993333	58.402583	68.528112	80.697691	95.338830
29	52.966286	57.423033	62.322712	73.639798	87.346529	103.96594
30	56.084938	61.007070	66.438848	79.058186	94.460786	113.28321
31	59.328335	64.752388	70.760790	84.801677	102.07304	123.34587
32	62.701469	68.666245	75.298829	90.889778	110.21815	134.21354
33	66.209527	72.756226	80.063771	97.343165	118.93343	145.95062
34	69.857909	77.030256	85.066959	104.18376	128.25877	158.62667
35	73.652225	81.496618	90.320307	111.43478	138.23688	172.31680
36	77.598314	86.163966	95.836323	119.12087	148.91346	187.10215
37	81.702246	91.041344	101.62814	127.26812	160.33740	203.07032
38	85.970336	96.138205	107.70955	135.90421	172.56102	220.31595
39	90.409150	101.46442	114.09502	145.05846	185.64029	238.94122
40	95.025516	107.03032	120.79977	154.76197	199.63511	259.05652
41	99.826536	112.84669	127.83976	165.04768	214.60957	280.78104
42	104.81960	118.92479	135.23175	175.95055	230.63224	304.24352
43	110.01238	125.27640	142.99334	187.50758	247.77650	329.58301
44	115.41288	131.91384	151.14301	199.75803	266.12085	356.94965
45	121.02939	138.84997	159.70016	212.74351	285.74931	386.50562
46	126.87057	146.09821	168.68516	226.50812	306.75176	418.42607
47	132.94539	153.67263	178.11942	241.09861	329.22439	452.90015
48	139.26321	161.58790	188.02539	256.56453	353.27009	490.13216
49	145.83373	169.85936	198.42666	272.95840	378.99900	530.34274
50	152.66708	178.50303	209.34800	290.33590	406.52893	573.77016

TABLE VII.

Showing the *Amount* of £1 *per Annum* forborn and improved for any number of years not exceeding 100.

Years.	1 per Cent.	1½ per Cent.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.
51	66.107810	75.788035	87.270989	100.92146	117.18077	136.58284
52	67.768888	77.924853	90.016409	104.44449	121.69620	142.36324
53	69.446577	80.093723	92.816737	108.05561	126.34708	148.34595
54	71.141043	82.295127	95.673072	111.75700	131.13749	154.53806
55	72.852454	84.529552	98.586534	115.55092	136.07162	160.94689
56	74.580979	86.797498	101.55826	119.43969	141.15377	167.58003
57	76.326789	89.099462	104.58943	123.42569	146.38838	174.45533
58	78.090057	91.435956	107.68122	127.51133	151.78003	181.55092
59	79.870958	93.807497	110.83484	131.69911	157.33343	188.90520
60	81.669668	96.214611	114.05154	135.99159	163.05344	196.51688
61	83.486365	98.657831	117.33257	140.39138	168.94504	204.39497
62	85.321229	101.13770	120.67922	144.90116	175.01339	212.54880
63	87.174442	103.65477	124.09281	149.52369	181.26379	220.98801
64	89.046187	106.20959	127.57466	154.26179	187.70171	229.72259
65	90.936649	108.80273	131.12616	159.11833	194.33276	238.76288
66	92.846016	111.43478	134.74868	164.09629	201.16274	248.11958
67	94.774477	114.10630	138.44365	169.19870	208.19762	257.80376
68	96.722223	116.81789	142.21253	174.42866	215.44355	267.82689
69	98.689446	119.57016	146.05678	179.78938	222.90686	278.20084
70	100.67634	122.36371	149.97791	185.28411	230.59406	288.93786
71	102.68311	125.19916	153.97747	190.91622	238.51189	300.05069
72	104.70994	128.07715	158.05702	196.68912	246.66724	311.55246
73	106.75704	130.99831	162.21816	202.60635	255.06726	323.45680
74	108.82461	133.96328	166.46252	208.67151	263.71928	335.77779
75	110.91286	136.97273	170.79177	214.88830	272.63086	348.53001
76	113.02198	140.02732	175.20761	221.26050	281.80978	361.72856
77	115.15220	143.12773	179.71176	227.79202	291.26407	375.38906
78	117.30373	146.27464	184.30600	234.48682	301.00200	389.52768
79	119.47676	149.46876	188.99212	241.34899	311.03206	404.16115
80	121.67153	152.71079	193.77196	248.38271	321.36302	419.30679
81	123.88825	156.00145	198.64740	255.59228	332.00391	434.98252
82	126.12713	159.34147	203.62034	262.98209	342.96403	451.20691
83	128.38840	162.73159	208.69275	270.55664	354.25295	467.99915
84	130.67228	166.17256	213.86661	278.32056	365.88054	485.37913
85	132.97901	169.66514	219.14394	286.27857	377.85695	503.36739
86	135.30880	173.21012	224.52682	294.43553	390.19266	521.98525
87	137.66188	176.80827	230.01735	302.79642	402.89844	541.25474
88	140.03850	180.46039	235.61770	311.36633	415.98539	561.19865
89	142.43889	184.16730	241.33006	320.15049	429.46496	581.84061
90	144.86328	187.92980	247.15666	329.15425	443.34890	603.20503
91	147.31191	191.74875	253.09979	338.38311	457.64937	625.31720
92	149.78503	195.62498	259.16179	347.84269	472.37885	648.20331
93	152.28288	199.55936	265.34502	357.53875	487.55022	671.89042
94	154.80571	203.55275	271.65192	367.47722	503.17672	696.40659
95	157.35376	207.60604	278.08496	377.66415	519.27203	721.78082
96	159.92730	211.72013	284.64666	388.10576	535.85019	748.04314
97	162.52657	215.89593	291.33959	398.80840	552.92569	775.22465
98	165.15184	220.13436	298.16638	409.77861	570.51346	803.35752
99	167.80335	224.43638	305.12971	421.02308	588.62887	832.47503
100	170.48139	228.80292	312.23231	432.54865	607.28773	862.61166

TABLE VII.

Showing the *Amount* of £1 *per Annum* forborn and improved for any number of years not exceeding 100.

Years.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
51	159.77377	187.53566	220.81540	308.75606	435.98595	620.67177
52	167.16472	196.97477	232.85617	328.28142	467.50497	671.32551
53	174.85131	206.83863	245.49897	348.97831	501.23032	726.03155
54	182.84536	217.14637	258.77392	370.91701	537.31644	785.11408
55	191.15917	227.91796	272.71262	394.17203	575.92859	848.92320
56	199.80554	239.17427	287.34825	418.82235	617.24359	917.83706
57	208.79776	250.93711	302.71566	444.95169	661.45065	992.26402
58	218.14967	263.22928	318.85144	472.64879	708.75219	1072.6451
59	227.87566	276.07460	335.79402	502.00772	759.36484	1159.4568
60	237.99069	289.49795	353.58372	533.12818	813.52038	1253.2133
61	248.51031	303.52536	372.26290	566.11587	871.46681	1354.4704
62	259.45073	318.18400	391.87605	601.08282	933.46949	1463.8280
63	270.82875	333.50228	412.46985	638.14779	999.81235	1581.9342
64	282.66190	349.50989	434.09334	677.43666	1070.7992	1709.4890
65	294.96838	366.23783	456.79801	719.08286	1146.7552	1847.2481
66	307.76712	383.71853	480.63791	763.22783	1228.0280	1996.0279
67	321.07780	401.98587	505.66981	810.02150	1314.9900	2156.7102
68	334.92091	421.07523	531.95330	859.62279	1408.0393	2330.2470
69	349.31775	441.02362	559.55096	912.20016	1507.6020	2517.6667
70	364.29046	461.86968	588.52851	967.93217	1614.1342	2720.0801
71	379.86208	483.65382	618.95494	1027.0081	1728.1236	2938.6865
72	396.05656	506.41824	650.90268	1089.6286	1850.0922	3174.7814
73	412.89882	530.20706	684.44782	1156.0063	1980.5987	3429.7639
74	430.41478	555.06638	719.67021	1226.3667	2120.2406	3705.1450
75	448.63137	581.04436	756.65372	1300.9487	2269.6574	4002.5566
76	467.57662	608.19136	795.48640	1380.0056	2429.5334	4323.7612
77	487.27969	636.55997	836.26072	1463.8059	2600.6008	4670.6620
78	507.77087	666.20517	879.07376	1552.6343	2783.6428	5045.3150
79	529.08171	697.18440	924.02745	1646.7924	2979.4978	5449.9402
80	551.24498	729.55770	971.22882	1746.5999	3189.0627	5886.9354
81	574.29478	763.38779	1020.7903	1852.3959	3413.2971	6358.8903
82	598.26657	798.74025	1072.8298	1964.5396	3653.2279	6868.6015
83	623.19723	835.68356	1127.4713	2083.4120	3909.9538	7419.0896
84	649.12512	874.28932	1184.8448	2209.4167	4184.6506	8013.6168
85	676.09012	914.63234	1245.0871	2342.9817	4478.5761	8655.7061
86	704.13373	956.79079	1308.3414	2484.5606	4793.0764	9349.1626
87	733.29908	1000.8464	1374.7585	2634.6343	5129.5918	10098.096
88	763.63104	1046.8845	1444.4964	2793.7123	5489.6632	10906.943
89	795.17628	1094.9943	1517.7212	2962.3351	5874.9397	11780.499
90	827.98333	1145.2690	1594.6073	3141.0752	6287.1854	12723.939
91	862.10267	1197.8061	1675.3377	3330.5397	6728.2884	13742.854
92	897.58677	1252.7074	1760.1045	3531.3721	7200.2686	14843.282
93	934.49024	1310.0792	1849.1098	3744.2544	7705.2874	16031.745
94	972.86985	1370.0328	1942.5653	3969.9097	8245.6575	17315.284
95	1012.7846	1432.6843	2040.6935	4209.1042	8823.8535	18701.507
96	1054.2960	1498.1551	2143.7282	4462.6505	9442.5233	20198.627
97	1097.4679	1566.5720	2251.9146	4731.4095	10104.500	21815.518
98	1142.3666	1638.0678	2365.5103	5016.2941	10812.815	23561.759
99	1189.0613	1712.7808	2484.7859	5318.2718	11570.712	25447.700
100	1237.6237	1790.8560	2610.0252	5638.3681	12381.662	27484.516

TABLE VIII.

Showing the *Present Value* of £1 *per Annum* for any number of years not exceeding 100.—See page 56.

Years.	1 per Cent.	1½ per Cent.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.
1	.990099	.985222	.980392	.975610	.970874	.966184
2	1.970395	1.955884	1.941561	1.927424	1.913470	1.899694
3	2.940985	2.912201	2.883883	2.856024	2.828611	2.801637
4	3.901965	3.854385	3.807729	3.761974	3.717098	3.673079
5	4.853431	4.782645	4.713460	4.645828	4.579707	4.515052
6	5.795476	5.697187	5.601431	5.508125	5.417191	5.328553
7	6.728194	6.598214	6.471991	6.349391	6.230283	6.114544
8	7.651677	7.485925	7.325481	7.170137	7.019692	6.873956
9	8.566017	8.360517	8.162237	7.970866	7.786109	7.607687
10	9.471304	9.222184	8.982585	8.752064	8.530203	8.316605
11	10.367628	10.071117	9.786848	9.514209	9.252624	9.001551
12	11.255077	10.907504	10.575341	10.257765	9.954004	9.663334
13	12.133739	11.731531	11.348374	10.983185	10.634955	10.302738
14	13.003702	12.543380	12.106249	11.690912	11.296073	10.920520
15	13.865051	13.343232	12.849264	12.381378	11.937935	11.517411
16	14.717872	14.131263	13.577709	13.055003	12.561102	12.094117
17	15.562249	14.907648	14.291872	13.712198	13.166118	12.651321
18	16.398266	15.672560	14.992031	14.353364	13.753513	13.189682
19	17.226006	16.426167	15.678462	14.978891	14.323799	13.709837
20	18.045550	17.168638	16.351433	15.589162	14.877475	14.212403
21	18.856980	17.900136	17.011209	16.184549	15.415024	14.697974
22	19.660376	18.620823	17.658048	16.765413	15.936917	15.167125
23	20.455818	19.330860	18.292204	17.332110	16.443608	15.620410
24	21.243384	20.030404	18.913926	17.884986	16.935542	16.058368
25	22.023152	20.719610	19.523456	18.424376	17.413148	16.481515
26	22.795200	21.398630	20.121036	18.950611	17.876842	16.890352
27	23.559604	22.067616	20.706898	19.464011	18.327031	17.285365
28	24.316440	22.726715	21.281272	19.964889	18.764108	17.667019
29	25.065782	23.376074	21.844385	20.453550	19.188455	18.035767
30	25.807705	24.015836	22.396456	20.930293	19.600441	18.392045
31	26.542282	24.646144	22.937702	21.395407	20.000428	18.736276
32	27.269586	25.267138	23.468335	21.849178	20.388766	19.068865
33	27.989689	25.878954	23.988564	22.291881	20.765792	19.390208
34	28.702662	26.481728	24.498592	22.723786	21.131837	19.700684
35	29.408576	27.075594	24.998619	23.145157	21.487220	20.000661
36	30.107501	27.660684	25.488842	23.556251	21.832252	20.290494
37	30.799506	28.237127	25.969453	23.957318	22.167235	20.570525
38	31.484659	28.805051	26.440641	24.348603	22.492462	20.841087
39	32.163029	29.364582	26.902589	24.730344	22.808215	21.102500
40	32.834682	29.915844	27.355479	25.102775	23.114772	21.355072
41	33.499685	30.458960	27.799489	25.466122	23.412400	21.599104
42	34.158104	30.994049	28.234794	25.820607	23.701359	21.834883
43	34.810004	31.521231	28.661562	26.166446	23.981902	22.062689
44	35.455449	32.040622	29.079963	26.503849	24.254274	22.282791
45	36.094504	32.552337	29.490160	26.833024	24.518713	22.495450
46	36.727232	33.056490	29.892314	27.154170	24.775449	22.700918
47	37.353695	33.553192	30.286582	27.467483	25.024708	22.899438
48	37.973955	34.042554	30.673120	27.773154	25.266707	23.091244
49	38.588074	34.524684	31.052078	28.071369	25.501657	23.276564
50	39.196113	34.999689	31.423606	28.362312	25.729764	23.455618

TABLE VIII.

Showing the *Present Value* of £1 *per Annum* for any number of years not exceeding 100.

Years.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
1	.961538	.956938	.952381	.943396	.934579	.925926
2	1.886095	1.872668	1.859410	1.833393	1.808018	1.783265
3	2.775091	2.748964	2.723248	2.673012	2.624316	2.577097
4	3.629895	3.587526	3.545951	3.465106	3.387211	3.312127
5	4.451822	4.389977	4.329477	4.212364	4.100197	3.992710
6	5.242137	5.157872	5.075692	4.917324	4.766540	4.622880
7	6.002055	5.892701	5.786373	5.582381	5.389289	5.206370
8	6.732745	6.595886	6.463213	6.209794	5.971299	5.746639
9	7.435332	7.268790	7.107822	6.801692	6.515232	6.246888
10	8.110896	7.912718	7.721735	7.360087	7.023582	6.710081
11	8.760477	8.528917	8.306414	7.886875	7.498674	7.138964
12	9.385074	9.118581	8.863252	8.383844	7.942686	7.536078
13	9.985648	9.682852	9.393573	8.852683	8.357651	7.903776
14	10.563123	10.222825	9.898641	9.294984	8.745468	8.244237
15	11.118387	10.739546	10.379658	9.712249	9.107914	8.559479
16	11.652296	11.234015	10.837770	10.105895	9.446649	8.851369
17	12.165669	11.707191	11.274066	10.477260	9.763223	9.121638
18	12.659297	12.159992	11.689587	10.827603	10.059087	9.371887
19	13.133939	12.593294	12.085321	11.158116	10.335595	9.603599
20	13.590326	13.007936	12.462210	11.469921	10.594014	9.818147
21	14.029160	13.404724	12.821153	11.764077	10.835527	10.016803
22	14.451115	13.784425	13.163003	12.041582	11.061241	10.200744
23	14.856842	14.147775	13.488574	12.303379	11.272187	10.371059
24	15.246963	14.495478	13.798642	12.550358	11.469334	10.528758
25	15.622080	14.828209	14.093945	12.783356	11.653583	10.674776
26	15.982769	15.146611	14.375185	13.003166	11.825779	10.809978
27	16.329586	15.451303	14.643034	13.210534	11.986709	10.935165
28	16.663063	15.742874	14.898127	13.406164	12.137111	11.051078
29	16.983715	16.021889	15.141074	13.590721	12.277674	11.158406
30	17.292033	16.288889	15.372451	13.764831	12.409041	11.257783
31	17.588494	16.544391	15.592811	13.929086	12.531814	11.349799
32	17.873552	16.788891	15.802677	14.084043	12.646555	11.434999
33	18.147646	17.022862	16.002549	14.230230	12.753790	11.513888
34	18.411198	17.246758	16.192904	14.368141	12.854009	11.586934
35	18.664613	17.461012	16.374194	14.498246	12.947672	11.654568
36	18.908282	17.666041	16.546852	14.620987	13.035208	11.717193
37	19.142579	17.862240	16.711287	14.736780	13.117017	11.775179
38	19.367864	18.049990	16.867893	14.846019	13.193473	11.828869
39	19.584485	18.229656	17.017041	14.949075	13.264928	11.878582
40	19.792774	18.401584	17.159086	15.046297	13.331709	11.924613
41	19.993052	18.566109	17.294368	15.138016	13.394120	11.967235
42	20.185627	18.723550	17.423208	15.224543	13.452449	12.006699
43	20.370795	18.874210	17.545912	15.306173	13.506962	12.043240
44	20.548841	19.018383	17.662773	15.383182	13.557908	12.077074
45	20.720040	19.156347	17.774070	15.455832	13.605522	12.108402
46	20.884654	19.288371	17.880067	15.524370	13.650020	12.137409
47	21.042936	19.414709	17.981016	15.589028	13.691608	12.164267
48	21.195131	19.535607	18.077158	15.650027	13.730474	12.189136
49	21.341472	19.651298	18.168722	15.707572	13.766799	12.212163
50	21.482185	19.762008	18.255925	15.761861	13.800746	12.233485

TABLE VIII.

Showing the *Present Value* of £1 *per Annum* for any number of years not exceeding 100.

Years.	1 per Cent.	1½ per Cent.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.
51	39.798132	35.467674	31.787849	28.646158	25.951227	23.628616
52	40.394190	35.928743	32.144950	28.923081	26.166240	23.795765
53	40.984346	36.382998	32.495049	29.193250	26.374990	23.957260
54	41.568659	36.830540	32.838283	29.456829	26.577660	24.113295
55	42.147187	37.271468	33.174788	29.713979	26.774428	24.264053
56	42.719987	37.705880	33.504694	29.964858	26.965464	24.409713
57	43.287116	38.133872	33.828131	30.209617	27.150936	24.550448
58	43.848630	38.555533	34.145227	30.448407	27.331005	24.686423
59	44.404584	38.970968	34.456104	30.681373	27.505831	24.817800
60	44.955034	39.380264	34.760887	30.908656	27.675564	24.944734
61	45.500034	39.783511	35.059693	31.130397	27.840353	25.067376
62	46.039638	40.180799	35.352640	31.346728	28.000343	25.185870
63	46.573899	40.572216	35.639843	31.557784	28.155673	25.300358
64	47.102870	40.957848	35.921415	31.763691	28.306478	25.410974
65	47.626604	41.337781	36.197466	31.964577	28.452891	25.517849
66	48.145152	41.712099	36.468104	32.160563	28.595040	25.621110
67	48.658566	42.080886	36.733435	32.351769	28.733049	25.720880
68	49.166897	42.444223	36.993564	32.538311	28.867038	25.817275
69	49.670195	42.802190	37.248592	32.720303	28.997124	25.910411
70	50.168510	43.154867	37.498619	32.897857	29.123421	26.000397
71	50.661891	43.502332	37.743744	33.071080	29.246040	26.087340
72	51.150387	43.844662	37.984063	33.240078	29.365087	26.171343
73	51.634046	44.181933	38.219670	33.404954	29.480667	26.252505
74	52.112917	44.514220	38.450657	33.565809	29.592881	26.330923
75	52.587047	44.841596	38.677114	33.722740	29.701826	26.406689
76	53.056482	45.164134	38.899132	33.875844	29.807598	26.479892
77	53.521269	45.481905	39.116796	34.025214	29.910290	26.550621
78	53.981454	45.794980	39.330192	34.170940	30.009990	26.618957
79	54.437083	46.103429	39.539404	34.313113	30.106786	26.684983
80	54.888201	46.407319	39.744514	34.451817	30.200763	26.748776
81	55.334852	46.706718	39.945602	34.587139	30.292003	26.810411
82	55.777081	47.001693	40.142747	34.719160	30.380586	26.869963
83	56.214932	47.292309	40.336026	34.847961	30.466588	26.927500
84	56.648448	47.578630	40.525516	34.973620	30.550086	26.983092
85	57.077671	47.860719	40.711290	35.096215	30.631151	27.036804
86	57.502644	48.138639	40.893422	35.215819	30.709855	27.088699
87	57.923410	48.412452	41.071982	35.332507	30.786267	27.138840
88	58.340010	48.682219	41.247041	35.446348	30.860454	27.187285
89	58.752485	48.947999	41.418668	35.557413	30.932479	27.234092
90	59.160876	49.209851	41.586929	35.665768	31.002407	27.279316
91	59.565224	49.467834	41.751891	35.771481	31.070298	27.323010
92	59.965568	49.722004	41.913619	35.874616	31.136212	27.365227
93	60.361948	49.972418	42.072175	35.975235	31.200206	27.406017
94	60.754404	50.219131	42.227623	36.073400	31 262336	27.445427
95	61.142974	50.462198	42.380023	36.169171	31.322656	27.483504
96	61.527697	50.701673	42.529434	36.262606	31.381219	27.520294
97	61.908611	50.937609	42.675916	36.353762	31.438077	27.555839
98	62.285753	51.170058	42.819525	36.442694	31.493279	27.590183
99	62.659161	51.399072	42.960319	36.529458	31.546872	27.623365
100	63.028872	51.624701	43.098352	36.614105	31.598905	27.655425
Perp.	100.000000	66.666667	50.000000	40.000000	33.333333	28.571429

TABLE VIII.

Showing the *Present Value* of £1 *per Annum* for any number of years not exceeding 100.

Years.	4 per Cent.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
51	21.617485	19.867950	18.338977	15.813076	13.832473	12.253227
52	21.747582	19.969330	18.418073	15.861393	13.862124	12.271506
53	21.872675	20.066345	18.493403	15.906974	13.889836	12.288432
54	21.992957	20.159181	18.565146	15.949976	13.915735	12.304103
55	22.108612	20.248021	18.633472	15.990543	13.939939	12.318614
56	22.219819	20.333034	18.698545	16.028814	13.962560	12.332050
57	22.326749	20.414387	18.760519	16.064919	13.983701	12.344491
58	22.429567	20.492236	18.819542	16.098980	14.003459	12.356010
59	22.528430	20.566733	18.875754	16.131113	14.021924	12.366676
60	22.623490	20.638022	18.929290	16.161428	14.039181	12.376552
61	22.714894	20.706241	18.980276	16.190026	14.055309	12.385696
62	22.802783	20.771523	19.028834	16.217006	14.070383	12.394163
63	22.887291	20.833993	19.075080	16.242458	14.084470	12.402003
64	22.968549	29.893773	19.119124	16.266470	14.097635	12.409262
65	23.046682	20.950979	19.161070	16.289123	14.109940	12.415983
66	23.121810	21.005722	19.201019	16.310493	14.121439	12.422207
67	23.194048	21.058107	19.239066	16.330654	14.132186	12.427969
68	23.263507	21.108236	19.275301	16.349673	14.142230	12.433305
69	23.330296	21.156207	19.309810	16.367617	14.151617	12.438245
70	23.394515	21.202112	19.342677	16.384544	14.160389	12.442820
71	23.456264	21.246040	19.373978	16.400513	14.168588	12.447055
72	23.515639	21.288077	19.403788	16.415578	14.176251	12.450977
73	23.572730	21.328303	19.432179	16.429791	14.183412	12.454608
74	23.627625	21.366797	19.459218	16.443199	14.190104	12.457971
75	23.680408	21.403634	19.484970	16.455848	14.196359	12.461084
76	23.731162	21.438884	19.509495	16.467781	14.202205	12.463967
77	23.779963	21.472616	19.532853	10.479039	14.207668	12.466636
78	23.826888	21.504896	19.555098	16.489659	14.212774	12.469107
79	23.872008	21.535785	19.576284	16.499679	14.217546	12.471396
80	23.915392	21.565345	19.596460	16.509131	14.222005	12.473514
81	23.957108	21.593632	19.615677	16.518048	14.226173	12.475476
82	23.997219	21.620700	19.633978	16.526460	14.230069	12.477293
83	24.035787	21.646603	19.651407	16.534396	14.233709	12.478975
84	24.072872	21.671390	19.668007	16.541883	14.237111	12.480532
85	24.108531	21.695110	19.683816	16.548947	14.240291	12.481974
86	24.142818	21.717809	19.698873	16.555610	14.243262	12.483310
87	24.175787	21.739530	19.713212	16.561896	14.246040	12.484546
88	24.207487	21.760316	19.726869	16.567827	14.248635	12.485691
89	24.237969	21.780207	19.739875	16.573421	14.251061	12.486751
90	24.267278	21.799241	19.752262	16.578699	14.253328	12.487732
91	24.295459	21.817455	19.764059	16.583679	14.255447	12.488641
92	24.322557	21.834885	19.775294	16.588376	14.257427	12.489482
93	24.348612	21.851565	19.785994	16.592808	14.259277	12.490261
94	24.373666	21.867526	19.796185	16.596988	14.261007	12.490983
95	24.397756	21.882800	19.805891	16.600932	14.262623	12.491651
96	24.420919	21.897417	19.815134	16.604653	14.264134	12.492269
97	24.443191	21.911403	19.823937	16.608163	14.265546	12.492842
98	24.464607	21.924788	19.832321	16.611475	14.266865	12.493372
99	24.485199	21.937596	19.840306	16.614599	14.268098	12.493863
100	24.504999	21.949853	19.847910	16.617546	14.269251	12.494318
Perp.	25.000000	22.222222	20.000000	16.666667	14.285714	12.500000

TABLE IX.

Showing the *Logarithm* of the Present Value of £1 due at the end of any number of years not exceeding 100.

Years.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.	4 per Cent.
1	$\bar{1}$.9913998	$\bar{1}$.9892761	$\bar{1}$.9871628	$\bar{1}$.9850596	$\bar{1}$.9829667
2	.9827997	.9785523	.9743256	.9701193	.9659333
3	.9741995	.9678284	.9614884	.9551789	.9489000
4	.9655993	.9571045	.9486512	.9402386	.9318666
5	.9569991	.9463806	.9358139	.9252983	.9148333
6	.9483990	.9356568	.9229767	.9103579	.8978000
7	.9397988	.9249329	.9101395	.8954176	.8807666
8	.9311986	.9142090	.8973023	.8804772	.8637333
9	.9225985	.9034852	.8844650	.8655369	.8467000
10	.9139983	.8927613	.8716278	.8505965	.8296667
11	.9053981	.8820374	.8587906	.8356562	.8126333
12	.8967979	.8713136	.8459534	.8207158	.7956000
13	.8881978	.8605897	.8331161	.8057755	.7785667
14	.8795976	.8498658	.8202789	.7908351	.7615333
15	.8709974	.8391420	.8074417	.7758948	.7445000
16	.8623973	.8284181	.7946045	.7609544	.7274667
17	.8537971	.8176942	.7817672	.7460141	.7104333
18	.8451969	.8069704	.7689300	.7310737	.6934000
19	.8365968	.7962465	.7560928	.7161334	.6763667
20	.8279966	.7855227	.7432556	.7011930	.6593333
21	.8193964	.7747988	.7304183	.6862527	.6422999
22	.8107962	.7640749	.7175811	.6713123	.6252666
23	.8021961	.7533511	.7047439	.6563720	.6082332
24	.7935959	.7426272	.6919067	.6414316	.5911999
25	.7849957	.7319033	.6790694	.6264913	.5741666
26	.7763955	.7211795	.6662322	.6115509	.5571333
27	.7677954	.7104556	.6533950	.5966106	.5401000
28	.7591952	.6997317	.6405578	.5816702	.5230667
29	.7505950	.6890079	.6277205	.5667299	.5060333
30	.7419949	.6782840	.6148833	.5517895	.4890000
31	.7333947	.6675601	.6020461	.5368492	.4719667
32	.7247945	.6568363	.5892089	.5219088	.4549333
33	.7161944	.6461124	.5763716	.5069685	.4379000
34	.7075942	.6353885	.5635344	.4920281	.4208667
35	.6989940	.6246647	.5506972	.4770878	.4038333
36	.6903938	.6139408	.5378600	.4621474	.3868000
37	.6817937	.6032169	.5250227	.4472071	.3697667
38	.6731935	.5924931	.5121855	.4322667	.3527333
39	.6645933	.5817692	.4993483	.4173264	.3357000
40	.6559932	.5710454	.4865111	.4023860	.3186667
41	.6473930	.5603215	.4736738	.3874457	.3016333
42	.6387928	.5495976	.4608366	.3725053	.2846000
43	.6301927	.5388738	.4479994	.3575650	.2675667
44	.6215925	.5281499	.4351622	.3426246	.2505333
45	.6129923	.5174260	.4223249	.3276843	.2335000
46	.6043921	.5067021	.4094877	.3127439	.2164667
47	.5957920	.4959783	.3966505	.2978036	.1994333
48	.5871918	.4852544	.3838133	.2828632	.1824000
49	.5785916	.4745305	.3709760	.2679229	.1653667
50	.5699914	.4638067	.3581388	.2529825	.1483333

TABLE IX.

Showing the *Logarithm* of the Present Value of £1 due at the end of any number of years not exceeding 100.

Years.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
1	$\bar{1}$.9808837	$\bar{1}$.9788107	$\bar{1}$.9746941	$\bar{1}$.9706162	$\bar{1}$.9665762
2	.9617674	.9576214	.9493882	.9412324	.9331524
3	.9426511	.9364321	.9240824	.9118487	.8997287
4	.9235348	.9152428	.8987765	.8824649	.8663049
5	.9044185	.8940535	.8734706	.8530811	.8328812
6	.8853022	.8728642	.8481648	.8236973	.7994574
7	.8661859	.8516749	.8228589	.7943135	.7660337
8	.8470696	.8304856	.7975530	.7649298	.7326099
9	.8279533	.8092963	.7722472	.7355460	.6991862
10	.8088371	.7881070	.7469413	.7061622	.6657624
11	.7897208	.7669177	.7216354	.6767784	.6323387
12	.7706045	.7457284	.6963296	.6473946	.5989149
13	.7514882	.7245391	.6710237	.6180109	.5654912
14	.7323719	.7033498	.6457178	.5886271	.5320674
15	.7132556	.6821605	.6204120	.5592433	.4986437
16	.6941393	.6609712	.5951061	.5298595	.4652199
17	.6750230	.6397819	.5698002	.5004757	.4317961
18	.6559067	.6185926	.5444943	.4710920	.3983724
19	.6367904	.5974033	.5191885	.4417082	.3649486
20	.6176742	.5762140	.4938826	.4123244	.3315249
21	.5985579	.5550247	.4685767	.3829406	.2981011
22	.5794416	.5338354	.4432709	.3535568	.2646774
23	.5603253	.5126461	.4179650	.3241731	.2312536
24	.5412090	.4914568	.3926591	.2947893	.1978299
25	.5220927	.4702675	.3673533	.2654055	.1644061
26	.5029764	.4490782	.3420474	.2360217	.1309824
27	.4838601	.4278889	.3167415	.2066379	.0975586
28	.4647438	.4066996	.2914357	.1772542	.0641349
29	.4456275	.3855103	.2661298	.1478704	$\bar{}$.0307111
30	.4265113	.3643210	.2408239	.1184866	$\bar{2}$.9972874
31	.4073950	.3431317	.2155181	.0891028	.9638636
32	.3882787	.3219424	.1902122	.0597190	.9304399
33	.3691624	.3007531	.1649063	.0303353	.8970161
34	.3500461	.2795638	.1396005	$\bar{}$.0009515	.8635923
35	.3309298	.2583745	.1142946	$\bar{2}$.9715677	.8301685
36	.3118135	.2371852	.0889887	.9421839	.7967448
37	.2926972	.2159959	.0636829	.9128001	.7633210
38	.2735809	.1948066	.0383770	.8834164	.7298973
39	.2544646	.1736173	$\bar{}$.0130711	.8540326	.6964735
40	.2353484	.1524280	$\bar{2}$.9877653	.8246488	.6630498
41	.2162321	.1312387	.9624594	.7952650	.6296260
42	.1971158	.1100495	.9371535	.7658812	.5962023
43	.1779995	.0888602	.9118477	.7364975	.5627785
44	.1588832	.0676709	.8865418	.7071137	.5293548
45	.1397669	.0464816	.8612359	.6777299	.4959310
46	.1206506	.0252923	.8359301	.6483461	.4625073
47	.1015343	$\bar{}$.0041030	.8106242	.6189623	.4290835
48	.0824180	$\bar{2}$.9829137	.7853183	.5895786	.3956598
49	.0633017	.9617244	.7600125	.5601948	.3622360
50	.0441855	.9405352	.7347066	.5308110	.3288122

TABLE IX.

Showing the *Logarithm* of the Present Value of £1 due at the end of any number of years not exceeding 100.

Years.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.	4 per Cent.
51	$\bar{1}$.5613912	$\bar{1}$.4530828	$\bar{1}$.3453016	$\bar{1}$.2380422	$\bar{1}$.1313000
52	.5527910	.4423590	.3324644	.2231018	.1142667
53	.5441909	.4316351	.3196271	.2081615	.0972333
54	.5355907	.4209112	.3067899	.1932211	.0802000
55	.5269905	.4101874	.2939527	.1782808	.0631667
56	.5183904	.3994635	.2811155	.1633404	.0461333
57	.5097902	.3887396	.2682782	.1484001	.0291000
58	.5011900	.3780158	.2554410	.1334597	$\bar{1}$.0120667
59	.4925898	.3672919	.2426038	.1185194	$\bar{2}$.9950333
60	.4839897	.3565681	.2297666	.1035790	.9780000
61	.4753895	.3458442	.2169293	.0886387	.9609667
62	.4667893	.3351203	.2040921	.0736983	.9439333
63	.4581892	.3243965	.1912549	.0587580	.9269000
64	.4495890	.3136726	.1784177	.0438176	.9098667
65	.4409888	.3029487	.1655804	.0288773	.8928333
66	.4323887	.2922249	.1527432	$\bar{1}$.0139369	.8758000
67	.4237885	.2815010	.1399060	$\bar{2}$.9989966	.8587667
68	.4151883	.2707771	.1270688	.9840562	.8417333
69	.4065882	.2600533	.1142315	.9691159	.8247000
70	.3979880	.2493294	.1013943	.9541755	.8076667
71	.3893878	.2386055	.0885571	.9392352	.7906333
72	.3807876	.2278817	.0757199	.9242948	.7736000
73	.3721874	.2171578	.0628826	.9093545	.7565667
74	.3635872	.2064339	.0500454	.8944141	.7395333
75	.3549870	.1957101	.0372082	.8794738	.7225000
76	.3463869	.1849862	.0243710	.8645334	.7054667
77	.3377867	.1742623	$\bar{1}$.0115337	.8495931	.6884333
78	.3291865	.1635385	$\bar{2}$.9986965	.8346527	.6714000
79	.3205864	.1528146	.9858593	.8197124	.6543667
80	.3119863	.1420908	.9730221	.8047720	.6373333
81	.3033861	.1313669	.9601848	.7898317	.6203000
82	.2947859	.1206430	.9473476	.7748913	.6032667
83	.2861858	.1099192	.9345104	.7599510	.5862333
84	.2775856	.0991953	.9216732	.7450106	.5692000
85	.2689854	.0884714	.9088359	.7300703	.5521667
86	.2603853	.0777476	.8959987	.7151299	.5351333
87	.2517851	.0670237	.8831615	.7001896	.5181000
88	.2431850	.0562998	.8703243	.6852492	.5010667
89	.2345848	.0455760	.8574870	.6703089	.4840333
90	.2259846	.0348521	.8446498	.6553685	.4670000
91	.2173844	.0241282	.8318126	.6404282	.4499667
92	.2087843	.0134044	.8189754	.6254878	.4329333
93	.2001841	$\bar{2}$.0026805	.8061381	.6105475	.4159000
94	.1915839	$\bar{2}$.9919567	.7933009	.5956071	.3988667
95	.1829838	.9812328	.7804637	.5806668	.3818333
96	.1743836	.9705090	.7676265	.5657265	.3648000
97	.1657834	.9597851	.7547892	.5507861	.3477667
98	.1571832	.9490612	.7419520	.5358458	.3307333
99	.1485831	.9383374	.7291148	.5209054	.3137000
100	.1399829	.9276135	.7162775	.5059650	.2966667

TABLE IX.

Showing the *Logarithm* of the Present Value of £1 due at the end of any number of years not exceeding 100.

Years.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
51	$\overline{1}$.0250692	$\overline{2}$.9193459	$\overline{2}$.7094007	$\overline{2}$.5014272	$\overline{2}$.2953884
52	$\overline{1}$.0059529	.8981566	.6840949	.4720434	.2619646
53	$\overline{2}$.9868366	.8769673	.6587890	.4426597	.2285409
54	.9677203	.8557780	.6334831	.4132759	.1951171
55	.9486040	.8345887	.6081773	.3838921	.1616934
56	.9294877	.8133994	.5828714	.3545083	.1282696
57	.9103714	.7922101	.5575655	.3251245	.0948459
58	.8912551	.7710208	.5322597	.2957408	.0614221
59	.8721388	.7498315	.5069538	.2663570	$\overline{1}$.0279984
60	.8530226	.7286422	.4816479	.2369732	$\overline{3}$.9945746
61	.8339063	.7074529	.4563421	.2075894	.9611509
62	.8147900	.6862636	.4310362	.1782057	.9277271
63	.7956737	.6650743	.4057303	.1488219	.8943033
64	.7765574	.6438850	.3804245	.1194381	.8608796
65	.7574411	.6226957	.3551186	.0900543	.8274558
66	.7383248	.6015064	.3298127	.0606705	.7940321
67	.7192085	.5803171	.3045069	.0312868	.7606083
68	.7000922	.5591278	.2792010	$\overline{1}$.0019030	.7271846
69	.6809759	.5379385	.2538951	$\overline{3}$.9725192	.6937608
70	.6618596	.5167492	.2285893	.9431354	.6603371
71	.6427433	.4955599	.2032834	.9137516	.6269133
72	.6236271	.4743706	.1779775	.8843679	.5934896
73	.6045108	.4531813	.1526717	.8549841	.5600658
74	.5853945	.4319920	.1273658	.8256003	.5266421
75	.5662782	.4108027	.1020599	.7962165	.4932183
76	.5471619	.3896134	.0767541	.7668327	.4597946
77	.5280456	.3684241	.0514482	.7374490	.4263708
78	.5089293	.3472348	.0261423	.7080652	.3929470
79	.4898130	.3260455	$\overline{1}$.0008365	.6786814	.3595232
80	.4706968	.3048562	$\overline{3}$.9755306	.6492976	.3260995
81	.4515805	.2836669	.9502247	.6199138	.2926757
82	.4324642	.2624776	.9249189	.5905301	.2592520
83	.4133479	.2412883	.8996130	.5611463	.2258282
84	.3942316	.2200990	.8743071	.5317625	.1924045
85	.3751153	.1989097	.8490013	.5023787	.1589807
86	.3559990	.1777204	.8236954	.4729949	.1255570
87	.3368827	.1565311	.7983895	.4436112	.0921332
88	.3177664	.1353418	.7730837	.4142274	.0587095
89	.2986501	.1141525	.7477778	.3848436	$\overline{1}$.0252857
90	.2795339	.0929632	.7224719	.3554598	$\overline{4}$.9918620
91	.2604176	.0717739	.6971661	.3260761	.9583382
92	.2413013	.0505846	.6718602	.2966923	.9249144
93	.2221850	.0293953	.6465543	.2673085	.8914907
94	.2030687	.0082060	.6212485	.2379247	.8580669
95	.1839524	$\overline{3}$.9870167	.5959426	.2085409	.8246432
96	.1648361	.9658274	.5706367	.1791572	.7912194
97	.1457198	.9446381	.5453309	.1497734	.7577957
98	.1266035	.9234488	.5200250	.1203896	.7243719
99	.1074872	.9022595	.4947191	.0910058	.6909482
100	.0883710	.8810702	.4694133	.0616221	.6575244

TABLE X.

Showing the Rate of Mortality among the Members of the Equitable Society, from the year 1768 to 1825.—*See page* 91.

Age.	Number Living.	Decrement.	Age.	Number Living.	Decrement.
10	2844	11	54	1785	41
11	2833	11	55	1744	42
12	2822	12	56	1702	43
13	2810	12	57	1659	44
14	2798	13	58	1615	45
15	2785	14	59	1570	46
16	2771	15	60	1524	46
17	2756	16	61	1478	46
18	2740	17	62	1432	47
19	2723	18	63	1385	48
20	2705	18	64	1337	49
21	2687	18	65	1288	50
22	2669	19	66	1238	51
23	2650	19	67	1187	52
24	2631	20	68	1135	53
25	2611	20	69	1082	54
26	2591	21	70	1028	54
27	2570	22	71	974	55
28	2548	23	72	919	55
29	2525	24	73	864	56
30	2501	24	74	808	56
31	2477	25	75	752	55
32	2452	26	76	697	55
33	2426	26	77	642	54
34	2400	26	78	588	54
35	2374	27	79	534	54
36	2347	27	80	480	54
37	2320	28	81	426	53
38	2292	28	82	373	52
39	2264	28	83	321	50
40	2236	28	84	271	47
41	2208	28	85	224	43
42	2180	28	86	181	38
43	2152	29	87	143	32
44	2123	30	88	111	26
45	2093	30	89	85	20
46	2063	30	90	65	16
47	2033	31	91	49	13
48	2002	32	92	36	11
49	1970	33	93	25	9
50	1937	35	94	16	7
51	1902	37	95	9	5
52	1865	39	96	4	3
53	1826	41	97	1	1

TABLE XI.

Showing the Logarithm, and Arithmetical Complement, of the *Number Living* at each age in Table X.

Age.	Logarithm λa.	Co-Logarithm $\varkappa a$.	Age.	Logarithm λa.	Co-Logarithm $\varkappa a$.
10	3.4539296	$\bar{4}$.5460704	54	3.2516382	$\bar{4}$.7483618
11	.4522466	.5477534	55	.2415465	.7584534
12	.4505570	.5494430	56	.2309596	.7690404
13	.4487063	.5512937	57	.2198464	.7801536
14	.4468477	.5531523	58	.2081725	.7918275
15	.4448252	.5551748	59	.1958997	.8041003
16	.4426365	.5573635	60	.1829850	.8170150
17	.4402792	.5597208	61	.1696744	.8303256
18	.4377506	.5622494	62	.1559430	.8440570
19	.4350476	.5649524	63	.1414498	.8585502
20	.4321673	.5678327	64	.1261314	.8738686
21	.4292677	.5707323	65	.1099159	.8900841
22	.4263486	.5736514	66	.0927206	.9072794
23	.4232459	.5767541	67	.0744507	.9255493
24	.4201208	.5798792	68	.0549959	.9450041
25	.4168069	.5831931	69	.0342273	.9657727
26	.4134674	.5865326	70	.0119931	.9880069
27	.4099331	.5900669	71	2.9885590	$\bar{3}$.0114410
28	.4061994	.5938006	72	.9633155	.0366845
29	.4022614	.5977386	73	.9365137	.0634863
30	.3981137	.6018863	74	.9074114	.0925886
31	.3939260	.6060740	75	.8762178	.1237822
32	.3895205	.6104795	76	.8432328	.1567672
33	.3848908	.6151092	77	.8075350	.1924650
34	.3802112	.6197888	78	.7693773	.2306227
35	.3754807	.6245193	79	.7275413	.2724587
36	.3705131	.6294869	80	.6812412	.3187588
37	.3654880	.6345120	81	.6294096	.3705904
38	.3602146	.6397854	82	.5717088	.4282912
39	.3548764	.6451236	83	.5065050	.4934950
40	.3494718	.6505282	84	.4329693	.5670307
41	.3439991	.6560009	85	.3502480	.6497520
42	.3384565	.6615435	86	.2576786	.7423214
43	.3328423	.6671577	87	.1553360	.8446640
44	.3269500	.6730500	88	.0453230	.9546770
45	.3207692	.6792308	89	1.9294189	$\bar{2}$.0705811
46	.3144992	.6855008	90	.8129134	.1870866
47	.3081374	.6918626	91	.6901961	.3098039
48	.3014641	.6985359	92	.5563025	.4436975
49	.2944662	.7055338	93	.3979400	.6020600
50	.2871296	.7128704	94	.2041200	.7958800
51	.2792105	.7207895	95	0.9542425	$\bar{1}$.0457575
52	.2706788	.7293212	96	.6020600	.3979400
53	.2615008	.7384992	97	.0000000	.0000000

TABLE XII.

Showing, out of the number of persons which entered upon each age, the proportion which died off during the year, and the proportion which survived that period, with its reciprocal, according to the experience of the Equitable.—*See pages* 104—5.

Age.	Died. $1-{}_1a$	Survived. ${}_1a$	Reciprocal. $\frac{1}{{}_1a}$	Age.	Died. $1-{}_1a$	Survived. ${}_1a$	Reciprocal. $\frac{1}{{}_1a}$
10	.003868	.996132	1.00388	54	.022969	.977031	1.02351
11	.003882	.996117	1.00390	55	.024083	.975917	1.02467
12	.004252	.995748	1.00427	56	.025264	.974736	1.02592
13	.004270	.995730	1.00429	57	.026522	.973478	1.02724
14	.004646	.995354	1.00467	58	.027864	.972136	1 02866
15	.005027	.994973	1.00505	59	.029299	.970701	1.03018
16	.005413	.994587	1.00544	60	.030184	.969816	1.03112
17	.005805	.994195	1.00584	61	.031123	.968877	1.03212
18	.006205	.993795	1.00624	62	.032821	.967179	1.03393
19	.006610	.993390	1.00666	63	.034657	.965343	1.03590
20	.006654	.993346	1.00670	64	.036649	.963351	1.03804
21	.006699	.993301	1.00675	65	.038820	.961180	1.04039
22	.007119	.992881	1.00717	66	.041195	.958805	1.04296
23	.007170	.992830	1.00722	67	.043807	.956193	1 04582
24	.007602	.992398	1.00766	68	.046696	.953304	1.04898
25	.007660	.992340	1.00772	69	.049908	.950092	1.05253
26	.008105	.991895	1.00817	70	.052529	.947471	1.05544
27	.008560	.991440	1.00864	71	.056468	.943532	1.05985
28	.009027	.990973	1.00911	72	.059848	.940152	1.06366
29	.009505	.990495	1.00960	73	.064815	.935185	1.06931
30	.009596	.990404	1.00969	74	.069307	.930693	1.07447
31	.010092	.989908	1.01020	75	.073117	.926883	1.07888
32	.010604	.989396	1.01072	76	.078910	.921090	1.08567
33	.010717	.989283	1.01083	77	.084112	.915888	1.09184
34	.010833	.989167	1.01095	78	.091837	.908163	1.10112
35	.011373	.988627	1.01150	79	.101124	.898876	1.11250
36	.011504	.988496	1.01164	80	.112500	.887500	1.12676
37	.012069	.987931	1.01222	81	.124423	.875577	1.14209
38	.012216	.987784	1.01237	82	.139410	.860590	1.16199
39	.012367	.987633	1.01252	83	.155763	.844237	1.18450
40	.012522	.987478	1.01268	84	.173432	.826568	1.20982
41	.012681	.987319	1.01284	85	.191964	.808036	1.23757
42	.012844	.987156	1.01301	86	.209945	.790055	1.26574
43	.013476	.986524	1.01366	87	.223776	.776224	1.28829
44	.014131	.985869	1.01433	88	.234234	.765766	1.30588
45	.014333	.985667	1.01454	89	.235294	.764706	1.30769
46	.014542	.985458	1.01476	90	.246154	.753846	1.32653
47	.015248	.984752	1.01549	91	.265306	.734694	1.36111
48	.015984	.984016	1.01624	92	.305556	.694444	1.44000
49	.016751	.983249	1.01704	93	.360000	.640000	1.56250
50	.018069	.981931	1.01840	94	.437500	.562500	1 77778
51	.019453	.980547	1.01984	95	.555556	.444444	2 25000
52	.020911	.979089	1.02136	96	.750000	.250000	4 00000
53	.022454	.977546	1.02297	97	1.00000	.000000	infinite.

TABLE XIII.

Showing the Logarithm and Arithmetical Complement of the proportion of the number living which *Survived* one year at each age as represented by Table XII.

Age.	Logarithm $\lambda_x a$	Co-Logarithm. $\mu_x a = \lambda\left(\frac{1}{x a}\right)$	Age.	Logarithm. $\lambda_x a$	Co-Logarithm. $\mu_x a = \lambda\left(\frac{1}{x a}\right)$
10	$\overline{1}$.9983170	0.0016830	54	$\overline{1}$.9899083	0.0100917
11	.9983104	.0016896	55	.9894131	.0105869
12	.9981493	.0018507	56	.9888868	.0111132
13	9981414	.0018586	57	.9883261	.0116739
14	.9979775	.0020225	58	.9877272	.0122728
15	.9978113	.0021887	59	.9870853	.0129147
16	.9976427	.0023573	60	.9866894	.0133106
17	.9974714	.0025286	61	.9862686	.0137314
18	.9972970	.0027030	62	.9855068	.0144932
19	.9971197	.0028803	63	.9846816	.0153184
20	.9971004	.0028996	64	.9837845	.0162155
21	.9970809	.0029191	65	.9828047	.0171953
22	.9968973	.0031027	66	.9817301	.0182699
23	.9968749	.0031251	67	.9805452	.0194548
24	.9966861	.0033139	68	.9792314	.0207686
25	.9966605	.0033395	69	.9777658	.0222342
26	.9964657	.0035343	70	.9765659	.0234341
27	.9962663	.0037337	71	.9747565	.0252435
28	.9960620	.0039380	72	.9731982	.0268018
29	.9958523	.0041477	73	.9708977	.0291023
30	.9958123	.0041877	74	.9688064	.0311936
31	.9955945	.0044055	75	.9670250	.0329750
32	.9953703	.0046297	76	.9643022	.0356978
33	.9953204	.0046796	77	.9618423	.0381577
34	.9952695	.0047305	78	.9581640	.0418360
35	.9950324	.0049676	79	.9536999	.0463001
36	.9949749	.0050251	80	.9481684	.0518316
37	.9947266	.0052734	81	.9422992	.0577008
38	.9946618	.0053382	82	.9347962	.0652038
39	.9945954	.0054046	83	.9264643	.0735357
40	.9945273	.0054727	84	.9172787	.0827213
41	.9944574	.0055426	85	.9074306	.0925694
42	.9943858	.0056142	86	.8976574	.1023426
43	.9941077	.0058923	87	.8899870	.1100130
44	.9938192	.0061808	88	.8840959	.1159041
45	.9937300	.0062700	89	.8834945	.1165055
46	.9936382	.0063618	90	.8772827	.1227173
47	.9933267	.0066733	91	.8661064	.1338936
48	.9930021	.0069979	92	.8416375	.1583625
49	.9926634	.0073366	93	.8061800	.1938200
50	.9920809	.0079191	94	.7501225	.2498775
51	.9914683	.0085317	95	.6478175	.3521825
52	.9908220	.0091780	96	.3979400	.6020600
53	.9901374	.0098626	97	.0000000	- - -

TABLE XIV.

Showing the Rate of Mortality among Two Joint Lives, according to the experience of the Equitable.—*See page* 110.

Younger Age.	Difference between the two Ages.						
	0 Years.	5 Years.	10 Years	15 Years.	20 Years.	25 Years.	30 Years.
10	8088	7920	7693	7425	7113	6751	6359
11	8025	7850	7612	7341	7017	6649	6255
12	7964	7777	7531	7252	6920	6547	6152
13	7896	7699	7447	7159	6817	6441	6048
14	7828	7619	7362	7065	6715	6334	5940
15	7756	7534	7272	6966	6612	6226	5828
16	7679	7446	7180	6863	6503	6118	5716
17	7595	7355	7083	6757	6394	6008	5603
18	7508	7261	6982	6647	6280	5896	5485
19	7414	7165	6876	6535	6165	5780	5364
20	7318	7063	6766	6422	6048	5661	5240
21	7218	6962	6655	6305	5932	5543	5110
22	7122	6859	6544	6192	5818	5426	4977
23	7024	6752	6429	6074	5703	5305	4839
24	6923	6644	6314	5956	5586	5183	4696
25	6818	6531	6198	5838	5465	5057	4553
26	6714	6417	6081	5720	5345	4928	4410
27	6605	6302	5962	5603	5225	4793	4264
28	6492	6181	5840	5483	5101	4652	4115
29	6376	6060	5716	5361	4974	4508	3964
30	6256	5937	5592	5235	4845	4362	3812
31	6135	5813	5468	5109	4711	4216	3660
32	6013	5689	5345	4985	4573	4068	3512
33	5884	5560	5221	4857	4429	3918	3360
34	5760	5434	5095	4728	4284	3768	3209
35	5635	5308	4968	4598	4140	3617	3057
36	5508	5181	4841	4464	3995	3468	2904
37	5382	5058	4717	4327	3849	3323	2754
38	5252	4932	4588	4185	3701	3174	2601
39	5126	4806	4460	4041	3554	3026	2449
40	4999	4680	4330	3899	3408	2879	2299
41	4876	4555	4199	3758	3263	2734	2151
42	4752	4431	4066	3616	3121	2587	2003
43	4631	4308	3930	3476	2981	2443	1859
44	4507	4182	3790	3333	2839	2297	1715
45	4380	4054	3650	3190	2695	2151	1574
46	4256	3924	3511	3048	2554	2009	1438
47	4133	3791	3373	2911	2412	1868	1305
48	4008	3656	3233	2773	2272	1730	1177
49	3881	3517	3093	2633	2132	1592	1052
50	3751	3378	2953	2493	1991	1457	930
51	3618	3237	2811	2354	1853	1326	810
52	3478	3093	2671	2214	1714	1197	696
53	3334	2949	2529	2073	1578	1074	586

TABLE XIV.

Showing the Rate of Mortality among Two Joint Lives, according to the experience of the Equitable.

Younger Age.	Difference between the two Ages.					
	35 Years.	40 Years.	45 Years.	50 Years.	55 Years.	60 Years.
10	5952	5509	4960	4334	3662	2923
11	5844	5389	4822	4186	3507	2759
12	5737	5263	4681	4042	3350	2593
13	5626	5131	4538	3892	3189	2428
14	5511	4994	4392	3740	3026	2261
15	5394	4857	4245	3586	2863	2094
16	5270	4716	4095	3430	2699	1931
17	5140	4572	3946	3271	2533	1769
18	5003	4425	3795	3110	2367	1611
19	4861	4275	3641	2946	2200	1454
20	4718	4123	3484	2781	2034	1298
21	4573	3971	3325	2617	1873	1145
22	4426	3822	3167	2453	1714	996
23	4280	3670	3008	2290	1558	851
24	4131	3517	2846	2126	1405	713
25	3979	3363	2684	1964	1253	585
26	3828	3207	2524	1806	1104	469
27	3680	3051	2362	1650	959	368
28	3528	2892	2202	1498	818	283
29	3377	2732	2040	1348	684	215
30	3221	2571	1881	1201	560	163
31	3065	2413	1726	1055	448	121
32	2910	2253	1574	915	351	88
33	2754	2096	1427	779	269	61
34	2597	1939	1282	650	204	38
35	2439	1785	1140	532	154	21
36	2286	1636	1000	425	115	9
37	2132	1489	865	332	84	2
38	1980	1348	736	254	57	
39	1829	1209	614	192	36	
40	1682	1073	501	145	20	
41	1539	941	400	108	9	
42	1400	813	312	78	2	
43	1265	691	239	54		
44	1134	575	180	34		
45	1005	469	136	19		
46	879	373	101	8		
47	758	291	73	2		
48	643	222	50			
49	534	167	32			
50	434	126	17			
51	344	93	8			
52	267	67	2			
53	203	46				

TABLE XIV.

Showing the Rate of Mortality among Two Joint Lives, according to the experience of the Equitable.

Younger Age.	Difference between the two Ages.								
	0 Years.	5 Years.	10 Years.	15 Years.	20 Years.	25 Years.	30 Years.	35 Years.	40 Years.
54	3186	2803	2386	1930	1442	953	484	152	29
55	3042	2658	2246	1793	1312	837	391	113	16
56	2896	2516	2107	1658	1186	725	308	83	7
57	2751	2376	1968	1525	1065	619	237	60	2
58	2608	2237	1833	1395	950	518	179	40	
59	2465	2099	1699	1269	838	426	133	25	
60	2322	1963	1566	1146	732	341	99	14	
61	2183	1829	1440	1030	630	268	72	6	
62	2051	1699	1316	919	534	205	52	1	
63	1917	1572	1197	814	445	154	35		
64	1787	1446	1080	714	362	114	21		
65	1658	1324	969	618	289	84	12		
66	1533	1206	863	527	224	61	5		
67	1408	1091	762	443	170	43	1		
68	1289	981	667	364	126	28			
69	1170	874	578	293	92	17			
70	1056	773	493	230	67	9			
71	949	679	415	176	48	4			
72	845	590	343	131	33	1			
73	746	508	277	96	22				
74	653	431	219	69	13				
75	566	361	168	49	7				
76	486	297	126	34	3				
77	412	239	92	23	1				
78	346	189	65	15					
79	285	145	45	9					
80	230	108	31	4					
81	181	77	21	2					
82	139	53	13						
83	103	36	8						
84	74	23	4						
85	50	15	2						
86	33	9	1						
87	20	5							
88	12	3							
89	7	1							
90	4	1							
91	2								
92	1								
93	1								
94									
95									
96									
97									

TABLE XV.

Showing the Rate of Mortality among Three Joint Lives of equal ages, according to the experience of the Equitable.—*See page* 115.

Common Age.	Combinations of Joint Lives existing.	Common Age.	Combinations of Joint Lives existing.
10	23001	50	7266
11	22735	51	6881
12	22474	52	6485
13	22188	53	6088
14	21902	54	5686
15	21600	55	5305
16	21278	56	4929
17	20932	57	4564
18	20572	58	4212
19	20188	59	3870
20	19796	60	3538
21	19394	61	3226
22	19008	62	2937
23	18613	63	2655
24	18215	64	2388
25	17802	65	2135
26	17396	66	1898
27	16975	67	1671
28	16541	68	1462
29	16099	69	1266
30	15646	70	1085
31	15196	71	924
32	14744	72	776
33	14275	73	645
34	13824	74	527
35	13377	75	425
36	12927	76	339
37	12487	77	265
38	12037	78	203
39	11604	79	152
40	11177	80	111
41	10765	81	77
42	10359	82	52
43	9966	83	33
44	9569	84	20
45	9167	85	11
46	8780	86	6
47	8402	87	3
48	8024	88	1
49	7646	89	1

TABLE XVI.

Being a preparatory Table for determining the Average Duration of Single Lives, according to the experience of the Equitable.—*See page* 120.

Age.	D	N	Age.	D	N
- -	- -	140299	54	1785	32002
10	2844	137455	55	1744	30258
11	2833	134622	56	1702	28556
12	2822	131800	57	1659	26897
13	2810	128990	58	1615	25282
14	2798	126192	59	1570	23712
15	2785	123407	60	1524	22188
16	2771	120636	61	1478	20710
17	2756	117880	62	1432	19278
18	2740	115140	63	1385	17893
19	2723	112417	64	1337	16556
20	2705	109712	65	1288	15268
21	2687	107025	66	1238	14030
22	2669	104356	67	1187	12843
23	2650	101706	68	1135	11708
24	2631	99075	69	1082	10626
25	2611	96464	70	1028	9598
26	2591	93873	71	974	8624
27	2570	91303	72	919	7705
28	2548	88755	73	864	6841
29	2525	86230	74	808	6033
30	2501	83729	75	752	5281
31	2477	81252	76	697	4584
32	2452	78800	77	642	3942
33	2426	76374	78	588	3354
34	2400	73974	79	534	2820
35	2374	71600	80	480	2340
36	2347	69253	81	426	1914
37	2320	66933	82	373	1541
38	2292	64641	83	321	1220
39	2264	62377	84	271	949
40	2236	60141	85	224	725
41	2208	57933	86	181	544
42	2180	55753	87	143	401
43	2152	53601	88	111	290
44	2123	51478	89	85	205
45	2093	49385	90	65	140
46	2063	47322	91	49	91
47	2033	45289	92	36	55
48	2002	43287	93	25	30
49	1970	41317	94	16	14
50	1937	39380	95	9	5
51	1902	37478	96	4	1
52	1865	35613	97	1	0
53	1826	33787			

TABLE XVII.

Showing the Average Duration (otherwise termed the *Expectation*) of a Single Life of any age, not under 10 years, according to the experience of the Equitable.—*See pages* 117—20.

Age.	Expectation.		Age.	Expectation.	
	Curtate.	Complete.		Curtate.	Complete.
10	48.33	48.83	54	17.93	18.43
11	47.52	48.02	55	17.35	17.85
12	46.70	47.20	56	16.78	17.28
13	45.90	46.40	57	16.21	16.71
14	45.10	45.60	58	15.65	16.15
15	44.31	44.81	59	15.10	15.60
16	43.54	44.04	60	14 56	15.06
17	42.77	43.27	61	14.01	14.51
18	42.02	42.52	62	13.46	13.96
19	41.28	41.78	63	12.92	13.42
20	40.56	41.06	64	12.38	12.88
21	39 83	40.33	65	11.85	12.35
22	39.10	39.60	66	11.33	11.83
23	38 38	38.88	67	10.82	11.32
24	37.66	38.16	68	10.32	10.82
25	36.94	37.44	69	9 82	10.32
26	36.23	36.73	70	9.34	9.84
27	35.52	36.02	71	8.86	9.36
28	34.83	35.33	72	8.38	8.88
29	34.15	34.65	73	7.92	8.42
30	33.48	33.98	74	7.47	7.97
31	32.80	33.30	75	7.02	7.52
32	32.14	32.64	76	6.58	7.08
33	31.48	31.98	77	6.14	6.64
34	30.82	31.32	78	5.70	6.20
35	30.16	30 66	79	5 28	5.78
36	29.51	30.01	80	4.88	5.38
37	28.85	29.35	81	4.50	5.00
38	28.20	28.70	82	4.13	4.63
39	27.55	28.05	83	3.80	4.30
40	26.90	27.40	84	3.50	4.00
41	26.24	26.74	85	3.23	3.73
42	25.57	26.07	86	3.00	3.50
43	24.90	25.40	87	2.81	3.31
44	24.25	24.75	88	2.61	3.11
45	23.60	24.10	89	2.41	2.91
46	22.94	23.44	90	2.15	2.65
47	22.28	22.78	91	1.86	2.36
48	21.62	22.12	92	1.53	2.03
49	20.97	21.47	93	1.20	1.70
50	20.33	20.83	94	.81	1.31
51	19.70	20.20	95	.55	1.05
52	19.09	19.59	96	.25	.75
53	18.50	19.00	97	.00	.50

TABLE XVIII.

Being a Form of preparatory Table, for determining the *Expectation* of Two Joint Lives, according to the experience of the Equitable.—*See page* 122.

Younger Age.	Difference between the two Ages.					
	19 Years.		20 Years.		21 Years.	
	D	N	D	N	D	N
	- -	214328	- -	209214	- -	204083
10	7181	207147	7113	202101	7045	197038
11	7086	200061	7017	195084	6947	190091
12	6990	193071	6920	188164	6846	183245
13	6891	186180	6817	181347	6744	176501
14	6787	179393	6715	174632	6641	169860
15	6684	172709	6612	168020	6536	163324
16	6578	166131	6503	161517	6428	156896
17	6468	159663	6394	155123	6316	150580
18	6357	153306	6280	148843	6203	144377
19	6241	147065	6165	142678	6089	138288
20	6124	140941	6048	136630	5973	132315
21	6007	134934	5932	130698	5857	126458
22	5893	129041	5818	124880	5743	120715
23	5777	123264	5703	119177	5626	115089
24	5662	117602	5586	113591	5507	109582
25	5543	112059	5465	108126	5387	104195
26	5423	106636	5345	102781	5268	98927
27	5302	101334	5225	97556	5145	93782
28	5180	96154	5101	92455	5019	88763
29	5055	91099	4974	87481	4892	83871
30	4927	86172	4845	82636	4757	79114
31	4797	81375	4711	77925	4619	74495
32	4664	76711	4573	73352	4477	70018
33	4524	72187	4429	68923	4330	65688
34	4382	67805	4284	64639	4186	61502
35	4238	63567	4140	60499	4041	57461
36	4093	59474	3995	56504	3893	53568
37	3949	55525	3849	52655	3747	49821
38	3802	51723	3701	48954	3598	46223
39	3656	48067	3554	45400	3450	42773
40	3510	44557	3408	41992	3304	39469
41	3365	41192	3263	38729	3161	36308
42	3222	37970	3121	35608	3019	33289
43	3082	34888	2981	32627	2878	30411

TABLE XVIII.

Being a Form of preparatory Table for determining the Expectation of Two Joint Lives, according to the experience of the Equitable.

Younger Age.	Difference between the two Ages.					
	19 Years.		20 Years.		21 Years.	
	𝔻	ℕ	𝔻	ℕ	𝔻	ℕ
44	2941	31947	2839	29788	2735	27676
45	2798	29149	2695	27093	2591	25085
46	2657	26492	2554	24539	2448	22637
47	2517	23975	2412	22127	2307	20330
48	2376	21599	2272	19855	2166	18164
49	2236	19363	2132	17723	2024	16140
50	2095	17268	1991	15732	1887	14253
51	1955	15313	1852	13879	1748	12505
52	1816	13497	1714	12165	1611	10894
53	1678	11819	1578	10588	1475	9418.9
54	1542	10276	1442	9145.5	1342	8076.5
55	1409	8867.3	1311	7834.0	1216	6860.9
56	1280	7587.4	1186	6647.7	1093	5768.2
57	1156	6431.1	1065	5582.6	975.4	4792.8
58	1037	5394.3	949.6	4633.0	862.4	3930.4
59	923.2	4471.1	838.4	3794.6	753.6	3176.8
60	813.8	3657.3	731.5	3063.1	649.2	2527.6
61	709.4	2947.9	629.6	2433.5	551.3	1976.3
62	610.0	2337.9	534.1	1899.4	459.6	1516.7
63	516.6	1821.3	444.6	1454.8	375.4	1141.3
64	429.1	1392.2	362.3	1092.5	299·4	841.92
65	349.1	1043.1	288.5	804.0	233.1	608.82
66	277.3	765.8	224.0	580.0	177.0	431.82
67	214.9	550.9	169.7	410.3	131.8	300.02
68	162.3	388.6	126.0	284.3	96.48	203.54
69	120.1	268.5	91.97	192.4	70.33	133.21
70	87.38	181.1	66.82	125.5	50.37	82.84
71	63.31	117.8	47.73	77.81	35.06	47.78
72	45.03	72.76	33.08	44.73	22.98	24.80
73	31.10	41.66	21.60	23.13	13.82	10.98
74	20.20	21.46	12.93	10.20	7.272	3.705
75	12.03	9.429	6.768	3.430	3.008	.697
76	6.273	3.156	2.788	.642	.697	.000
77	2.568	.588	.642	.000		
78	.588	.000				

TABLE XIX.

Showing the Curtate Expectation of Two Joint Lives, according to the experience of the Equitable.—*See page* 122.

Younger Age.	Difference between the two Ages.						
	0 Years.	5 Years.	10 Years.	15 Years.	20 Years.	25 Years.	30 Years.
10	36.51	34.61	32.67	30.58	28.41	26.18	23.82
11	35.80	33.92	32.01	29.93	27.80	25.58	23.22
12	35.07	33.24	31.35	29.30	27.19	24.98	22.60
13	34.37	32.58	30.70	28.69	26.60	24.39	22.00
14	33.67	31.92	30.06	28.08	26.00	23.80	21.39
15	32.98	31.28	29.43	27.46	25.41	23.21	20.80
16	32.31	30.65	28.80	26.87	24.83	22.62	20.21
17	31.67	30.03	28.20	26.30	24.26	22.04	19.62
18	31.03	29.42	27.61	25.73	23.70	21.46	19.04
19	30.43	28.81	27.04	25.17	23.14	20.89	18.47
20	29.83	28.23	26.48	24.62	22.59	20.32	17.91
21	29.24	27.64	25.93	24.07	22.03	19.76	17.37
22	28.64	27.05	25.37	23.51	21.46	19.19	16.83
23	28.03	26.48	24.82	22.97	20.89	18.62	16.31
24	27.44	25.91	24.26	22.42	20.33	18.06	15.81
25	26.87	25.36	23.72	21.88	19.78	17.52	15.30
26	26.28	24.81	23.17	21.33	19.22	16.98	14.80
27	25.72	24.26	22.64	20.77	18.67	16.45	14.31
28	25.16	23.74	22.11	20.23	18.12	15.94	13.82
29	24.62	23.21	21.59	19.69	17.58	15.45	13.35
30	24.09	22.69	21.07	19.16	17.05	14.97	12.88
31	23.57	22.18	20.55	18.64	16.53	14.49	12.42
32	23.05	21.66	20.02	18.10	16.04	14.02	11.95
33	22.55	21.16	19.49	17.58	15.56	13.55	11.49
34	22.04	20.65	18.97	17.06	15.08	13.09	11.02
35	21.53	20.14	18.46	16.54	14.61	12.64	10.57
36	21.02	19.64	17.95	16.03	14.14	12.18	10.13
37	20.52	19.11	17.42	15.54	13.67	11.72	9.68
38	20.03	18.60	16.91	15.07	13.22	11.26	9.25
39	19.52	17.10	16.39	14.61	12.77	10.81	8.83
40	19.01	17.58	15.89	14.14	12.32	10.37	8.40
41	18.49	17.06	15.38	13.67	11.87	9.93	7.98
42	17.98	16.54	14.89	13.21	11.41	9.48	7.57
43	17.45	16.01	14.40	12.74	10.96	9.04	7.16
44	16.93	15.50	13.93	12.28	10.49	8.62	6.76
45	16.42	14.98	13.47	11.83	10.05	8.20	6.36
46	15.90	14.48	13.00	11.38	9.60	7.78	5.97
47	15.37	13.98	12.53	10.93	9.17	7.37	5.57
48	14.85	13.51	12.08	10.47	8.73	6.96	5.18
49	14.33	13.04	11.62	10.03	8.31	6.57	4.80
50	13.83	12.57	11.17	9.59	7.90	6.19	4.43
51	13.34	12.12	10.74	9.16	7.49	5.79	4.08
52	12.88	11.69	10.30	8.73	7.09	5.41	3.75
53	12.43	11.26	9.88	8.33	6.71	5.03	3.46

TABLE XIX.

Showing the Curtate Expectation of Two Joint Lives, according to the experience of the Equitable.

Younger Age.	Difference between the two Ages.					
	35 Years.	40 Years.	45 Years.	50 Years.	55 Years.	60 Years.
10	21.28	18.64	16.14	13 75	11.31	9.01
11	20.68	18.05	15.61	13.21	10.81	8.54
12	20.06	17.49	15.08	12.69	10.32	8.09
13	19.46	16.94	14.55	12.17	9.84	7.64
14	18.86	16.40	14.03	11.67	9.37	7.20
15	18.27	15 87	13.52	11.16	8.91	6.77
16	17.70	15.34	13.02	10.68	8.45	6.35
17	17.15	14.82	12.51	10.20	8.00	5.93
18	16.62	14.32	12.00	9.73	7.56	5.51
19	16.11	13.82	11.51	9.27	7.14	5.11
20	15.59	13.33	11.03	8.82	6.72	4.72
21	15.09	12.84	10.56	8.37	6.30	4.35
22	14.59	12.35	10.09	7.93	5.88	4.00
23	14.09	11.85	9.62	7.50	5.47	3.68
24	13.60	11.36	9.17	7.07	5.06	3.40
25	13.12	10.88	8.72	6.66	4.68	3.14
26	12.63	10.41	8.28	6.24	4.32	2.92
27	12.14	9.95	7.84	5.83	3.96	2.72
28	11.66	9.49	7.41	5.42	3.66	2.54
29	11.19	9.05	7.00	5.03	3.37	2.35
30	10.73	8.62	6.59	4.65	3.12	2.10
31	10.28	8.19	6.18	4.29	2.90	1.82
32	9.82	7.76	5.78	3.94	2.71	1.49
33	9.38	7.35	5.38	3.63	2.53	1.17
34	8.95	6.94	4.99	3.35	2.34	.86
35	8.53	6.54	4.62	3.10	2.09	.55
36	8.10	6.14	4.26	2.88	1.81	.25
37	7.69	5.74	3.92	2.69	1.49	
38	7.27	5.34	3.62	2.52	1.17	
39	6.87	4.96	3.35	2.33	.85	
40	6.48	4.59	3.09	2.08	.54	
41	6.08	4.24	2.87	1.80	.23	
42	5.68	3.90	2.68	1.48		
43	5.29	3.59	2.50	1.17		
44	4.90	3.31	2.32	.85		
45	4.53	3.06	2.07	.54		
46	4.18	2.85	1.79	.24		
47	3.84	2.66	1.48			
48	3.54	2.48	1.16			
49	3.26	2.29	.85			
50	3.01	2.05	.54			
51	2.80	1.77	.24			
52	2.61	1.46				
53	2.44	1.15				

TABLE XIX.

Showing the Curtate Expectation of Two Joint Lives, according to the experience of the Equitable.

Younger Age.	Difference between the two Ages.								
	0 Years.	5 Years.	10 Years.	15 Years.	20 Years.	25 Years.	30 Years.	35 Years.	40 Years.
54	12.01	10.84	9.47	7.95	6.34	4.67	3.19	2.26	.83
55	11.58	10.44	9.06	7.56	5.97	4.31	2.95	2.02	.53
56	11.17	10.03	8.66	7.17	5.60	3.98	2.74	1.75	.26
57	10.75	9.62	8.27	6.80	5.24	3.67	2.56	1.44	
58	10.34	9.21	7.88	6.43	4.87	3.38	2.39	1.16	
59	9.94	8.82	7.51	6.07	4.52	3.12	2.22	.84	
60	9.56	8.43	7.14	5.72	4.18	2.88	1.99	.54	
61	9.17	8.05	6.77	5.37	3.86	2.67	1.73	.24	
62	8.76	7.66	6.41	5.03	3.55	2.51	1.42		
63	8.37	7.29	6.05	4.67	3.27	2.34	1.12		
64	7.97	6.93	5.70	4.32	3.01	2.15	.82		
65	7.60	6.56	5.36	3.98	2.79	1.94	.52		
66	7.22	6.21	5.02	3.67	2.58	1.68	.23		
67	6.86	5.86	4.68	3.37	2.42	1.39			
68	6.49	5.52	4.35	3.11	2.25	1.10			
69	6.15	5.19	4.02	2.85	2.09	.81			
70	5.81	4.86	3.71	2.64	1.88	.52			
71	5.47	4.55	3.41	2.44	1.63	.23			
72	5.15	4.23	3.13	2.28	1.35				
73	4.83	3.91	2.87	2.13	1.07				
74	4.52	3.61	2.63	1.97	.78				
75	4.22	3.32	2.43	1.75	.50				
76	3.92	3.05	2.24	1.54	.23				
77	3.61	2.77	2.08	1.28					
78	3.31	2.51	1.93	1.01					
79	3.01	2.28	1.77	.73					
80	2.72	2.07	1.58	.48					
81	2.46	1.87	1.37	.21					
82	2.21	1.73	1.13						
83	1.99	1.58	.89						
84	1.79	1.45	.66						
85	1.62	1.30	.43						
86	1.49	1.13	.19						
87	1.40	.95							
88	1.31	.78							
89	1.23	.60							
90	1.10	.39							
91	.95	.18							
92	.76								
93	.57								
94	.37								
95	.20								
96	.06								

TABLE XX.

Showing the Curtate Expectation of Three Joint Lives of equal ages, according to the experience of the Equitable.—*See page* 125.

Common Age.	Expectation.	Common Age.	Expectation.
10	29.90	54	9.09
11	29.25	55	8.75
12	28.59	56	8.42
13	27.96	57	8.09
14	27.33	58	7.77
15	26.71	59	7.45
16	26.11	60	7.15
17	25.55	61	6.84
18	24.99	62	6.52
19	24.46	63	6.21
20	23.95	64	5.90
21	23.45	65	5.60
22	22.93	66	5.30
23	22.41	67	5.02
24	21.90	68	4.74
25	21.41	69	4.47
26	20.91	70	4.21
27	20.42	71	3.94
28	19.96	72	3.70
29	19.51	73	3.45
30	19.07	74	3.22
31	18.64	75	2.99
32	18.21	76	2.76
33	17.81	77	2.54
34	17.39	78	2.30
35	16.97	79	2.07
36	16.56	80	1.85
37	16.15	81	1.65
38	15.75	82	1.46
39	15.34	83	1.28
40	14.92	84	1.13
41	14.49	85	1.00
42	14.06	86	.90
43	13.62	87	.83
44	13.18	88	.79
45	12.76	89	.75
46	12.32	90	.68
47	11.88	91	.58
48	11.44	92	.44
49	11.00	93	.32
50	10.58	94	.19
51	10.17	95	.09
52	9.79	96	.00
53	9.43		

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities &c. on Single Lives, according to the experience of the Equitable.—*See page* 219.

($2\frac{1}{2}$ per Cent.)

Age.	D	N	S	M	R
		60122.404	1291906		
10	2221.726	57900.678	1231784	755.3274	28612.484
11	2159.156	55741.522	1173884	746.9438	27857.157
12	2098.315	53643.207	1118142	738.7646	27110.213
13	2038.430	51604.777	1064499	730.0596	26371.449
14	1980.220	49624.557	1012894	721.5669	25641.389
15	1922.947	47701.610	963269.5	712.5908	24919.822
16	1866.615	45834.995	915567.9	703.1600	24207.231
17	1811.230	44023.765	869732.9	693.3020	23504.071
18	1756.794	42266.971	825709.1	683.0433	22810.769
19	1703.313	40563.658	783442.2	672.4094	22127.726
20	1650.783	38912.875	742878.5	661.4245	21455.317
21	1599.801	37313.074	703965.6	650.7076	20793.892
22	1550.328	35762.746	666652.6	640.2520	20143.185
23	1501.747	34260.999	630889 8	629.4848	19502.933
24	1454.614	32806.385	596628.8	618.9802	18873.448
25	1408.350	31398.035	563822.4	608.1924	18254.468
26	1363.475	30034.560	532424.4	597.6677	17646.275
27	1319.438	28715.122	502389.8	586.8863	17048.607
28	1276.236	27438.886	473674.7	575.8670	16461.721
29	1233.869	26205.017	446235.8	564.6278	15885.854
30	1192.335	25012.682	420030.8	553.1859	15321.226
31	1152.090	23860.592	395018.1	542.0232	14768·040
32	1112.646	22747.946	371157.5	530.6789	14226.017
33	1073.997	21673.949	348409.6	519.1686	13695,338
34	1036.572	20637.377	326735.6	507.9391	13176.170
35	1000.334	19637.043	306098.3	496.9835	12668.231
36	964.837	18672.206	286461.2	485.8840	12171.247
37	930.475	17741.731	267789.0	475.0553	11685.363
38	896.824	16844.907	250047.3	464.0994	11210.308
39	864.261	15980.646	233202.4	453.4107	10746,208
40	832.755	15147.891	217221.7	442.9827	10292.798
41	802.269	14345.622	202073.8	432.8091	9849.815
42	772.777	13572.845	187728.2	422.8836	9417.006
43	744.246	12828.599	174155.4	413.2002	8994.122
44	716.308	12112.291	161326.8	403.4155	8580.922
45	688.960	11423.331	149214.5	393.5403	8177.507
46	662.523	10760.808	137791.1	383.9059	7783.966
47	636.965	10123.843	127030.3	374.5065	7400.060
48	611.953	9511.890	116906.5	365.0307	7025.554
49	587.485	8924.405	107394.6	355.4878	6660 523
50	563.554	8360.851	98470.20	345.8867	6305.035
51	539.875	7820.976	90109.35	335.9521	5959.149
52	516.461	7304.515	82288.37	325.7060	5623.197
53	493.328	6811.187	74983.86	315.1695	5297.491

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

($2\frac{1}{2}$ per Cent.)

Age	D	N	S	M	R
54	470.488	6340.699	68172.67	304.3627	4982.321
55	448.471	5892.228	61831.97	293.8195	4677.958
56	426.996	5465.232	55939.74	283.2826	4384.139
57	406.056	5059.176	50474.51	272.7579	4100.856
58	385.646	4673.530	45415.33	262.2511	3828.098
59	365.756	4307.774	40741.80	251.7677	3565.847
60	346.381	3961.393	36434.03	241.3126	3314.080
61	327.732	3633.661	32472.64	231.1126	3072.767
62	309.788	3323.873	28838.98	221.1613	2841.654
63	292.311	3031.562	25515.10	211.2417	2620.493
64	275.298	2756.264	22483.54	201.3582	2409.251
65	258.739	2497.525	19727.28	191.5149	2207.893
66	242.629	2254.896	17229.75	181.7156	2016.378
67	226.961	2027.935	14974.86	171.9641	1834.663
68	211.725	1816 210	12946.92	162.2639	1662.699
69	196.915	1619.295	11130.71	152.6183	1500.435
70	182.525	1436.770	9511.416	143.0304	1347.816
71	168.719	1268.051	8074.647	133.6763	1204.786
72	155.309	1112.741	6806.596	124.3814	1071.110
73	142 453	970.2886	5693.854	115.3132	946.728
74	129.971	840.3178	4723.566	106.3053	831.415
75	118.012	722.3056	3883.248	97.5171	725.110
76	106.714	615.5921	3160.942	89.0964	627.593
77	95.8955	519.6966	2545.350	80.8810	538.496
78	85.6869	434.0097	2025.654	73.0118	457.615
79	75.9199	358.0898	1591.644	65.3345	384.604
80	66.5784	291.5114	1233.554	57.8444	319.269
81	57.6471	233.8643	942.043	50.5370	261.425
82	49.2439	184.6204	708.179	43.5398	210.888
83	41.3451	143.2753	523.558	36.8421	167.348
84	34.0536	109.2217	380.283	30.5591	130.506
85	27.4613	81.7604	271.061	24.7972	99.947
86	21.6485	60.1119	189.301	19.6542	75.149
87	16.6862	43.4257	129.189	15.2202	55.495
88	12.6363	30.7894	85.7632	11.5773	40.275
89	9.4405	21.3489	54.9738	8.6896	28.698
90	7.0431	14.3058	33.6249	6.5225	20.008
91	5.1799	9.12581	19.3191	4.8311	13.486
92	3.7129	5.41295	10.1933	3.4903	8.6546
93	2.5155	2.89747	4.7804	2.3835	5.1643
94	1.5706	1.32683	1.8829	1.5000	2.7809
95	.86194	.46490	.55605	.8296	1.2809
96	.37374	.09116	.09116	.3624	.4513
97	.09116	.00000	.00000	.0889	.0889

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities &c. on Single Lives, according to the experience of the Equitable.

(3 per Cent.)

Age.	D	N	S	M	R
		51998.642	1049902.4		
10	2116.203	49882.439	997903.7	601.6785	21418.90
11	2046.619	47835.820	948021.3	593.7319	20817.23
12	1979.295	45856.525	900185.5	586.0167	20223.49
13	1913.473	43943.052	854328.9	577.8453	19637.48
14	1849.808	42093.244	810385.9	569.9119	19059.63
15	1787.585	40305.659	768292.7	561 5677	18489.72
16	1726.795	38578.864	727987.0	552.8434	17928.15
17	1667.424	36911.440	689408.1	543.7681	17375.31
18	1609.463	35301.977	652496.7	534.3698	16831.54
19	1552.889	33749.088	617194.7	524.6749	16297.17
20	1497.693	32251.395	583445.6	514.7087	15772.50
21	1444.394	30807.001	551194.2	505.0329	15257.79
22	1392.932	29414.069	520387.2	495.6389	14752.75
23	1342.734	28071.335	490973.2	486.0118	14257.11
24	1294.278	26777.057	462901.8	476.6651	13771.10
25	1247.030	25530.027	436124.8	467.1130	13294.44
26	1201.434	24328.593	410594.7	457.8391	12827.32
27	1156.986	23171.607	386266.2	448.3851	12369.49
28	1113.672	22057.935	363094.5	438.7695	11921.10
29	1071.474	20986.461	341036.6	429.0096	11482.33
30	1030.380	19956.081	320050.2	419.1229	11053.32
31	990.767	18965.314	300094.1	409.5224	10634.20
32	952.203	18013.111	281129.8	399.8140	10224.68
33	914.664	17098.447	263116.6	390.0114	9824.86
34	878.508	16219.939	246018.2	380.4943	9434.85
35	843.679	15376.260	229798.3	371.2543	9054.36
36	809.790	14566.470	214422.0	361.9385	8683.10
37	777.161	13789.309	199855.5	352.8939	8321.16
38	745.417	13043.892	186066.2	343.7876	7968.27
39	714.867	12329.025	173022.3	334.9465	7624.48
40	685.461	11643.564	160693.3	326.3630	7289.54
41	657.163	10986.401	149049.7	318.0294	6963.17
42	629.930	10356.471	138063.3	309.9386	6645.14
43	603.728	9752.743	127706.9	302.0834	6335.20
44	578.245	9174.498	117954.1	294.1847	6033.12
45	553.470	8621.028	108779.6	286.2515	5738.94
46	529.648	8091.380	100158.6	278.5494	5452.69
47	506.744	7584.636	92067.2	271.0716	5174.14
48	484.482	7100.154	84482.6	263.5696	4903.06
49	462.852	6637.302	77382.4	256.0512	4639.49
50	441.843	6195.459	70745.1	248.5237	4383.44
51	421.223	5774.236	64549.7	240.7725	4134.92
52	400.999	5373.237	58775.4	232.8170	3894.15
53	381.177	4992.060	53402.2	224.6757	3661.33

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

(3 per Cent.)

Age.	D	N	S	M	R
54	361.766	4630.294	48410.1	216.3662	3436.65
55	343.161	4287.133	43779.9	208.2988	3220.29
56	325.143	3961.990	39492.7	200.2753	3011.99
57	307.698	3654.292	35530.7	192.3000	2811.71
58	290.813	3363.479	31876.4	184.3769	2619.41
59	274.475	3089.004	28513.0	176.5098	2435.04
60	258.674	2830.330	25424.0	168.7021	2258.53
61	243.558	2586.772	22593.6	161.1218	2089.83
62	229.106	2357.666	20006.9	153.7623	1928.70
63	215.132	2142.534	17649.2	146.4618	1774.94
64	201.628	1940.906	15506.7	139.2232	1628.48
65	188.580	1752.326	13565.7	132.0490	1489.26
66	175.980	1576.346	11813.4	124.9415	1357.21
67	163.816	1412.530	10237.1	117.9030	1232.27
68	152.077	1260.453	8824.54	110.9355	1114.36
69	140.752	1119.701	7564.09	104.0410	1003.43
70	129.833	989.8681	6444.39	97.2209	899.386
71	119.431	870.4373	5454.52	90.5995	802.166
72	109.404	761.0331	4584.08	84.0519	711.566
73	99.8611	661.1720	3823.05	77.6950	627.514
74	90.6689	570.5031	3161.88	71.4110	549.819
75	81.9267	488.5764	2591.38	65.3101	478.408
76	73.7231	414.8533	2102.80	59.4926	413.098
77	65.9276	348.9257	1687.95	53.8445	353.605
78	58.6236	290.3021	1339.02	48.4607	299.761
79	51.6891	238.6130	1048.72	43.2337	251.300
80	45.1090	193.5040	810.106	38.1589	208.067
81	38.8682	154.6358	616.602	33.2319	169.908
82	33.0410	121.5948	461.966	28.5371	136.676
83	27.6066	93.9882	340.371	24.0650	108.139
84	22.6277	71.3605	246.383	19.8901	84.0736
85	18.1585	53.2020	175.022	16.0801	64.1835
86	14.2454	38.9566	121.820	12.6958	48.1034
87	10.9269	28.0297	82.8638	9.7921	35.4076
88	8.23470	19.7950	54.8341	7.4181	25.6155
89	6.12221	13.6728	35.0391	5.5455	18.1974
90	4.54532	9.1275	21.3663	4.1469	12.6519
91	3.32666	5.8008	12.2388	3.0607	8.5050
92	2.37290	3.4279	6.4379	2.2039	5.4443
93	1.59985	1.8281	3.0100	1.5000	3.2404
94	.99408	.83399	1.1820	.94083	1.7404
95	.54288	.29111	.34797	.51859	.79956
96	.23425	.05686	.05686	.22577	.28097
97	.05686	.00000	.00000	.05520	.05520

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

($3\frac{1}{2}$ per Cent.)

Age.	D	N	S	M	R
		45276.549	860039.9		
10	2016.165	43260.384	814763.4	485.0732	16193.06
11	1940.452	41319.932	771503.0	477.5388	15707.99
12	1867.551	39452.381	730183.1	470.2592	15230.45
13	1796.725	37655.656	690730.7	462.5864	14760.19
14	1728.553	35927.103	653075.0	455.1730	14297.60
15	1662.341	34264.762	617147.9	447.4134	13842.43
16	1598.052	32666.710	582883.2	439.3395	13395.02
17	1535.654	31131.056	550216.5	430.9815	12955.68
18	1475.109	29655.947	519085.4	422.3677	12524.70
19	1416.384	28239.563	489429.5	413.5250	12102.33
20	1359.441	26880.122	461189.9	404.4788	11688.80
21	1304.730	25575.392	434309.8	395.7385	11284.33
22	1252.164	24323.228	408734.4	387.2938	10888.59
23	1201.208	23122.020	384411.1	378.6814	10501.29
24	1152.265	21969.755	361289.1	370.3602	10122.61
25	1104.836	20864.919	339319.4	361.8973	9752.25
26	1059.299	19805.620	318454.5	353.7205	9390.35
27	1015.181	18790.439	298648.8	345.4253	9036.63
28	972.454	17817.985	279858.4	337.0289	8691.21
29	931.089	16886.896	262040.4	328.5477	8354.18
30	891.051	15995.845	245153.5	319.9970	8025.63
31	852.657	15143.188	229157.7	311.7355	7705.64
32	815.511	14327.677	214014.5	303.4207	7393.90
33	779.578	13548.099	199686.8	295.0658	7090.48
34	745.142	12802.957	186138.7	286.9935	6795.41
35	712.145	12090.812	173335.7	279.1942	6508.42
36	680.238	11410.574	161244.9	271.3687	6229.23
37	649.675	10760.899	149834.4	263.8079	5957.86
38	620.127	10140.772	139073.5	256.2322	5694.05
39	591.840	9548.932	128932.7	248.9126	5437.82
40	564.750	8984.182	119383.8	241.8406	5188.90
41	538.820	8445.362	110399.6	235.0078	4947.06
42	513.998	7931.364	101954.2	228.4060	4712.06
43	490.239	7441.125	94022.8	222.0275	4483.65
44	467.276	6973.849	86581.7	215.6446	4261.62
45	445.095	6528.754	79607.9	209.2648	4045.98
46	423.880	6104.874	73079.1	203.1008	3836.71
47	403.592	5701.282	66974.2	197.1452	3633.61
48	383.996	5317.286	61273.0	191.1992	3436.47
49	365.080	4952.206	55955.7	185.2690	3245.27
50	346.826	4605.380	51003.5	179.3602	3060.00
51	329.042	4276.338	46398.1	173.3053	2880.64
52	311.730	3964.608	42121.7	167.1209	2707.33
53	294.891	3669.717	38157.1	160.8226	2540.21

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TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

($3\frac{1}{2}$ per Cent.)

Age.	D	N	S	M	R
54	278.522	3391.195	34487.42	154.4252	2379.390
55	262.922	3128.273	31096.23	148.2441	2224.965
56	247.913	2880.360	27967.96	142.1264	2076.721
57	233.477	2646.883	25087.60	136.0748	1934.595
58	219.600	2427.283	22440.71	130.0919	1798.520
59	206.262	2221.021	20013.43	124.1799	1668.428
60	193.448	2027.573	17792.41	118.3409	1544.248
61	181.265	1846.308	15764.83	112.6994	1425.907
62	169.685	1676.623	13918.53	107.2487	1313.208
63	158.563	1518.060	12241.90	101.8678	1205.959
64	147.893	1370.167	10723.84	96.5583	1104.091
65	137.654	1232.513	9353.67	91.3215	1007.533
66	127.837	1104.676	8121.16	86.1584	916.212
67	118.425	986.251	7016.49	81.0701	830.053
68	109.409	876.842	6030.23	76.0575	748.983
69	100.772	776.070	5153.39	71.1213	672.926
70	92.5050	683.565	4377.32	66.2621	601.804
71	84.6825	598.883	3693.76	61.5671	535.542
72	77.1987	521.684	3094.87	56.9469	473.975
73	70.1239	451.560	2573.19	52.4830	417.028
74	63.3617	388.198	2121.63	48.0916	364.545
75	56.9760	331.222	1733.43	43.8487	316.454
76	51.0232	280.199	1402.21	39.8225	272.605
77	45.4073	234.792	1122.01	35.9325	232.782
78	40.1821	194.610	887.217	32.2423	196.850
79	35.2579	159.352	692.607	28.6769	164.608
80	30.6206	128.731	533.255	25.2320	135.931
81	26.2569	102.474	404.523	21.9037	110.699
82	22.2126	80.2619	302.049	18.7474	88.795
83	18.4697	61.7922	221.787	15.7554	70.048
84	15.0654	46.7268	159.995	12.9758	54.292
85	12.0314	34.6954	113.268	10.4513	41.316
86	9.3932	25.3022	78.5725	8.2198	30.865
87	7.1701	18.1321	53.2703	6.3145	22.645
88	5.3774	12.7547	35.1382	4.7642	16.331
89	3.9786	8.77612	22.3835	3.5473	11.567
90	2.9396	5.83656	13.6074	2.6428	8.0193
91	2.1411	3.69550	7.7708	1.9437	5.3765
92	1.5198	2.17569	4.0753	1.3949	3.4328
93	1.0197	1.15596	1.8996	.94616	2.0379
94	.63056	.52540	.74365	.59147	1.0917
95	.34269	.18271	.21825	.32493	.50025
96	.14716	.03555	.03555	.14098	.17532
97	.03555	.00000	.00000	.03434	.03434

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

(4 per Cent.)

Age.	D	N	S	M	R
		39668.456	709902.0		
10	1921.303	37747.153	670233.5	395.5927	12364.51
11	1840.262	35906.890	632486.4	388.4473	11968.92
12	1762.613	34144.277	596579.5	381.5767	11580.47
13	1687.613	32456.664	562435.2	374.3699	11198.89
14	1615.774	30840.890	529978.5	367.4402	10824.52
15	1546.413	29294.477	499137.6	360.2217	10457.08
16	1479.459	27815.018	469843.2	352.7470	10096.86
17	1414.856	26400.162	442028.1	345.0464	9744.11
18	1352.541	25047.621	415628.0	337.1484	9399.07
19	1292.450	23755.171	390580.4	329.0795	9061.92
20	1234.527	22520.644	366825.2	320.8646	8732.84
21	1179.146	21341.498	344304.5	312.9657	8411.98
22	1126.197	20215.301	322963.0	305.3705	8099.01
23	1075.174	19140.127	302747.7	297.6617	7793.64
24	1026.409	18113.718	283607.6	290.2494	7495.98
25	979.430	17134.288	265493.9	282.7471	7205.73
26	934.545	16199.743	248359.6	275.5333	6922.98
27	891.320	15308.423	232159.9	268.2502	6647.45
28	849.699	14458.724	216851.4	260.9138	6379.20
29	809.644	13649.080	202392.7	253.5388	6118.28
30	771.106	12877.974	188743.6	246.1392	5864.74
31	734.331	12143.643	175865.7	239.0242	5618.61
32	698.962	11444.681	163722.0	231.8977	5379.58
33	664.952	10779.729	152277.3	224.7713	5147.68
34	632.525	10147.204	141497.6	217.9190	4922.91
35	601.608	9545.596	131350.4	211.3302	4704.99
36	571.890	8973.706	121804.8	204.7512	4493.66
37	543.569	8430.137	112831.1	198.4253	4288.91
38	516.352	7913.785	104401.0	192.1174	4090.49
39	490.429	7423.356	96487.2	186.0520	3898.37
40	465.733	6957.623	89063.8	180.2200	3712.32
41	442.214	6515.409	82106.2	174.6122	3532.10
42	419.814	6095.595	75590.8	169.2201	3357.49
43	398.481	5697.114	69495.2	164.0354	3188.27
44	377.992	5319.122	63798.1	158.8721	3024.23
45	358.316	4960.806	58479.0	153.7362	2865.36
46	339.599	4621.207	53518.2	148.7978	2711.62
47	321.789	4299.418	48896.9	144.0493	2562.82
48	304.694	3994.724	44597.5	139.3313	2418.77
49	288.292	3706.432	40602.8	134.6484	2279.44
50	272.561	3433.871	36896.4	130.0049	2144.79
51	257.343	3176.528	33462.5	125.2694	2014.79
52	242.630	2933.898	30286.0	120.4559	1889.52
53	228.419	2705.479	27352.1	115.5773	1769.06

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

(4 per Cent.)

Age.	D	N	S	M	R
54	214.702	2490.777	24646.60	110.6457	1653.487
55	201.703	2289.074	22155.82	105.9038	1542.841
56	189.274	2099.800	19866.75	101.2331	1436.938
57	177.397	1922.403	17766.95	96.6351	1335.704
58	166.049	1756.354	15844.55	92.1112	1239.069
59	155.215	1601.139	14088.19	87.6624	1146.958
60	144.871	1456.268	12487.05	83.2896	1059.296
61	135.095	1321.173	11030.79	79.0850	976.006
62	125.857	1195.316	9709.61	75.0421	896.921
63	117.043	1078.273	8514.30	71.0703	821.879
64	108.641	969.632	7436.02	67.1700	750.809
65	100.635	868.997	6466.39	63.3415	683.639
66	93.0090	775.988	5597.39	59.5851	620.297
67	85.7450	690.243	4821.41	55.9010	560.712
68	78.8370	611.406	4131.16	52.2891	504.811
69	72.2630	539.143	3519.76	48.7494	452.522
70	66.0170	473.126	2980.61	45.2816	403.773
71	60.1435	412.982	2507.49	41.9471	358.491
72	54.5646	358.418	2094.51	38.6815	316.544
73	49.3267	309.091	1736.09	35.5414	277.862
74	44.3551	264.736	1427.00	32.4673	242.321
75	39.6936	225.042	1162.26	29.5114	209.854
76	35.3756	189.666	937.220	26.7199	180.342
77	31.3302	158.336	747.554	24.0358	153.622
78	27.5913	130.745	589.217	21.5019	129.587
79	24.0941	106.651	458.472	19.0654	108.084
80	20.8243	85.8266	351.822	16.7227	89.0193
81	17.7710	68.0556	265.995	14.4701	72.2966
82	14.9614	53.0942	197.939	12.3442	57.8265
83	12.3807	40.7135	144.845	10.3386	45.4823
84	10.0501	30.6634	104.132	8.4843	35.1437
85	7.9876	22.6758	73.468	6.8083	26.6594
86	6.2060	16.4698	50.792	5.3340	19.8511
87	4.7146	11.7552	34.323	4.0812	14.5171
88	3.5188	8.2364	22.567	3.0668	10.4359
89	2.5909	5.6455	14.331	2.2743	7.3691
90	1.9051	3.7404	8.6854	1.6881	5.0948
91	1.3809	2.3595	4.9450	1.2372	3.4067
92	.97553	1.3840	2.5855	.88490	2.1695
93	.65140	.73258	1.2015	.59817	1.2846
94	.40085	.33173	.46893	.37269	.6864
95	.21681	.11492	.13720	.20406	.3137
96	.09265	.02227	.02227	.08824	.1097
97	.02227	.00000	.00000	.02142	.0214

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

($4\frac{1}{2}$ per Cent.)

Age.	D	N	S	M	R
		34953.239	590256.8		
10	1831.331	33121.908	555303.5	325.8605	9430.303
11	1745.691	31376.217	522181.6	319.0823	9104.443
12	1664.031	29712.186	490805.4	312.5960	8785.360
13	1585.605	28126.581	461093.2	305.8248	8472.764
14	1510.843	26615.738	432966.7	299.3452	8166.939
15	1439.066	25176.672	406350.9	292.6278	7867.594
16	1370.172	23806.500	381174.2	285.7053	7574.966
17	1304.073	22502.427	357367.7	278.6076	7289.261
18	1240.672	21261.755	334865.3	271.3628	7010.654
19	1179.881	20081.874	313603.6	263.9967	6739.291
20	1121.609	18960.265	293521.7	256.5332	6475.294
21	1066.165	17894.100	274561.4	249.3911	6218.761
22	1013.422	16880.678	256667.3	242.5565	5969.370
23	962.878	15917.800	239786.6	235.6528	5726.813
24	914.807	15002.993	223868.8	229.0465	5491.160
25	868.761	14134.232	208865.9	222.3919	5262.114
26	824.979	13309.253	194731.6	216.0239	5039.722
27	783.056	12526.197	181422.4	209.9301	4823.698
28	742.923	11783.274	168896.2	203.5156	4613.768
29	704.513	11078.761	157112.9	197.0983	4410.252
30	667.767	10410.994	146034.1	190.6903	4213.154
31	632.879	9778.115	135623.1	184.5583	4022.464
32	599.514	9178.601	125845.0	178.4458	3837.906
33	567.612	8610.989	116666.4	172.3626	3659.460
34	537.350	8073.639	108055.4	166.5414	3487.097
35	508.639	7565.000	99981.8	160.9708	3320.556
36	481.200	7083.800	92416.8	155.4351	3159.585
37	455.182	6628.618	85333.0	150.1378	3004.150
38	430.323	6198.295	78704.4	144.8808	2854.012
39	406.761	5791.534	72506.1	139.8502	2709.131
40	384.433	5407.101	66714.6	135.0362	2569.281
41	363.271	5043.830	61307.5	130.4295	2434.245
42	343.219	4700.611	56263.6	126.0212	2303.815
43	324.222	4376.389	51563.0	121.8027	2177.794
44	306.078	4070.311	47186.6	117.6217	2055.991
45	288.758	3781.553	43116.3	113.4828	1938.370
46	272.363	3509.190	39334.8	109.5221	1824.887
47	256.845	3252.345	35825.6	105.7320	1715.365
48	242.038	3010.307	32573.2	101.9842	1609.633
49	227.913	2782.394	29562.9	98.2820	1507.649
50	214.445	2567.949	26780.5	94.6286	1409.367
51	201.502	2366.447	24212.6	90.9206	1314.738
52	189.074	2177.373	21846.1	87.1695	1223.817
53	177.147	2000.226	19668.8	83.3860	1136.648

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

($4\frac{1}{2}$ per Cent.)

Age.	D	N	S	M	R
54	165.713	1834.513	17668.53	79.5797	1053.262
55	154.935	1679.578	15834.01	75.9373	973.682
56	144.692	1534.886	14154.43	72.3668	897.745
57	134.965	1399.921	12619.55	68.8686	825.378
58	125.725	1274.196	11219.63	65.4432	756.510
59	116.960	1157.236	9945.43	62.0908	691.066
60	108.645	1048.591	8788.20	58.8115	628.976
61	100.828	947.763	7739.60	55.6734	670.164
62	93.481	854.282	6791.84	52.6705	614.491
63	86.521	767.761	5937.56	49.7344	561.820
64	79.925	687.836	5169.80	46.8650	512.086
65	73.681	614.155	4481.96	44.0620	465.221
66	67.772	546.383	3867.81	41.3248	421.159
67	62.180	484.203	3321.42	38.6531	379.834
68	56.897	427.306	2837.22	36.0464	341.181
69	51.905	375.401	2409.92	33.5039	305.134
70	47.190	328.211	2034.52	31.0250	271.631
71	42.786	285.425	1706.30	28.6529	240.606
72	38.632	246.793	1420.88	26.3408	211.953
73	34.755	212.038	1174.09	24.1284	185.612
74	31.103	180.935	962.048	21.9728	161.483
75	27.701	153.234	781.113	19.9100	139.511
76	24.569	128.665	627.879	17.9712	119.601
77	21.656	107.009	499.214	16.1159	101.629
78	18.981	88.0282	392.205	14.3728	85.5136
79	16.495	71.5329	304.177	12.7047	71.1408
80	14.188	57.3446	232.644	11.1085	58.4361
81	12.050	45.2944	175.300	9.5810	47.3276
82	10.097	35.1977	130.005	8.1463	37.7466
83	8.3149	26.8828	94.8077	6.7993	29.6003
84	6.7173	20.1655	67.9249	5.5599	22.8010
85	5.3133	14.8522	47.7594	4.4451	17.2411
86	4.1085	10.7437	32.9072	3.4690	12.7960
87	3.1061	7.63760	22.1635	2.6436	9.3270
88	2.3073	5.33027	14.5259	1.9784	6.6834
89	1.6907	3.63953	9.19559	1.4613	4.7050
90	1.2372	2.40232	5.55606	1.0806	3.2437
91	.89254	1.50978	3.15374	.7891	2.1631
92	.62748	.88230	1.64396	.5625	1.3740
93	.41700	.46530	.76166	.3790	.81152
94	.25538	.20992	.29635	.2353	.43252
95	.13747	.07245	.08644	.1284	.19717
96	.05846	.01399	.01399	.0553	.06874
97	.01399	.00000	.00000	.0134	.01339

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

(5 per Cent.)

Age.	D	N	S	M	R
		30959.563	494490.9		
10	1745.968	29213.595	463531.4	271.7015	7426.605
11	1656.395	27557.200	434317.8	265.2700	7154.903
12	1571.395	25985.805	406760.6	259.1448	6889.633
13	1490.202	24495.603	380774.8	252.7810	6630.489
14	1413.179	23082.424	356279.2	246.7202	6377.708
15	1339.632	21742.792	333196.7	240.4670	6130.987
16	1269.428	20473.364	311454.0	234.0535	5890.520
17	1202.434	19270.930	290980.6	227.5090	5656.467
18	1138.528	18132.402	271709.7	220.8607	5428.958
19	1077.584	17054.818	253577.3	214.1333	5208.097
20	1019.484	16035.334	236522.4	207.3494	4993.964
21	964.476	15070.858	220487.1	200.8885	4786.615
22	912.397	14158.461	205416.2	194.7352	4585.726
23	862.764	13295.697	191257.8	188.5494	4390.991
24	815.789	12479.908	177962.1	182.6582	4202.441
25	771.036	11708.872	165212.2	176.7521	4019.783
26	728.696	10980.176	153503.3	171.1273	3843.031
27	688.369	10291.807	142523.1	165.5025	3671.904
28	649.979	9641.828	132231.3	159.8904	3506.401
29	613.439	9028.389	122589.5	154.3027	3346.511
30	578.674	8449.715	113561.1	148.7497	3192.208
31	545.829	7903.886	105111.4	143.4611	3043.459
32	514.591	7389.295	97207.5	138.2145	2899.997
33	484.891	6904.404	89818.2	133.0178	2761.783
34	456.852	6447.552	82913.8	128.0686	2628 765
35	430.382	6017.170	76436.3	123.3551	2500.697
36	405.225	5611.945	70419.1	118.6934	2377.341
37	381.492	5230.453	64807.1	114.2537	2258.648
38	358.938	4871.515	59576.7	109.8688	2144.394
39	337.670	4533.845	54705.2	105.6927	2034.526
40	317.614	4216.231	50171.3	101.7155	1928.833
41	298.702	3917.529	45955.1	97.9277	1827.117
42	280.871	3636.658	42037.6	94.3202	1729.190
43	264.058	3372.600	38400.9	90.8845	1634.869
44	248.095	3124.505	35028.3	87.4956	1543.985
45	232.944	2891.561	31903.8	84.1567	1456.489
46	218.671	2672.890	29012.2	80.9768	1372.333
47	205.229	2467.661	26339.4	77.9484	1291.356
48	192.476	2275.185	23871.7	74.9680	1213.407
49	180.381	2094.804	21596.5	72.0380	1138.439
50	168.914	1925.890	19501.7	69.1603	1066.401
51	157.963	1767.927	17575.8	66.2535	997.241
52	147.513	1620.414	15807.9	63.3270	930.988
53	137.553	1482.861	14187.5	60.3891	867.661

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

(5 per Cent.)

Age.	D	N	S	M	R
54	128.061	1354.800	12704.62	57.4477	807.272
55	119.160	1235.640	11349.82	54.6464	749.824
56	110.754	1124.886	10114.18	51.9134	695.177
57	102.814	1022.072	8989.29	49.2485	643.264
58	95.322	926.750	7967.22	46.6515	594.016
59	88.253	838.497	7040.47	44.1219	547.364
60	81.589	756.908	6201.97	41.6593	503.242
61	75.356	681.552	5445.06	39.3140	461.583
62	69.535	612.017	4763.51	37.0804	422.269
63	64.050	547.967	4151.49	34.9069	385.189
64	58.886	489.081	3603.53	32.7928	350.282
65	54.025	435.056	3114.45	30.7375	317.489
66	49.456	385.600	2679.39	28.7400	286.751
67	45.161	340.439	2293.79	26.7996	258.011
68	41.127	299.312	1953.35	24.9153	231.212
69	37.338	261.974	1654.04	23.0863	206.296
70	33.785	228.189	1392.07	21.3116	183.210
71	30.487	197.702	1163.88	19.6213	161.898
72	27.396	170.306	966.174	17.9816	142.277
73	24.530	145.776	795.868	16.4200	124.296
74	21.847	123.929	650.092	14.9058	107.876
75	19.366	104.563	526.163	13.4637	92.9698
76	17.094	87.4692	421.600	12.1148	79.5061
77	14.995	72.4740	334.131	10.8301	67.3913
78	13.080	59.3940	261.657	9.6288	56.5612
79	11.313	48.0807	202.263	8.4848	46.9324
80	9.6850	38.3957	154.182	7.3952	38.4476
81	8.1860	30.2097	115.786	6.3576	31.0524
82	6.8263	23.3834	85.5766	5.3876	24.6948
83	5.5950	17.7884	62.1932	4.4812	19.3072
84	4.4986	13.2898	44.4048	3.6512	14.8260
85	3.5412	9.7486	31.1150	2.9082	11.1748
86	2.7252	7.0234	21.3664	2.2608	8.2666
87	2.0505	4.9729	14.3430	1.7160	6.0058
88	1.5160	3.4569	9.3701	1.2790	4.2898
89	1.1055	2.3514	5.9132	.9409	3.0108
90	.80516	1.5462	3.5619	.6932	2.0699
91	.57805	.96814	2.0157	.5045	1.3767
92	.40446	.56368	1.0476	.3584	.87220
93	.26750	.29618	.48388	.2407	.51380
94	.16306	.13312	.18770	.1489	.27313
95	.08735	.04578	.05458	.0810	.12418
96	.03697	.00880	.00880	.0348	.04317
97	.00880	.00000	.00000	.0084	.00838

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

(6 per Cent.)

Age.	D	N	S	M	R
		24630.133	353203.6		
10	1588.075	23042.058	328573.5	193.9144	4637.416
11	1492.389	21549.669	305531.4	188.1197	4443.501
12	1402.446	20147.223	283981.8	182.6530	4255.381
13	1317.437	18829.786	263834.6	177.0269	4072.728
14	1237.558	17592.228	245004.8	171.7193	3895.702
15	1162.083	16430.145	227412.5	166.2948	3723.982
16	1090.793	15339.352	210982.4	160.7837	3557.687
17	1023.479	14315.873	195643.0	155.2133	3396.904
18	959.943	13355.930	181327.2	149.6079	3241.690
19	899.987	12455.943	167971.2	143.9892	3092.083
20	843.433	11612.510	155515.3	138.3767	2948.093
21	790.394	10822.116	143902.8	133.0819	2809.717
22	740.661	10081.455	133080.7	128.0868	2676.635
23	693.762	9387.693	122999.2	123.1127	2548.548
24	649.801	8737.892	113611.5	118.4202	2425.435
25	608.360	8129.532	104873.6	113.7602	2307.015
26	569.528	7560.004	96744.1	109.3640	2193.255
27	532.935	7027.069	89184.1	105.0092	2083.891
28	498.465	6528.604	82157.0	100.7053	1978.882
29	466.007	6062.597	75628.4	96.4605	1878.176
30	435.449	5627.148	69565.8	92.2819	1781.716
31	406.859	5220.289	63938.7	88.3398	1689.434
32	379.955	4840.334	58718.4	84.4659	1601.094
33	354.647	4485.687	53878.1	80.6651	1516.628
34	330.989	4154.698	49392.4	77.0794	1435.963
35	308.869	3845.829	45237.7	73.6967	1358.884
36	288.073	3557.756	41391.8	70.3827	1285.187
37	268.640	3289.116	37834.1	67.2563	1214.804
38	250.375	3038.741	34545.0	64.1977	1147.548
39	233.318	2805.423	31506.2	61.3122	1083.350
40	217.388	2588.035	28700.8	58.5900	1022.038
41	202.516	2385.519	26112.8	56.0219	963.448
42	188.629	2196.890	23727.2	53.5992	907.426
43	175.668	2021.222	21530.4	51.3136	853.827
44	163.490	1857.732	19509.1	49.0804	802.513
45	152.057	1705.675	17651.4	46.9009	753.433
46	141.394	1564.281	15945.7	44.8448	706.532
47	131.450	1432.831	14381.4	42.9051	661.687
48	122.118	1310.713	12948.6	41.0142	618.782
49	113.365	1197.348	11637.9	39.1727	577.768
50	105.154	1092.194	10440.6	37.3813	538.595
51	97.411	994.783	9348.4	35.5887	501.214
52	90.110	904.673	8353.6	33.8010	465.625
53	83.233	821.440	7448.9	32.0233	431.824

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

(6 per Cent.)

Age.	D	N	S	M	R
54	76.757	744.683	6627.47	30.2603	399.801
55	70.748	673.935	5882.78	28.5970	369.541
56	65.137	608.798	5208.85	26.9897	340.944
57	59.898	548.900	4600.05	25.4372	313.954
58	55.009	493.891	4051.15	23.9386	288.517
59	50.449	443.442	3557.26	22.4926	264.578
60	46.198	397.244	3113.82	21.0981	242.086
61	42.266	354.978	2716.58	19.7827	220.988
62	38.635	316.343	2361.60	18.5416	201.205
63	35.252	281.091	2045.25	17.3453	182.663
64	32.104	248.987	1764.16	16.1927	165.318
65	29.177	219.810	1515.18	15.0827	149.125
66	26.455	193.355	1295.37	14.0142	134.043
67	23.931	169.424	1102.01	12.9859	120.028
68	21.588	147.836	932.587	11.9969	107.042
69	19.414	128.422	784.751	11.0459	95.0456
70	17.400	111.022	656.329	10.1318	83.9997
71	15.554	95.4680	545.308	9.2695	73.8679
72	13.845	81.6233	449.840	8.4409	64.5984
73	12.280	69.3433	368.216	7.6591	56.1575
74	10.834	58.5097	298.873	6.9083	48.4984
75	9.5121	48.9976	240.363	6.2000	41.5901
76	8.3173	40.6803	191.366	5.5436	35.3901
77	7.2276	33.4527	150.685	4.9244	29.8465
78	6.2446	27.2081	117.233	4.3509	24.9221
79	5.3501	21.8580	90.0246	3.8099	20.5712
80	4.5370	17.3210	68.1666	3.2995	16.7613
81	3.7986	13.5224	50.8456	2.8180	13.4618
82	3.1376	10.3848	37.3232	2.3722	10.6438
83	2.5474	7.8374	26.9384	1.9595	8.2716
84	2.0290	5.8084	19.1010	1.5851	6.3121
85	1.5821	4.2263	13.2926	1.2532	4.7270
86	1.2060	3.0203	9.0663	.9667	3.4738
87	.89880	2.1215	6.0460	.7279	2.5071
88	.65820	1.4633	3.9245	.5381	1.7792
89	.47558	.98772	2.4612	.3927	1.2411
90	.34307	.64465	1.4735	.2871	.84840
91	.24397	.40068	.82885	.2075	.56130
92	.16909	.23159	.42817	.1464	.35380
93	.11080	.12079	.19658	.0977	.20735
94	.06690	.05389	.07579	.0601	.10966
95	.03550	.01839	.02190	.0324	.04960
96	.01488	.00351	.00351	.0138	.01715
97	.00351	.00000	.00000	.0033	.00331

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

(7 per Cent.)

Age.	D	N	Age.	D	N
		19913.077			
10	1445.744	18467.333	54	46.228	413.407
11	1345.938	17121.395	55	42 212	371.195
12	1253.002	15868.393	56	38.501	332.694
13	1166.049	14702.344	57	35.073	297.621
14	1085.111	13617.233	58	31.910	265.711
15	1009.411	12607.822	59	28.990	236.721
16	938.635	11669.187	60	26.300	210.421
17	872.478	10796.709	61	23.836	186.585
18	810.667	9986.042	62	21.584	165.001
19	752.932	9233.110	63	19.509	145.492
20	699.023	8534.087	64	17.603	127.889
21	648.946	7885.141	65	15.847	112.042
22	602.428	7282.713	66	14.235	97.8071
23	559.009	6723.704	67	12.756	85.0511
24	518.693	6205.011	68	11.399	73.6521
25	481.073	5723.938	69	10.156	63 4961
26	446.157	5277.781	70	9.0180	54.4781
27	413.590	4864.191	71	7.9858	46.4923
28	383.224	4480.967	72	7.0413	39.4510
29	354.922	4126.045	73	6.1871	33.2639
30	328.549	3797.496	74	5.4079	27.8560
31	304.108	3493.388	75	4.7038	23.1522
32	281.344	3212.044	76	4.0746	19.0776
33	260.152	2951.892	77	3 5072	15.5704
34	240.526	2711.366	78	3.0023	12.5681
35	222.355	2489.011	79	2.5483	10.0198
36	205.445	2283.566	80	2.1408	7.8790
37	189.797	2093.769	81	1.7756	6.1034
38	175.239	1918.530	82	1.4529	4.6505
39	161.774	1756.756	83	1.1684	3.4821
40	149.319	1607.437	84	.9219	2.5602
41	137.805	1469.632	85	.7123	1.8479
42	127.157	1342.475	86	.5380	1.3099
43	117.312	1225.163	87	.3971	.9128
44	108.159	1117.004	88	.2882	.6246
45	99.654	1017.350	89	.2062	.4184
46	91.800	925 550	90	.1474	.2711
47	84.546	841.004	91	.1038	.1672
48	77.812	763.192	92	.0713	.0960
49	71.558	691.634	93	.0463	.0497
50	65.756	625.878	94	.0277	.0220
51	60.344	565.534	95	.0145	.0075
52	55.299	510.235	96	.0060	.0014
53	50.600	459.635	97	.0014	.0000

TABLE XXI.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the experience of the Equitable.

(8 per Cent.)

Age.	D	N	Age.	D	N
		16320.012			
10	1317.320	15002.692	54	27.974	231.672
11	1215.024	13787.668	55	25.307	206.365
12	1120.655	12667.013	56	22.868	183.497
13	1033.231	11633.782	57	20.640	162.857
14	952.619	10681.163	58	18.603	144.254
15	877.948	9803.215	59	16.745	127.509
16	808.827	8994.388	60	15.050	112.459
17	744.860	8249.528	61	13.515	98.944
18	685.682	7563.846	62	12.125	86.819
19	630.951	6932 895	63	10.858	75.961
20	580.353	6352.542	64	9.7050	66.256
21	533.788	5818.754	65	8.6570	57.599
22	490.938	5327.816	66	7.7050	49.894
23	451.335	4876.481	67	6.8390	43.055
24	414.906	4461.575	68	6.0570	36.998
25	381.253	4080.322	69	5.3450	31.653
26	350.308	3730.014	70	4 7010	26.952
27	321.731	3408.283	71	4.1258	22.826
28	295.348	3112.935	72	3.6043	19.222
29	271.003	2841.932	73	3.1373	16.085
30	248.542	2593.390	74	2.7165	13.368
31	227.923	2365.467	75	2.3410	11.027
32	208.910	2156.557	76	2.0095	9.0179
33	191.385	1965.172	77	1.7135	7.3044
34	175.308	1789.864	78	1.4530	5.8514
35	160.565	1629.299	79	1.2217	4.6297
36	146.981	1482.318	80	1.0171	3.6126
37	134.528	1347.790	81	.8358	2.7768
38	123.057	1224.733	82	.6777	2.0991
39	112.551	1112.182	83	.5399	1.5592
40	102.925	1009.257	84	.4220	1.1372
41	94.107	915.150	85	.3230	.8142
42	86.031	829.119	86	.2416	.5726
43	78.636	750.483	87	.1767	.3959
44	71.829	678.654	88	.1271	.2688
45	65.569	613.085	89	.0901	.1787
46	59.841	553.244	90	.0638	.1149
47	54.604	498.640	91	.0445	.0704
48	49.788	448.852	92	.0303	.0401
49	45.362	403.490	93	.0195	.0206
50	41.299	362.191	94	.0115	.0091
51	37.549	324.642	95	.0060	.0030
52	34.092	290.550	96	.0025	.0006
53	30.904	259.646	97	.0006	.0000

TABLE XXII.

Being a form of preparatory Table for determining the Values of Annuities on Two Joint Lives, according to the Rate of Mortality among the Members of the Equitable, reckoning Interest at 3 per Cent.—*See page* 227.

Difference of Age 19 *Years.*

Ages.	D	N	Ages.	D	N
		56326927	44 & 63	456725	3927151
10 & 29	3047274	53279653	45 - 64	422006	3505145
11 - 30	2919063	50360590	46 - 65	389041	3116104
12 - 31	2795947	47564643	47 - 66	357768	2758336
13 - 32	2675689	44888954	48 - 67	327960	2430376
14 - 33	2559235	42329719	49 - 68	299592	2130784
15 - 34	2446644	39883075	50 - 69	272639	1858145
16 - 35	2337838	37545237	51 - 70	246944	1611201
17 - 36	2231785	35313452	52 - 71	222738	1388463
18 - 37	2129420	33184032	53 - 72	199773	1188690
19 - 38	2029774	31154258	54 - 73	178252	1010438
20 - 39	1933713	29220545	55 - 74	158126	852312
21 - 40	1841834	27378711	56 - 75	139439	712873
22 - 41	1753967	25624744	57 - 76	122307	590566
23 - 42	1669318	23955426	58 - 77	106473	484093
24 - 43	1588409	22367017	59 - 78	92039.3	392054
25 - 44	1509799	20857218	60 - 79	78774.5	313280
26 - 45	1434041	19423177	61 - 80	66671.1	246608
27 - 46	1361194	18061983	62 - 81	55659.3	190949
28 - 47	1291182	16770801	63 - 82	45762.1	145187
29 - 48	1223316	15547485	64 - 83	36910.2	108277
30 - 49	1157592	14389893	65 - 84	29144.6	79132.3
31 - 50	1094446	13295447	66 - 85	22480.4	56651.9
32 - 51	1032839	12262608	67 - 86	16909.4	39742.5
33 - 52	972823	11289785	68 - 87	12402.0	27340.5
34 - 53	914827	10374958	69 - 88	8909.93	18430.5
35 - 54	858833	9516125	70 - 89	6293.60	12136.9
36 - 55	805401	8710724	71 - 90	4427.13	7709.81
37 - 56	754333	7956391	72 - 91	3057.20	4652.61
38 - 57	705244	7251147	73 - 92	2050.18	2602.43
39 - 58	658400	6592747	74 - 93	1292.67	1309.76
40 - 59	613727	5979020	75 - 94	747.55	562.21
41 - 60	571151	5407869	76 - 95	378.39	183.82
42 - 61	530958	4876911	77 - 96	150.39	33.43
43 - 62	493035	4383876	78 - 97	33.43	0

TABLE XXII.

Being a form of preparatory Table for determining the Values of Annuities on Two Joint Lives, according to the Rate of Mortality among the Members of the Equitable, reckoning Interest at 3 per Cent.

Difference of Age 20 *Years.*

Ages.	D	N	Ages.	D	N
		53711197			
10 & 30	2930398	50780799	44 & 64	428055	3577809
11 - 31	2806847	47973952	45 - 65	394698	3183111
12 - 32	2687115	45286837	46 - 66	363047	2820064
13 - 33	2570210	42716627	47 - 67	333038	2487026
14 - 34	2458065	40258562	48 - 68	304459	2182567
15 - 35	2349650	37908912	49 - 69	277284	1905283
16 - 36	2243931	35664981	50 - 70	251488	1653795
17 - 37	2141854	33523127	51 - 71	227157	1426638
18 - 38	2042446	31480681	52 - 72	204039	1222599
19 - 39	1946580	29534101	53 - 73	182346	1040253
20 - 40	1854173	27679928	54 - 74	161843	878410
21 - 41	1765796	25914132	55 - 75	142880	735530
22 - 42	1681286	24232846	56 - 76	125477	610053
23 - 43	1599880	22632966	57 - 77	109374	500679
24 - 44	1521363	21111603	58 - 78	94677.4	406001
25 - 45	1445110	19666493	59 - 79	81152.2	324849
26 - 46	1372316	18294177	60 - 80	68746.1	256103
27 - 47	1302330	16991847	61 - 81	57447.2	198656
28 - 48	1234459	15757388	62 - 82	47315.1	151341
29 - 49	1168701	14588687	63 - 83	38235.4	113105
30 - 50	1105050	13483637	64 - 84	30253.4	82852.0
31 - 51	1043370	12440267	65 - 85	23388.4	59463.6
32 - 52	983249	11457018	66 - 86	17635.9	41827.7
33 - 53	924738	10532280	67 - 87	12970.2	28857.5
34 - 54	868239	9664041	68 - 88	9346.4	19511.1
35 - 55	814666	8849375	69 - 89	6624.2	12886.9
36 - 56	763112	8086263	70 - 90	4672.6	8214.37
37 - 57	713859	7372404	71 - 91	3240.2	4974.20
38 - 58	666543	6705861	72 - 92	2180.7	2793.51
39 - 59	621412	6084449	73 - 93	1382.3	1411.24
40 - 60	578394	5506055	74 - 94	803.22	608.03
41 - 61	537778	4968277	75 - 95	408.25	199.78
42 - 62	499449	4468828	76 - 96	163.27	36.50
43 - 63	462964	4005864	77 - 97	36.50	0

TABLE XXII.

Being a form of preparatory Table for determining the Values of Annuities on Two Joint Lives, according to the Rate of Mortality among the Members of the Equitable, reckoning Interest at 3 per Cent.

Difference of Age 21 *Years.*

Ages.	D	N	Ages.	D	N
		51188355	43 & 64	433902	3648623
10 & 31	2817745	48370610	44 - 65	400356	3248267
11 - 32	2697590	45673020	45 - 66	368326	2879941
12 - 33	2581186	43091834	46 - 67	337953	2541988
13 - 34	2468607	40623227	47 - 68	309173	2232815
14 - 35	2360617	38262610	48 - 69	281788	1951027
15 - 36	2255268	36007342	49 - 70	255772	1695255
16 - 37	2153511	33853831	50 - 71	231337	1463918
17 - 38	2054373	31799458	51 - 72	208087	1255831
18 - 39	1958733	29840725	52 - 73	186241	1069590
19 - 40	1866510	27974215	53 - 74	165561	904029
20 - 41	1777625	26196590	54 - 75	146239	757790
21 - 42	1692625	24503965	55 - 76	128573	629217
22 - 43	1611351	22892614	56 - 77	112209	517008
23 - 44	1532350	21360264	57 - 78	97256.9	419751
24 - 45	1456179	19904085	58 - 79	83478.2	336273
25 - 46	1382909	18521176	59 - 80	70821.1	265452
26 - 47	1312971	17208205	60 - 81	59235.1	206217
27 - 48	1245118	15963087	61 - 82	48834.9	157382
28 - 49	1179347	14783740	62 - 83	39532.9	117849
29 - 50	1115654	13668086	63 - 84	31339.5	86509.7
30 - 51	1053479	12614607	64 - 85	24278.1	62231.6
31 - 52	993274	11621333	65 - 86	18348.2	43883.4
32 - 53	934649	10686684	66 - 87	13527.5	30355.9
33 - 54	877645	9809039	67 - 88	9774.6	20581.3
34 - 55	823589	8985450	68 - 89	6948.7	13632.6
35 - 56	771891	8213559	69 - 90	4918.0	8714.59
36 - 57	722166	7491393	70 - 91	3419.8	5294.78
37 - 58	674686	6816707	71 - 92	2311.2	2983.58
38 - 59	629098	6187609	72 - 93	1470.3	1513.32
39 - 60	585636	5601973	73 - 94	858.88	654.44
40 - 61	544597	5057376	74 - 95	438.65	215.79
41 - 62	505864	4551512	75 - 96	176.16	39.63
42 - 63	468987	4082525	76 - 97	39.63	0

TABLE XXIII.

Being a specimen of preparatory Table for determining the Values of Annuities on Three Joint Lives, according to the rate of mortality among the members of the Equitable, reckoning Interest at 3 per Cent. and supposing the Three Lives of *Equal Ages*.

Common Age.	D	N	Common Age.	D	N
		32094604	53	127095	963932
10	1711626	30382978	54	115267	848665
11	1642595	28740383	55	104374	744291
12	1576247	27164136	56	94187.6	650104
13	1510897	25653239	57	84687.1	565417
14	1448178	24205061	58	75850.5	489566
15	1386491	22818570	59	67655.5	421911
16	1325910	21492660	60	60078.8	361832
17	1266499	20226161	61	53205.0	308627
18	1208319	19017842	62	46980.9	261646
19	1151425	17866417	63	41267.1	220379
20	1095866	16770551	64	36042.2	184337
21	1042849	15727702	65	31284.4	153052
22	992263	14735439	66	26971.5	126081
23	942935	13792504	67	23081.2	102999
24	895920	12896584	68	19591.0	83408.5
25	850139	12046445	69	16478.3	66930.2
26	806556	11239889	70	13720.7	53209.5
27	764178	10475711	71	13330.1	39879.4
28	723029	9752682	72	9239.9	30639.6
29	683132	9069550	73	7454.6	23185.0
30	644502	8425048	74	5919.4	17265.5
31	607889	7817159	75	4633.0	12632.5
32	572493	7244666	76	3581.5	9051 01
33	538324	6706342	77	2717.3	6333.70
34	506020	6200322	78	2026.9	4306.82
35	475488	5724834	79	1473.9	2832.87
36	446066	5278768	80	1039.3	1793.56
37	418299	4860469	81	705.36	1088.20
38	391588	4468881	82	459.70	628.50
39	366419	4102462	83	284.46	344.03
40	342710	3759752	84	166.18	177.85
41	320384	3439368	85	91.11	86.74
42	299368	3140000	86	46.67	40.07
43	279593	2860407	87	22.34	17.73
44	260623	2599784	88	10.15	7.580
45	242456	2357328	89	4.423	3.157
46	225416	2131912	90	1.920	1.236
47	209441	1922471	91	.7987	.4377
48	194180	1728291	92	.3075	.1302
49	179628	1548663	93	.1000	.0302
50	165778	1382885	94	.0254	.0048
51	152382	1230503	95	.0044	.0004
52	139476	1091027	96	.0004	0

TABLE XXIV.

Showing the Values of Annuities on Single Lives, according to the experience of the Equitable.—*See page* 227.

Age.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.	4 per Cent.
10	29.0178	26.0611	23.5717	21.4568	19.6465
11	28.7131	25.8164	23.3731	21.2940	19.5118
12	28.4015	25.5649	23.1681	21.1252	19.3714
13	28.0932	25.3159	22.9651	20.9580	19.2323
14	27.7782	25.0601	22.7554	20.7845	19.0874
15	27.4660	24.8065	22.5475	20.6124	18.9435
16	27.1569	24.5552	22.3413	20.4416	18.8008
17	26.8508	24.3060	22.1368	20.2722	18.6593
18	26.5477	24.0592	21.9340	20.1042	18.5189
19	26.2477	23.8146	21.7331	19.9378	18.3799
20	25.9508	23.5724	21.5340	19.7729	18.2424
21	25.6472	23.3235	21.3286	19.6021	18.0991
22	25.3365	23.0679	21.1167	19.4250	17.9500
23	25.0285	22.8141	20.9061	19.2490	17.8019
24	24.7135	22.5533	20.6889	19.0666	17.6477
25	24.4008	22.2942	20.4727	18.8851	17.4941
26	24.0810	22.0280	20.2496	18.6969	17.3344
27	23.6196	21.7632	20.0276	18.5095	17.1750
28	23.4418	21.4998	19.8065	18.3227	17.0163
29	23.1285	21.2382	19.5865	18.1367	16.8581
30	22.8174	20.9779	19.3677	17.9517	16.7007
31	22.4993	20.7107	19.1421	17.7600	16.5370
32	22.1833	20.4449	18.9173	17.5690	16.3739
33	21.8694	20.1807	18.6937	17.3789	16.2113
34	21.5485	19.9093	18.4631	17.1819	16.0424
35	21.2201	19.6305	18.2252	16.9780	15.8668
36	20.8936	19.3527	17.9879	16.7744	15.6914
37	20.5594	19.0674	17.7432	16.5636	15.5089
38	20.2268	18.7829	17.4988	16.3527	15.3263
39	19.8865	18.4905	17.2466	16.1344	15.1365
40	19.5382	18.1901	16.9865	15.9082	14.9390
41	19.1817	17.8813	16.7179	15.6737	14.7337
42	18.8166	17.5637	16.4406	15.4307	14.5198
43	18.4427	17.2371	16.1542	15.1787	14.2971
44	18.0685	16.9094	15.8661	14.9246	14.0721
45	17.6941	16.5805	15.5763	14.6682	13.8447
46	17.3104	16.2422	15.2769	14.4024	13.6078
47	16.9172	15.8939	14.9674	14.1264	13.3610
48	16.5227	15.5435	14.6552	13.8473	13.1106
49	16.1269	15.1909	14.3400	13.5647	12.8566
50	15.7297	14.8359	14.0218	13.2787	12.5986
51	15.3395	14.4865	13.7083	12.9963	12.3436
52	14.9567	14.1434	13.3996	12.7180	12.0921
53	14.5817	13.8066	13.0964	12.4443	11.8444

TABLE XXIV.

Showing the Values of Annuities on Single Lives, according to the experience of the Equitable.

Age.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
10	18.0863	16.7320	14.5094	12.774	11.389
11	17.9736	16.6370	14.4397	12.721	11.348
12	17.8556	16.5368	14.3658	12.664	11.303
13	17.7388	16.4378	14.2928	12.609	11.259
14	17.6165	16.3337	14.2153	12.549	11.213
15	17.4952	16.2304	14.1385	12.490	11.166
16	17.3748	16.1280	14.0626	12.432	11.120
17	17.2555	16.0266	13.9875	12.375	11.075
18	17.1373	15.9262	13.9133	12.318	11.031
19	17.0203	15.8269	13.8401	12.263	10.988
20	16.9045	15.7289	13.7682	12.209	10.946
21	16.7837	15.6260	13.6920	12.151	10.901
22	16.6571	15.5179	13.6114	12.089	10.852
23	16.5315	15.4106	13.5316	12.028	10.805
24	16.4002	15.2980	13.4471	11.963	10.753
25	16.2695	15.1859	13.3631	11.898	10.702
26	16.1329	15.0683	13.2742	11.829	10.648
27	15.9966	14.9510	13.1856	11.761	10.594
28	15.8607	14.8341	13.0974	11.693	10.540
29	15.7254	14.7177	13.0097	11.625	10.487
30	15.5908	14.6019	12.9227	11.558	10.434
31	15.4502	14.4805	12.8307	11.487	10.378
32	15.3101	14.3596	12.7392	11.417	10.323
33	15.1705	14.2392	12.6483	11.347	10.268
34	15.0250	14.1130	12.5525	11.273	10.210
35	14.8731	13.9810	12.4514	11.194	10.147
36	14.7212	13.8489	12.3503	11.115	10.085
37	14.5626	13.7106	12.2436	11.031	10.019
38	14.4039	13.5720	12.1368	10.948	9.953
39	14.2382	13.4269	12.0241	10.859	9.882
40	14.0652	13.2747	11.9051	10.765	9.806
41	13.8845	13.1152	11.7795	10.665	9.725
42	13.6957	12.9479	11.6467	10.558	9.637
43	13.4982	12.7722	11.5061	10.444	9.544
44	13.2984	12.5940	11.3630	10.328	9.448
45	13.0959	12.4132	11.2174	10.209	9.350
46	12.8842	12.2234	11.0634	10.082	9.245
47	12.6626	12.0240	10.9003	9.947	9.132
48	12.4375	11.8207	10.7332	9.808	9.015
49	12.2082	11.6134	10.5620	9.665	8.895
50	11.9749	11.4017	10.3865	9.518	8.770
51	11.7441	11.1921	10.2123	9.372	8.646
52	11.5160	10.9849	10.0398	9.227	8.523
53	11.2913	10.7805	9.8694	9.084	8.401

TABLE XXIV.

Showing the Values of Annuities on Single Lives, according to the experience of the Equitable.

Age.	2 per Cent.	2½ per Cent.	3 per Cent.	3½ per Cent.	4 per Cent.
54	14.2150	13.4769	12.7992	12.1758	11.6010
55	13.8401	13.1385	12.4930	11.8981	11.3487
56	13.4653	12.7994	12.1854	11.6184	11.0940
57	13.0906	12.4593	11.8762	11.3368	10.8367
58	12.7162	12.1188	11.5658	11.0532	10.5773
59	12.3423	11.7777	11.2542	10.7680	10.3157
60	11.9691	11.4366	10.9417	10.4813	10.0521
61	11.5885	11.0873	10.6208	10.1857	9.7796
62	11.1999	10.7297	10.2908	9.8809	9.4975
63	10.8116	10.3710	9.9592	9.5738	9.2126
64	10.4238	10.0120	9.6262	9.2646	8.9250
65	10.0367	9.6526	9.2922	8.9536	8.6351
66	9.6509	9.2935	8.9575	8.6413	8.3434
67	9.2669	8.9351	8.6227	8.3280	8.0499
68	8.8853	8.5781	8.2883	8.0144	7.7554
69	8.5069	8.2233	7.9551	7.7012	7.4607
70	8.1329	7.8716	7.6241	7.3894	7.1667
71	7.7554	7.5157	7.2882	7.0721	6.8666
72	7.3840	7.1647	6.9561	6.7577	6.5686
73	7.0111	6.8113	6.6209	6.4395	6.2662
74	6.6469	6.4654	6.2922	6.1270	5.9685
75	6.2847	6.1206	5.9636	5.8133	5.6695
76	5.9163	5.7687	5.6272	5.4916	5.3616
77	5.5516	5.4194	5.2926	5.1708	5.0537
78	5.1827	5.0651	4.9520	4.8432	4.7386
79	4.8209	4.7167	4.6163	4.5196	4.4264
80	4.4705	4.3785	4.2897	4.2041	4.1214
81	4.1380	4.0568	3.9784	3.9028	3.8296
82	3.8204	3.7491	3.6801	3.6133	3.5487
83	3.5281	3.4653	3.4046	3.3456	3.2885
84	3.2626	3.2073	3.1537	3.1016	3.0510
85	3.0262	2.9773	2.9299	2.8837	2.8388
86	2.8200	2.7767	2.7347	2.6937	2.6538
87	2.6407	2.6025	2.5652	2.5288	2.4934
88	2.4700	2.4366	2.4039	2.3719	2.3407
89	2.2901	2.2614	2.2333	2.2058	2.1790
90	2.0547	2.0312	2.0080	1.9855	1.9634
91	1.7801	1.7618	1.7438	1.7260	1.7087
92	1.4714	1.4579	1.4447	1.4315	1.4187
93	1.1612	1.1518	1.1427	1.1336	1.1246
94	0.8507	.8448	.8389	.8333	.8275
95	0.5425	.5394	.5362	.5331	.5301
96	0.2451	.2439	.2427	.2415	.2404
97	0	0	0	0	0

TABLE XXIV.

Showing the Values of Annuities on Single Lives, according to the experience of the Equitable.

Age.	4½ per Cent.	5 per Cent.	6 per Cent.	7 per Cent.	8 per Cent.
54	11.0704	10.5794	9.7018	8.943	8.282
55	10.8405	10.3695	9.5257	8.794	8.155
56	10.6079	10.1567	9.3465	8.641	8.024
57	10.3726	9.9410	9.1640	8.486	7.891
58	10.1347	9.7224	8.9785	8.327	7.754
59	9.8943	9.5011	8.7900	8.166	7.615
60	9.6516	9.2773	8.5986	8.001	7.472
61	9.3999	9.0444	8.3982	7.827	7.321
62	9.1384	8.8016	8.1881	7.644	7.161
63	8.8737	8.5553	7.9739	7.457	6.997
64	8.6059	8.3056	7.7558	7.266	6.827
65	8.3353	8.0526	7.5339	7.070	6.654
66	8.0622	7.7968	7.3085	6.871	6.476
67	7.7869	7.5383	7.0799	6.667	6.295
68	7.5102	7.2779	6.8485	6.461	6.110
69	7.2326	7.0161	6.6150	6.251	5.922
70	6.9550	6.7539	6.3802	6.041	5.732
71	6.6710	6.4848	6.1379	5.822	5.533
72	6.3884	6.2165	5.8956	5.602	5.334
73	6.1008	5.9428	5.6472	5.376	5.127
74	5.8172	5.6724	5.4008	5.148	4.921
75	5.5317	5.3997	5.1512	4.922	4.710
76	5.2368	5.1170	4.8912	4.682	4.489
77	4.9413	4.8330	4.6288	4.439	4.263
78	4.6378	4.5408	4.3571	4.186	4.026
79	4.3366	4.2500	4.0855	3.932	3.789
80	4.0416	3.9645	3.8179	3.681	3.552
81	3.7588	3.6905	3.5600	3.438	3.323
82	3.4861	3.4254	3.3097	3.201	3.098
83	3.2331	3.1793	3.0766	2.980	2.888
84	3.0019	2.9543	2.8629	2.777	2.695
85	2.7953	2.7528	2.6714	2.594	2.521
86	2.6150	2.5772	2.5044	2.435	2.369
87	2.4588	2.4251	2.3601	2.298	2.239
88	2.3102	2.2895	2.2229	2.168	2.115
89	2.1526	2.1269	2.0770	2.029	1.983
90	1.9417	1.9204	1.8791	1.839	1.801
91	1.6916	1.6749	1.6422	1.611	1.580
92	1.4061	1.3937	1.3694	1.346	1.323
93	1.1159	1.1072	1.0902	1.074	1.058
94	.8220	.8165	.8056	.795	.785
95	.5271	.5240	.5182	.512	.507
96	.2392	.2381	.2358	.234	.231
97	0	0	0	0	0

TABLE XXV.

Showing the Value of an Annuity on a Single Life, allowing the Purchaser a given rate of interest on the Sum advanced, beside the Premium necessary to secure his Capital by a Life Assurance, according to the Rates charged by the Equitable.—*See page* 247.

Age.	5 per Cent.	6 per Cent.	7 per Cent.
21	13.316	11.685	10.408
22	13.229	11.615	10.354
23	13.141	11.547	10.297
24	13.051	11.476	10.239
25	12.958	11.402	10.180
26	12.863	11.328	10.119
27	12.766	11.251	10.057
28	12.667	11.172	9.992
29	12.565	11.091	9.926
30	12.461	11.009	9.859
31	12.354	10.924	9.789
32	12.244	10.836	9.717
33	12.132	10.746	9.644
34	12.016	10.654	9.568
35	11.898	10.558	9.489
36	11.777	10.461	9.409
37	11.652	10.360	9.326
38	11.524	10.256	9.240
39	11.392	10.150	9.152
40	11.256	10.040	9.061
41	11.119	9.929	8.968
42	10.981	9.817	8.875
43	10.842	9.703	8.780
44	10.700	9.587	8.683
45	10.553	9.467	8.582
46	10.403	9.343	8.479
47	10.248	9.216	8.372
48	10.090	9.085	8.261
49	9.927	8.950	8.147
50	9.762	8.813	8.032
51	9.599	8.677	7.917
52	9.435	8.541	7.800
53	9.267	8.400	7.681
54	9.096	8.257	7.558
55	8.921	8.109	7.432
56	8.742	7.958	7.302
57	8.559	7.803	7.169
58	8.372	7.644	7.032
59	8.181	7.482	6.891
60	7.986	7.315	6.747
61	7.787	7.145	6.599
62	7.585	6.970	6.447
63	7.375	6.790	6.288
64	7.161	6.603	6.126
65	6.939	6.410	5.956

TABLE XXVI.

Showing the Annuity to be required on a Single Life for every £100 advanced, so as to allow the Purchaser a given rate of interest beside the Premium necessary to secure his Capital by a Life Assurance at the Rates charged by the Equitable.—*See page* 252.

Age.	5 per Cent.	6 per Cent.	7 per Cent.
21	7.510	8.558	9.607
22	7.559	8.609	9.658
23	7.610	8.661	9.711
24	7.663	8.714	9.766
25	7.717	8.770	9.823
26	7.774	8.828	9.882
27	7.833	8.888	9.944
28	7.895	8.951	10.008
29	7.959	9.016	10.074
30	8.026	9.084	10.143
31	8.095	9.155	10.216
32	8.167	9.229	10.291
33	8.243	9.306	10.370
34	8.322	9.387	10.452
35	8.405	9.471	10.538
36	8.492	9.560	10.628
37	8.583	9.653	10.723
38	8.678	9.750	10.822
39	8.779	9.852	10.926
40	8.884	9.960	11.036
41	8.994	10.072	11.150
42	9.107	10.187	11.268
43	9.223	10.306	11.389
44	9.346	10.431	11.517
45	9.476	10.564	11.652
46	9.613	10.703	11.794
47	9.758	10.851	11.945
48	9.911	11.007	12.105
49	10.074	11.174	12.274
50	10.244	11.347	12.451
51	10.418	11.524	12.632
52	10.599	11.709	12.820
53	10.790	11.904	13.019
54	10.994	12.112	13.231
55	11.210	12.332	13.456
56	11.439	12.566	13.695
57	11.684	12.816	13.949
58	11.945	13.082	14.221
59	12.223	13.366	14.511
60	12.521	13.671	14.821
61	12.841	13.996	15.154
62	13.184	14.346	15.511
63	13.559	14.730	15.902
64	13.965	15.144	16.324
65	14.412	15.600	16.791

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.—*See pages* 227—233.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
10	10	21.8858	20.0962	18.5409	17.1815	14.9309	13.1567
11	11	21.6075	19.8602	18.3392	17.0078	14.7994	13.0547
12	12	21.3207	19.6157	18.1293	16.8262	14.6608	12.9460
13	13	21.0407	19.3771	17.9245	16.6490	14.5256	12.8403
14	14	20.7521	19.1300	17.7113	16.4639	14.3829	12.7277
15	10	21.1424	19.4645	17.9999	16.7147	14.5759	12.8796
	15	20.4700	18.8883	17.5027	16.2827	14.2434	12.6176
16	11	20.8650	19.2279	17.7967	16.5389	14.4417	12.7746
	16	20.1943	18.6520	17.2988	16.1055	14.1071	12.5101
17	12	20.5869	18.9901	17.5920	16.3615	14.3057	12.6679
	17	19.9251	18.4212	17.0997	15.9325	13.9741	12.4055
18	13	20.3154	18.7581	17.3923	16.1883	14.1733	12.5641
	18	19.6624	18.1962	16.9055	15.7639	13.8447	12.3038
19	14	20.0432	18.5249	17.1911	16.0136	14.0391	12.4586
	19	19.4064	17.9768	16.7161	15.5998	13.7190	12.2054
20	10	20.3797	18.8180	17.4479	16.2401	14.2179	12.6027
	15	19.7775	18.2972	16.9948	15.8433	13.9084	12.3560
	20	19.1572	17.7633	16.5324	15.4405	13.5973	12.1105
21	11	20.1108	18.5881	17.2501	16.0688	14.0872	12.5005
	16	19.5108	18.0683	16.7570	15.6712	13.7759	12.2517
	21	18.9001	17.5422	16.3411	15.2739	13.4691	12.0097
22	12	19.8335	18.3500	17.0444	15.8899	13.9493	12.3920
	17	19.2431	17.8378	16.5973	15.4973	13.6415	12.1456
	22	18.6348	17.3130	16.1419	15.0999	13.3339	11.9026
23	13	19.5625	18.1174	16.8433	15.7151	13.8149	12.2862
	18	18.9816	17.6127	16.4024	15.3275	13.5105	12.0423
	23	18.3755	17.0890	15.9484	14.9299	13.2021	11.7983
24	14	19.2837	17.8762	16.6340	15.5323	13.6729	12.1736
	19	18.7190	17.3862	16.2058	15.1560	13.3777	11.9373
	24	18.1079	16.8568	15.7447	14.7521	13.0631	11.6875
25	10	19.5424	18.1062	16.8378	15.7141	13.8197	12.2940
	15	19.0095	17.6402	16.4292	15.3533	13.5341	12.0636
	20	18.4626	17.1650	16.0139	14.9886	13.2484	11.8354
	25	17.8458	16.6295	15.5463	14.5779	12.9272	11.5793
26	11	19.2640	17.8663	16.6298	15.5328	13.6794	12.1832
	16	18.7343	17.4021	16.2219	15.1719	13.3928	11.9512

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
26	21	18.1979	16.9358	15.8143	14.8138	13.1121	11.7270
	26	17.5757	16.3939	15.3398	14.3963	12.7840	11.4642
27	12	18.9845	17.6120	16.4201	15.3496	13.5372	12.0705
	17	18.4649	17.1690	16.0190	14.9944	13.2545	11.8413
	22	17.9322	16.7050	15.6128	14.6370	12.9738	11.6167
	27	17.3107	16.1628	15.1373	14.2178	12.6434	11.3515
28	13	18.7110	17.3880	16.2148	15.1701	13.3980	11.9603
	18	18.2015	16.9409	15.8205	14.8206	13.1194	11.7341
	23	17.6721	16.4791	15.4156	14.4639	12.8386	11.5091
	28	17.0512	15.9364	14.9388	14.0429	12.5058	11.2413
29	14	18.4365	17.1503	16.0078	14.9890	13.2570	11.8483
	19	17.9440	16.7180	15.6265	14.6509	12.9876	11.6298
	24	17.4109	16.2518	15.2168	14.2890	12.7016	11.3997
	29	16.7973	15.7149	14.7446	13.8719	12.3714	11.1338
30	10	18.6342	17.3290	16.1698	15.1364	13.3801	11.9522
	15	18.1678	16.9175	15.8051	14.8115	13.1190	11.7389
	20	17.6926	16.5005	15.4373	14.4855	12.8595	11.5287
	25	17.1554	16.0294	15.0223	14.1182	12.5678	11.2931
	30	16.5493	15.4984	14.5549	13.7050	12.2405	11.0295
31	11	18.3600	17.0918	15.9636	14.9561	13.2402	11.8418
	16	17.8974	16.6828	15.6003	14.6318	12.9787	11.6272
	21	17.4333	16.2751	15.2405	14.3128	12.7246	11.4215
	26	16.8917	15.7990	14.8199	13.9397	12.4269	11.1800
	31	16.2934	15.2743	15.3577	13.5308	12.1028	10.9189
32	12	18.0850	16.8533	15.7558	14.7741	13.0988	11 7297
	17	17.6327	16.4529	15.3997	14.4558	12.8415	11.5183
	22	17.1731	16.0485	15.0422	14.1385	12.5881	11.3128
	27	16.6335	15.5732	14.6216	13.7647	12.2889	11.0694
	32	16.0429	15.0549	14.1648	13.3604	11.9684	10.8112
33	13	17.8158	16.6199	15.5525	14.5961	12.9605	11.6204
	18	17.3739	16.2281	15.2035	14.2840	12.7076	11.4123
	23	16.9187	15.8269	14.8423	13.9682	12.4549	11.2070
	28	16.3807	15.3523	14.4276	13.5936	12.1543	10.9617
	33	15.7984	14.8407	13.9765	13.1943	11.8376	10.7068
34	14	17.5382	16.3782	15.3409	14.4103	12.8149	11.5044
	19	17.1137	16.0016	15.0054	14.1101	12.5718	11.3044
	24	16.6561	15.5973	14.6467	13.7904	12.3148	11.0948
	29	16.1268	15.1297	14.2319	13.4207	12.0178	10.8523
	34	15.5461	14.6189	13.7808	13.0210	11.7003	10.5965
35	10	17.6618	16.4935.	154486	14.5107	12.9028	11.5815
	15	17.2584	16.1339	151267	14.2215	12.6665	11.3858

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
35	20	16.8516	15.7730	14.8052	13.9339	12.4337	11.1946
	25	16.3917	15.3656	14.4427	13.6101	12.1723	10.9804
	30	15.8714	14.9055	14.0342	13.2458	11.8793	10.7410
	35	15.2857	14.3891	13.5773	12.8401	11.5558	10.4797
36	11	17.3825	16.2504	15.2360	14.3240	12.7570	11.4658
	16	16.9837	15.8940	14.9163	14.0360	12.5208	11.2695
	21	16.5887	15.5432	14.6035	13.7562	12.2941	11.0832
	26	16.1262	15.1322	14.2369	13.4279	12.0278	10.8639
	31	15.6148	14.6797	13.8348	13.0691	11.7389	10.6280
	36	15.0304	14.1638	13.3777	12.6626	11.4144	10.3655
37	12	17.0949	15.9987	15.0150	14.1291	12.6035	11.3431
	17	16.7068	15.6515	14.7030	13.8477	12.3723	11.1504
	22	16.3172	15.3050	14.3937	13.5705	12.1471	10.9650
	27	15.8581	14.8964	14.0285	13.2429	11.8805	10.7450
	32	15.3565	14.4520	13.6334	12.8905	11.5965	10.5130
	37	14.7669	13.9302	13.1700	12.4775	11.2657	10.2447
38	13	16.8120	15.7512	14.7975	13.9373	12.4525	11.2226
	18	16.4348	15.4132	14.4934	13.6627	12.2264	11.0337
	23	16.0508	15.0712	14.1875	13.3881	12.0028	10.8493
	28	15.5951	14.6648	13.8238	13.0613	11.7360	10.6284
	33	15.1035	14.2289	13.4360	12.7151	11.4572	10.4008
	38	14.5080	13.7008	12.9661	12.2956	11.1198	10.1263
39	14	16.5203	15.4949	14.5713	13.7369	12.2937	11.0947
	19	16.1605	15.1723	14.2810	13.4748	12.0776	10.9143
	24	15.7758	14.8288	13.9731	13.1977	11.8510	10.7266
	29	15.3301	14.4308	13.6165	12.8770	11.5888	10.5093
	34	14.8423	13.9978	13.2308	12.5323	11.3107	10.2821
	39	14.2409	13.4631	12.7548	12.1059	10.9664	10.0010
40	10	16.5687	15.5443	14.6210	13.7864	12.3414	11.1400
	15	16.2262	15.2350	14.3415	13.5328	12.1310	10.9632
	20	15.8836	14.9285	14.0656	13.2837	11.9257	10.7920
	25	15.4981	14.5833	13.7554	13.0039	11.6958	10.6007
	30	15.0628	14.1943	13.4065	12.6900	11.4388	10.3876
	35	14.5726	13.7581	13.0172	12.3413	11.1567	10.1564
	40	13.9648	13.2164	12.5330	11.9071	10.8049	9.8683
41	11	16.2650	15.2766	14.3841	13.5765	12.1738	11.0046
	16	15.9269	14.9713	14.1076	13.3247	11.9642	10.8279
	21	15.5976	14.6756	13.8412	13.0839	11.7657	10.6622
	26	15.2112	14.3287	13.5286	12.8013	11.5323	10.4671
	31	14.7867	13.9490	13.1878	12.4944	11.2809	10.2585
	36	14.3003	13.5157	12.8010	12.1472	10.9995	10.0277
	41	13.6793	12.9604	12.3026	11.6995	10.6347	9.7274

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
42	12	15.9516	14.9991	14.1375	13.3561	11.9970	10.8607
	17	15.6248	14.7035	13.8694	13.1121	11.7930	10.6882
	22	15.3021	14.4133	13.6075	12.8750	11.5971	10.5243
	27	14.9208	14.0703	13.2979	12.7165	11.3647	10.3294
	32	14.5075	13.7004	12.9657	12.2952	11.1195	10.1260
	37	14.0188	13.2641	12.5749	11.9443	10.8340	9.8913
	42	13.3837	12.6943	12.0624	11.4821	10.4551	9.5776
43	13	15.6339	14.7169	13.8861	13.1312	11.8153	10.7119
	18	15.3186	14.4313	13.6265	12.8946	11.6171	10.5439
	23	15.0026	14.1467	13.3693	12.6615	11.4238	10.3819
	28	14.6266	13.8077	13.0627	12.3833	11.1926	10.1875
	33	14.2251	13.4482	12.7398	12.0922	10.9541	9.9897
	38	13.7341	13.0088	12.3455	11.7374	10.6645	9.7509
	43	13.0777	12.4176	11.8116	11.2541	10.2654	9.4182
44	14	15.3134	14.4314	13.6309	12.9023	11.6295	10.5592
	19	15.0154	14.1613	13.3853	12.6784	11.4417	10.4000
	24	14.7003	13.8768	13.1276	12.4442	11.2467	10.2358
	29	14.3329	13.5476	12.8295	12.1735	11.0212	10.0459
	34	13.9401	13.1930	12.5106	11.8858	10.7852	9.8500
	39	13.4463	12.7501	12.1123	11.5267	10.4911	9.6067
	44	12.7733	12.1419	11.5613	11.0262	10.0751	9.2578
45	10	15.2812	14.4079	13.6142	12.8915	11.6271	10.5623
	15	14.9954	14.1478	13.3770	12.6743	11.4438	10.4061
	20	14.7153	13.8937	13.1459	12.4636	11.2671	10.2564
	25	14.4009	13.6090	12.8877	12.2281	11.0700	10.0897
	30	14.0475	13.2898	12.5981	11.9651	10.8508	9.9049
	35	13.6521	12.9345	12.2779	11.6758	10.6126	9.7068
	40	13.1551	12.4876	11.8751	11.3119	10.3134	9.4584
	45	12.4706	11.8673	11.3114	10.7984	9.8843	9.0966
46	11	14.9527	14.1144	13.3513	12.6550	11.4341	10.4029
	16	14.6726	13.8588	13.1175	12.4405	11.2523	10.2474
	21	14.4049	13.6158	12.8963	12.2387	11.0829	10.1038
	26	14.0911	13.3309	12.6368	12.0017	10.8836	9.9343
	31	13.7496	13.0221	12.3568	11.7470	10.6873	9.7551
	36	13.3602	12.6718	12.0407	11.4611	10.4353	9.5588
	41	12.8535	12.2148	11.6276	11.0868	10.1259	9.3007
	46	12.1569	11.5814	11.0503	10.5593	9.6826	8.9249
47	12	14.6133	13.8099	13.0771	12.4074	11.2304	10.2325
	17	14.3445	13.5640	12.8519	12.2006	11.0546	10.0826
	22	14.0840	13.3272	12.6360	12.0032	10.8884	9.9414
	27	13.7763	13.0473	12.3805	11.7694	10.6911	9.7732
	32	13.4471	12.7495	12.1103	11.5235	10.4859	9.6000
	37	13.0580	12.3986	11.7932	11.2362	10.2482	9.4016

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
47	42	12.5409	11.9309	11.3691	10.8507	9.9276	9.1328
	47	11.8313	11.2835	10.7771	10.3082	9.4690	8.7417
48	13	14.2755	13.5061	12.8031	12.1595	11.0257	10.0625
	18	14.0178	13.2701	12.5866	11.9603	10.8558	9.9164
	23	13.7648	13.0396	12.3760	11.7675	10.6931	9.7777
	28	13.4632	12.7646	12.1246	11.5371	10.4980	9.6108
	33	13.1467	12.4782	11.8647	11.3005	10.3005	9.4443
	38	12.7578	12.1268	11.5464	11.0116	10.0607	9.2436
	43	12.2233	11.6413	11.1046	10.6086	9.7232	8.9585
	48	11.5055	10.9847	10.5024	10.0551	9.2527	8.5554
49	14	13.9339	13.1979	12.5242	11.9064	10.8154	9.8859
	19	13.6929	12.9770	12.3214	11.7196	10.6561	9.7488
	24	13.4416	12.7475	12.1112	11.5268	10.4925	9.6088
	29	13.1517	12.4828	11.8690	11.3045	10.3039	9.4473
	34	12.8427	12.2028	11.6146	11.0728	10.1103	9.2838
	39	12.4536	11.8505	11.2948	10.7820	9.8681	9.0805
	44	11.9064	11.3519	10.8396	10.3653	9.5169	8.7821
	49	11.1792	10.6848	10.2260	9.7997	9.0335	8.3657
50	10	13.8262	13.1037	12.4418	11.8341	10.7593	9.8420
	15	13.5934	12.8900	12.2450	11.6524	10.6036	9.7074
	20	13.3693	12.6845	12.0562	11.4786	10.4553	9.5798
	25	13.1197	12.4559	11.8463	11.2856	10.2907	9.4383
	30	12.8416	12.2018	11.6136	11.0718	10.1091	9.2824
	35	12.5346	11.9231	11.3598	10.8402	9.9149	9.1181
	40	12.1449	11.5694	11.0382	10.5471	9.6700	8.9119
	45	11.5898	11.0621	10.5736	10.1207	9.3086	8.6034
	50	10.8527	10.3835	9.9476	9.5420	8.8112	8.1724
51	11	13.4886	12.7986	12.1652	11.5826	10.5499	9.6657
	16	13.2613	12.5893	11.9720	11.4039	10.3959	9.5321
	21	13.0492	12.3946	11.7929	11.2388	10.2549	9.4107
	26	12.8009	12.1665	11.5830	11.0452	10.0890	9.2673
	31	12.5348	11.9232	11.3599	10.8401	9.9146	9.1175
	36	12.2350	11.6506	11.1116	10.6133	9.7242	8.9563
	41	11.8383	11.2896	10.7823	10.3125	9.4715	8.7425
	46	11.2741	10.7724	10.3071	9.8751	9.0987	8.4224
	51	10.5371	10.0922	9.6781	9.2923	8.5953	7.9845
52	12	13.1551	12.4965	11.8908	11.3328	10.3412	9.4896
	17	12.9380	12.2962	11.7056	11.1611	10.1929	9.3606
	22	12.7328	12.1075	11.5317	11.0007	10.0554	9.2418
	27	12.4905	11.8845	11.3261	10.8106	9.8919	9.1001
	32	12.2367	11.6522	11.1130	10.6147	9.7251	8.9568
	37	11.9385	11.3806	10.8651	10.3879	9.5341	8.7947
	42	11.5340	11.0113	10.5273	10.0783	9.2726	8.5723

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent	6 per Cent.
Older.	Younger.						
52	47	10.9592	10.4826	10.0401	9.6284	8.8869	8.2393
	52	10.2334	9.8115	9.4183	9.0512	8.3868	7.8027
53	13	12.8308	12.2024	11.6235	11.0892	10.1375	9.3178
	18	12.6238	12.0111	11.4463	10.9247	9.9950	9.1933
	23	12.4255	11.8284	11.2777	10.7688	9.8609	9.0773
	28	12.1891	11.6105	11.0763	10.5823	9.6999	8.9371
	33	11.9478	11.3895	10.8735	10.3959	9.5412	8.8009
	38	11.6510	11.1187	10.6259	10.1689	9.3495	8.6378
	43	11.2320	10.7346	10.2732	9.8445	9.0736	8.4014
	48	10.6506	10.1985	9.7777	9.3858	8.6782	8.0583
	53	9.9420	9.5422	9.1687	8.8197	8.1863	7.6279
54	14	12.5114	11.9123	11.3595	10.8483	9.9356	9.1470
	19	12.3192	11.7347	11.1947	10.6953	9.8028	9.0310
	24	12.1227	11.5531	11.0267	10.5396	9.6683	8.9140
	29	11.8973	11.3449	10.8341	10.3609	9.5138	8.7793
	34	11.6632	11.1306	10.6373	10.1798	9.3594	8.6467
	39	11.3677	10.8602	10.3895	9.9524	9.1667	8.4823
	44	10.9381	10.4651	10.0257	9 6117	8.8792	8.2345
	49	10.3492	9.9203	9.5206	9.1476	8.4728	7.8799
	54	9.6641	9.2851	8.9306	8.5986	7.9949	7.4613
55	10	12.3730	11.7893	11.2499	10.7506	9.8574	9.0840
	15	12.1870	11.6168	11.0896	10.6013	9.7274	8.9701
	20	12.0100	11.4532	10.9378	10.4603	9.6050	8.8631
	25	11.8153	11.2727	10.7705	10.3048	9.4700	8.7451
	30	11.6011	11.0747	10.5871	10.1346	9.3224	8.6161
	35	11.3701	10.8626	10.3919	9.9546	9.1686	8.4837
	40	11.0751	10.5923	10.1438	9.7265	8.9747	8.3178
	45	10.6396	10.1906	9.7727	9.3830	8.6791	8.0618
	50	10.0422	9.6363	9.2573	8.9031	8.2607	7.6948
	55	9.3770	9.0186	8.6829	8 3680	7.7941	7.2853
56	11	12.0457	11.4910	10.9773	10.5010	9.6468	8.9050
	16	11.8645	11.3225	10.8204	10.3546	9.5187	8.7922
	21	11.6985	11.1689	10.6777	10.2219	9.4034	8.6912
	26	11.5054	10.9893	10 5107	10.0662	9.2675	8.5719
	31	11.3027	10.8017	10.3368	9.9046	9.1273	8.4492
	36	11.0793	10.5964	10.1478	9.7303	8.9780	8.3207
	41	10.7797	10.3211	9.8943	9.4966	8.7784	8.1490
	46	10.3372	9.9118	9.5150	9.1446	8.4737	7.8838
	51	9.7416	9.3574	8.9984	8.6622	8 0513	7.5115
	56	9.0916	8.7533	8.4359	8.1375	7.5927	7.1082
57	12	11.7163	11.1898	10.7014	10.2478	9.4322	8.7217
	17	11.5443	11.0296	10.5519	10.1080	9.3095	8.6133
	22	11.3848	10.8817	10.4143	9.9798	9.1978	8.5152

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
57	27	11.1975	10.7072	10.2517	9.8279	9.0647	8.3979
	32	11.0067	10.5305	10.0878	9.6756	8.9323	8.2821
	37	10.7862	10.3275	9.9006	9.5026	8.7838	8.1538
	42	10.4811	10.0463	9.6409	9.2626	8.5777	7.9757
	47	10.0307	9.6283	9.2524	8.9009	8.2628	7.6999
	52	9.4470	9.0841	8.7442	8.4256	7.8451	7.3306
	57	8.8082	8.4893	8.1895	7.9074	7.3909	6.9304
58	13	11.3891	10.8901	10.4263	9.9948	9.2171	8.5374
	18	11.2263	10.7381	10.2843	9.8618	9.0999	8.4336
	23	11.0732	10.5960	10.1519	9.7382	8.9919	8.3386
	28	10.8919	10.4267	9.9937	9.5902	8.8616	8.2232
	33	10.7134	10.2613	9 8403	9.4476	8.7377	8.1148
	38	10.4959	10.0607	9.6549	9.2760	8.5900	7.9869
	43	10.1795	9.7680	9.3836	9.0244	8.3723	7.7976
	48	9.7251	9.3451	8.9895	8.6563	8.0503	7.5141
	53	9.1593	8.8169	8.4955	8.1936	7.6424	7.1526
	58	8.5271	8.2269	7.9443	7.6779	7.1891	6.7520
59	14	11.0599	10.5878	10.1482	9.7384	8.9980	8.3490
	19	10.9106	10.4483	10.0177	9.6161	8.8902	8.2533
	24	10.7597	10.3078	9.8864	9.4933	8.7822	8.1579
	29	10.5889	10.1479	9.7369	9.3531	8.6586	8.0481
	34	10.4184	9.9899	9.5901	9.2166	8.5398	7.9441
	39	10.2035	9.7913	9.4056	9.0463	8.3928	7.8165
	44	9.8796	9.4907	9.1269	8.7861	8.1664	7.6185
	49	9.4205	9.0621	8.7262	8.4111	7.8363	7.3264
	54	8.8793	8.5562	8.2526	7.9669	7.4442	6.9782
	59	8.2485	7.9476	7.7004	7.4493	6.9875	6.5732
60	10	10.8772	10.4219	9.9973	9.6010	8.8835	8.2529
	15	10.7332	10.2870	9.8709	9.4823	8.7785	8.1596
	20	10.5976	10.1603	9.7523	9.3711	8.6804	8.0725
	25	10.4486	10.0213	9.6221	9.2490	8.5724	7.9766
	30	10.2885	9.8712	9.4815	9.1171	8.4558	7.8728
	35	10.1217	9.7163	9.3373	8.9827	8.3386	7.7699
	40	9.9092	9.5196	9.1550	8.8134	8.1921	7.6425
	45	9.5818	9.2149	8.8709	8.5483	7.9602	7.4386
	50	9.1169	8.7796	8.4628	8.1651	7.6209	7.1367
	55	8.5964	8.2924	8.0061	7.7364	7.2416	6.7994
	60	7.9728	7.7082	7.4584	7.2220	6.7864	6.3945
61	11	10.5408	10.1116	9.7107	9.3358	8.6554	8.0554
	16	10.4011	9.9806	9.5876	9.2199	8.5524	7.9634
	21	10.2756	9.8632	9.4775	9.1166	8.4611	7.8823
	26	10.1284	9.7253	9.3480	8.9948	8.3529	7.7857
	31	9.9793	9.5853	9.2168	8.8715	8.2436	7.6883
	36	9.8207	9.4379	9.0795	8.7436	8.1318	7.5901

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
61	41	9.6058	9.2385	8.8942	8.5711	7.9819	7.4591
	46	9.2744	8.9290	8.6049	8.3003	7.7437	7.2485
	51	8.8130	8.4960	8.1978	7.9171	7.4028	6.9438
	56	8.3098	8.0243	7.7551	7.5010	7.0338	6.6150
	61	7.6887	7.4414	7.2074	6.9857	6.5762	6.2068
62	12	10.1948	9.7914	9.4139	9.0601	8.4166	7.8474
	17	10.0635	9.6680	9.2976	8.9506	8.3189	7.7598
	22	9.9442	9.5561	9.1926	8.8519	8.2314	7.6818
	27	9.8027	9.4233	9.0676	8.7340	8.1262	7.5875
	32	9.6650	9.2939	8.9462	8.6199	8.0249	7.4972
	37	9.5105	9.1501	8.8121	8.4947	7.9153	7.4006
	42	9.2928	8.9475	8.6233	8.3185	7.7613	7.2655
	47	8.9562	8.6324	8.3278	8.0410	7.5159	7.0471
	52	8.5085	8.2111	7.9310	7.6669	7.1818	6.7476
	57	8.0190	7.7516	7.4991	7.2603	6.8204	6.4248
	62	7.3954	7.1650	6.9466	6.7394	6.3557	6.0086
63	13	9.8504	9.4719	9.1170	8.7839	8.1764	7.6372
	18	9.7274	9.3560	9.0077	8.6807	8.0840	7.5542
	23	9.6142	9.2498	8.9078	8.5866	8.0003	7.4794
	28	9.4784	9.1221	8.7873	8.4727	7.8982	7.3875
	33	9.3525	9.0037	8.6761	8.3682	7.8055	7.3048
	38	9.2022	8.8635	8.5452	8.2458	7.6980	7.2100
	43	8.9763	8.6527	8.3480	8.0612	7.5356	7.0664
	48	8.6387	8.3355	8.0497	7.7803	7.2858	6.8432
	53	8.2097	7.9313	7.6684	7.4202	6.9633	6.5532
	58	7.7299	7.4800	7.2435	7.0196	6.6061	6.2332
	63	7.1035	6.8893	6.6860	6.4928	6.1342	5.8088
64	14	9.5041	9.1497	8.8168	8.5039	7.9316	7.4221
	19	9.3930	9.0450	8.7180	8.4104	7.8478	7.3467
	24	9.2821	8.9406	8.6190	8.3174	7.7647	7.2721
	29	9.1559	8.8217	8.5071	8.2111	7.6691	7.1858
	34	9.0381	8.7108	8.4029	8.1131	7.5820	7.1079
	39	8.8917	8.5741	8.2751	7.9934	7.4767	7.0149
	44	8.6614	8.3583	8.0726	7.8033	7.3084	6.8652
	49	8.3216	8.0382	7.7708	7.5182	7.0535	6.6363
	54	7.9173	7.6569	7.4106	7.1777	6.7480	6.3610
	59	7.4429	7.2098	6.9888	6.7793	6.3914	6.0406
	64	6.8133	6.6147	6.4258	6.2460	5.9117	5.6073
65	10	9.2654	8.9284	8.6114	8.3129	7.7659	7.2776
	15	9.1594	8.8283	8.5168	8.2233	7.6853	7.2049
	20	9.0606	8.7351	8.4287	8.1400	7.6106	7.1375
	25	8.9518	8.6323	8.3315	8.0481	7.5280	7.0630
	30	8.8353	8.5226	8.2276	7.9495	7.4392	6.9826
	35	8.7218	8.4155	8.1268	7.8545	7.3545	6.9067

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		2½ per Cent.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
65	40	8.5792	8.2821	8.0001	7.7374	7.2512	6.8153
	45	8.3478	8.0647	7.7974	7.5448	7.0799	6.6623
	50	8.0049	7.7408	7.4909	7.2547	6.8189	6.4265
	55	7.6220	7.3791	7.1489	6.9309	6.5278	6.1637
	60	7.1582	6.9413	6.7353	6.5396	6.1766	5.8472
	65	6.5251	6.3413	6.1663	5.9995	5.6885	5.4046
66	11	8.9189	8.6048	8.3088	8.0295	7.5164	7.0570
	16	8.8169	8.5083	8.2172	7.9426	7.4379	6.9857
	21	8.7270	8.4233	8.1369	7.8665	7.3696	6.9241
	26	8.6198	8.3218	8.0406	7.7752	7.2871	6.8493
	31	8.5132	8.2213	7.9453	7.6847	7.2053	6.7751
	36	8.4080	8.1218	7.8516	7.5964	7.1265	6.7044
	41	8.2649	7.9876	7.7258	7.4781	7.0218	6.6113
	46	8.0315	7.7678	7.5183	7.2823	6.8466	6.4541
	51	7.6935	7.4476	7.2147	6.9940	6.5861	6.2176
	56	7.3287	7.1025	6.8879	6.6843	6.3070	5.9652
	61	6.8711	6.6697	6.4783	6.2961	5.9573	5.6491
	66	6.2393	6.0699	5.9081	5.7537	5.4652	5.2011
67	12	8.5718	8.2798	8.0040	7.7434	7.2634	6.8322
	17	8.4770	8.1898	7.9185	7.6621	7.1897	6.7651
	22	8.3923	8.1098	7.8427	7.5902	7.1250	6.7065
	27	8.2903	8.0128	7.7506	7.5026	7.0454	6.6341
	32	8.1937	7.9218	7.6642	7.4206	6.9711	6.5665
	37	8.0930	7.8264	7.5742	7.3356	6.8951	6.4983
	42	7.9489	7.6910	7.4468	7.2156	6.7884	6.4030
	47	7.7128	7.4677	7.2356	7.0156	6.6085	6.2406
	52	7.3879	7.1594	6.9426	6.7368	6.3556	6.0102
	57	7.0378	6.8277	6.6289	6.4383	6.0859	5.7657
	62	6.5815	6.3952	6.2177	6.0486	5.7335	5.4459
	67	5.9567	5.8007	5.6516	5.5091	5.2421	4.9970
68	13	8.2279	7.9570	7.7007	7.4581	7.0101	6.6063
	18	8.1401	7.8734	7.6212	7.3823	6.9412	6.5433
	23	8.0608	7.7984	7.5500	7.3147	6.8801	6.4879
	28	7.9636	7.7058	7.4618	7.2306	6.8034	6.4176
	33	7.8775	7.6247	7.3847	7.1574	6.7370	6.3574
	38	7.7814	7.5335	7.2986	7.0761	6.6641	6.2918
	43	7.6318	7.3924	7.1655	6.9501	6.5513	6.1905
	48	7.3958	7.1687	6.9532	6.7486	6.3692	6.0251
	53	7.0886	6.8767	6.6753	6.4838	6.1281	5.8050
	58	6.7497	6.5551	6.3698	6.1934	5.8650	5.5658
	63	6.2945	6.1226	5.9586	5.8020	5.5096	5.2420
	68	5.6779	5.5348	5.3977	5.2664	5.0201	4.7934
69	14	7.8847	7.6340	7.3965	7.1712	6.7543	6.3772
	19	7.8069	7.5600	7.3259	7.1040	6.6929	6.3211

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
69	24	7.7297	7.4866	7.2562	7.0377	6.6326	6.2661
	29	7.6405	7.4016	7.1751	6.9601	6.5617	6.2011
	34	7.5617	7.3274	7.1045	6.8929	6.5008	6.1456
	39	7.4700	7.2402	7.0221	6.8150	6.4308	6.0825
	44	7.3179	7.0963	6.8858	6.6858	6.3145	5.9773
	49	7.0812	6.8712	6.6717	6.4819	6.1292	5.8084
	54	6.7968	6.6006	6.4138	6.2359	5.9048	5.6030
	59	6.4650	6.2855	6.1139	5.9503	5.6451	5.3661
	64	6.0109	5.8527	5.7014	5.5569	5.2864	5.0380
	69	5.4040	5.2730	5.1473	5.0268	4.8002	4.5908
70	10	7.6192	7.3847	7.1619	6.9502	6.5576	6.2015
	15	7.5460	7.3147	7.0951	6.8865	6.4994	6.1481
	20	7.4785	7.2504	7.0338	6.8280	6.4460	6.0992
	25	7.4030	7.1785	6.9652	6.7625	6.3862	6.0446
	30	7.3220	7.1011	6.8913	6.6918	6.3213	5.9848
	35	7.2472	7.0306	6.8241	6.6278	6.2631	5.9316
	40	7.1600	6.9475	6.7283	6.5533	6.1961	5.8711
	45	7.0080	6.8034	6.6087	6.4234	6.0785	5.7644
	50	6.7696	6.5760	6.3918	6.2162	5.8891	5.5907
	55	6.5051	6.3240	6.1512	5.9865	5.6791	5.3982
	60	6.1860	6.0198	5.8613	5.7101	5.4270	5.1678
	65	5.7317	5.5863	5.4473	5.3142	5.0645	4.8346
	70	5.1363	5.0167	4.9019	4.7916	4.5836	4.3910
71	11	7.2747	7.0591	6.8538	6.6586	6.2954	5.9650
	16	7.2047	6.9920	6.7897	6.5972	6.2391	5.9131
	21	7.1446	6.9347	6.7350	6.5450	6.1914	5.8693
	26	7.0706	6.8640	6.6674	6.4803	6.1320	5.8147
	31	6.9979	6.7945	6.6008	6.4165	6.0733	5.7605
	36	6.9365	6.7310	6.5403	6.3588	6.0207	5.7124
	41	6.8440	6.6484	6.4621	6.2845	5.9536	5.6517
	46	6.6911	6.5035	6.3242	6.1532	5.8342	5.5428
	51	6.4583	6.2804	6.1107	5.9488	5.6464	5.3698
	56	6.2111	6.0445	5.8853	5.7333	5.4490	5.1883
	61	5.9001	5.7478	5.6021	5.4627	5.2014	4.9613
	66	5.4509	5.3181	5.1908	5.0688	4.8392	4.6273
	71	4.8646	4.7561	4.6516	4.5511	4.3612	4.1848
72	12	6.9336	6.7360	6.5476	6.3680	6.0331	5.7275
	17	6.8694	6.6744	6.4885	6.3113	5.9809	5.6791
	22	6.8138	6.6213	6.4381	6.2628	5.9365	5.6384
	27	6.7438	6.5543	6.3735	6.2013	5.8797	5.5858
	32	6.6797	6.4927	6.3146	6.1446	5.8275	5.5376
	37	6.6173	6.4333	6.2579	6.0906	5.7780	5.4922
	42	6.5308	6.3509	6.1795	6.0160	5.7105	5.4309
	47	6.3768	6.2043	6.0397	5.8824	5.5884	5.3189

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
72	52	6.1551	5.9920	5.8361	5.6871	5.4082	5.1522
	57	5.9222	5.7695	5.6232	5.4833	5.2210	4.9798
	62	5.6154	5.4761	5.3426	5.2146	4.9743	4.7528
	67	5.1760	5.0549	4.9387	4.8271	4.6167	4.4219
	72	4.6010	4.5027	4.4079	4.3166	4.1438	3.9828
73	13	6.5916	6.4113	6.2389	6.0744	5.7668	5.4852
	18	6.5331	6.3549	6.1848	6.0224	5.7187	5.4405
	23	6.4821	6.3061	6.1381	5.9776	5.6776	5.4027
	28	6.4159	6.2426	6.0771	5.9190	5.6233	5.3523
	33	6.3604	6.1895	6.0261	5.8701	5.5781	5.3104
	38	6.3026	6.1342	5.9734	5.8196	5.5319	5.2680
	43	6.2124	6.0484	5.8915	5.7416	5.4607	5.2029
	48	6.0599	5.9025	5.7519	5.6079	5.3379	5.0898
	53	5.8539	5.7048	5.5621	5.4255	5.1692	4.9332
	58	5.6326	5.4930	5.3592	5.2309	4.9899	4.7676
	63	5.3300	5.2030	5.0812	4.9642	4.7441	4.5405
	68	4.9016	4.7917	4.6861	4.5844	4.3923	4.2140
	73	4.3355	4.2470	4.1615	4.0791	3.9226	3.7764
74	14	6.2556	6.0916	5.9345	5.7842	5.5026	5.2439
	19	6.2052	6.0429	5.8877	5.7391	5.4608	5.2051
	24	6.1559	5.9956	5.8423	5.6956	5.4207	5.1680
	29	6.0962	5.9382	5.7870	5.6426	5.3712	5.1219
	34	6.0469	5.8908	5.7416	5.5987	5.3308	5.0844
	39	5.9934	5.8397	5.6927	5.5519	5.2879	5.0450
	44	5.9021	5.7526	5.6094	5.4723	5.2149	4.9779
	49	5.7498	5.6065	5.4692	5.3377	5.0906	4.8629
	54	5.5635	5.4275	5.2971	5.1721	4.9371	4.7201
	59	5.3505	5.2234	5.1012	4.9839	4.7631	4.5588
	64	5.0516	4.9363	4.8254	4.7189	4.5178	4.3313
	69	4.6356	4.5361	4.4403	4.3479	4.1731	4.0104
	74	4.0813	4.0018	3.9249	3.8506	3.7094	3.5771
75	10	5.9691	5.8183	5.6737	5.5356	5.2759	5.0366
	15	5.9220	5.7730	5.6304	5.4937	5.2370	5.0003
	20	5.8795	5.7322	5.5911	5.4559	5.2019	4.9677
	25	5.8316	5.6861	5.5468	5.4133	5.1624	4.9311
	30	5.7783	5.6349	5.4973	5.3656	5.1179	4.8895
	35	5.7325	5.5908	5.4549	5.3248	5.0800	4.8542
	40	5.6833	5.5438	5.4099	5.2817	5.0405	4.8178
	45	5.5934	5.4576	5.3275	5.2026	4.9677	4.7507
	50	5.4403	5.3104	5.1858	5.0662	4.8411	4.6328
	55	5.2713	5.1479	5.0293	4.9154	4.7009	4.5023
	60	5.0706	4.9552	4.8441	4.7374	4.5359	4.3490
	65	4.7751	4.6708	4.5704	4.4737	4.2908	4.1207
	70	4.3734	4.2838	4.1973	4.1138	3.9554	3.8075

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
75	75	3.8295	3.7586	3.6899	3.6233	3.4965	3.3774
76	11	5.6267	5.4908	5.3605	5.2354	5.0001	4.7825
	16	5.5818	5.4479	5.3190	5.1954	4.9627	4.7475
	21	5.5456	5.4127	5.2852	5.1629	4.9325	4.7194
	26	5.4988	5.3676	5.2418	5.1209	4.8934	4.6830
	31	5.4521	5.3226	5.1982	5.0789	4.8540	4.6461
	36	5.4127	5.2844	5.1615	5.0435	4.8211	4.6153
	41	5.3648	5.2388	5.1178	5.0016	4.7826	4.5798
	46	5.2756	5.1531	5.0356	4.9226	4.7096	4.5122
	51	5.1270	5.0100	4.8974	4.7893	4.5851	4.3958
	56	4.9733	4.8619	4.7547	4.6516	4.4569	4.2761
	61	4.7820	4.6780	4.5776	4.4811	4.2984	4.1285
	66	4.4940	4.4002	4.3097	4.2225	4.0572	3.9030
	71	4.1047	4.0244	3.9468	3.8719	3.7294	3.5959
	76	3.5692	3.5064	3.4455	3.3864	3.2736	3.1674
77	12	5.2858	5.1640	5.0469	4.9344	4.7220	4.5252
	17	5.2453	5.1252	5.0093	4.8981	4.6880	4.4933
	22	5.2128	5.0935	4.9789	4.8677	4.6607	4.4678
	27	5.1692	5.0514	4.9381	4.8293	4.6239	4.4332
	32	5.1290	5.0126	4.9006	4.7930	4.5898	4.4013
	37	5.0931	4.9780	4.8673	4.7609	4.5598	4.3732
	42	5.0467	4.9334	4.8245	4.7198	4.5219	4.3382
	47	4.9574	4.8475	4.7418	4.6401	4.4479	4.2693
	52	4.8187	4.7135	4.6123	4.5148	4.3306	4.1591
	57	4.6778	4.5777	4.4812	4.3882	4.2130	4.0485
	62	4.4925	4.3991	4.3090	4.2221	4.0574	3.9037
	67	4.2158	4.1319	4.0507	3.9724	3.8237	3.6846
	72	3.8411	3.7696	3.7004	3.6333	3.5057	3.3858
	77	3.3121	3.2569	3.2032	3.1511	3.0515	2.9573
78	13	4.9408	4.8322	4.7276	4.6269	4.4366	4.2596
	18	4.9044	4.7974	4.6939	4.5943	4.4059	4.2306
	23	4.8757	4.7692	4.6667	4.5681	4.3815	4.2078
	28	4.8349	4.7298	4.6285	4.5311	4.3467	4.1751
	33	4.8016	4.6975	4.5973	4.5008	4.3183	4.1484
	38	4.7695	4.6666	4.5675	4.4721	4.2914	4.1232
	43	4.7214	4.6203	4.5229	4.4291	4.2515	4.0861
	48	4.6338	4.5358	4.4414	4.3505	4.1781	4.0176
	53	4.5079	4.4140	4.3234	4.2361	4.0707	3.9164
	58	4.3778	4.2883	4.2019	4.1186	3.9607	3.8132
	63	4.1982	4.1151	4.0346	3.9569	3.8094	3.6713
	68	3.9343	3.8596	3.7873	3.7174	3.5844	3.4597
	73	3.5724	3.5091	3.4479	3.3884	3.2749	3.1680
	78	3.0471	2.9990	2.9525	2.9068	2.8196	2.7370
79	14	4.6004	4.5039	4.4110	4.3213	4.1515	3.9931

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		2½ per Cent.	3 per Cent.	3½ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
79	19	4.5699	4.4749	4.3828	4.2941	4.1258	3.9688
	24	4.5427	4.4481	4.3569	4.2690	4.1023	3.9468
	29	4.5066	4.4132	4.3230	4.2361	4.0713	3.9176
	34	4.4780	4.3854	4.2961	4.2101	4.0468	3.8944
	39	4.4497	4.3581	4.2698	4.1846	4.0230	3.8722
	44	4.4016	4.3117	4.2250	4.1414	3.9826	3.8344
	49	4.3150	4.2279	4.1440	4.0630	3.9092	3.7654
	54	4.2047	4.1211	4.0404	3.9625	3.8146	3.6762
	59	4.0826	4.0030	3.9260	3.8518	3.7105	3.5783
	64	3.9096	3.8348	3.7632	3.6941	3.5624	3.4389
	69	3.6579	3.5918	3.5277	3.4656	3.3472	3.2359
	74	3.3114	3.2557	3.2016	3.1492	3.0488	2.9540
	79	2.7869	2.7453	2.7048	2.6654	2.5896	2.5177
80	10	4.2971	4.2110	4.1279	4.0476	3.8951	3.7526
	15	4.2704	4.1851	4.1027	4.0231	3.8721	3.7308
	20	4.2459	4.1619	4.0801	4.0013	3.8515	3.7114
	25	4.2198	4.1360	4.0552	3.9771	3.8287	3.6899
	30	4.1883	4.1055	4.0255	3.9482	3.8015	3.6641
	35	4.1622	4.0802	4.0009	3.9244	3.7789	3.6428
	40	4.1376	4.0564	3.9779	3.9022	3.7582	3.6233
	45	4.0911	4.0115	3.9345	3.8602	3.7189	3.5865
	50	4.0042	3.9272	3.8529	3.7809	3.6442	2.5161
	55	3.9074	3.8333	3.7616	3.6924	3.5607	3.4371
	60	3.7960	3.7253	3.6571	3.5910	3.4652	3.3470
	65	3.6267	3.5613	3.4980	3.4366	3.3197	3.2098
	70	3.3903	3.3319	3.2753	3.2203	3.1154	3.0164
	75	3.0572	3.0084	2.9610	2.9149	2.8266	2.7429
	80	2.5355	2.4997	2.4648	2.4308	2.3653	2.3030
81	11	3.9822	3.9061	3.8326	3.7615	3.6262	3.4994
	16	3.9569	3.8816	3.8087	3.7383	3.6042	3.4785
	21	3.9367	3.8625	3.7901	3.7203	3.5872	3.4624
	26	3.9112	3.8371	3.7656	3.6965	3.5647	3.4412
	31	3.8841	3.8109	3.7400	3.6715	3.5411	3.4187
	36	3.8624	3.7898	3.7196	3.6516	3.5223	3.4009
	41	3.8393	3.7675	3.6979	3.6307	3.5027	3.3824
	46	3.7938	3.7233	3.6552	3.5892	3.4638	3.3459
	51	3.7096	3.6417	3.5759	3.5122	3.3908	3.2768
	56	3.6241	3.5586	3.4951	3.4337	3.3166	3.2064
	61	3.5205	3.4581	3.3976	3.3390	3.2273	3.1220
	66	3.3578	3.3000	3.2441	3.1898	3.0861	2.9884
	71	3.1327	3.0813	3.0313	2.9829	2.8901	2.8025
	76	2.8095	2.7670	2.7256	2.6853	2.6080	2.5346
	81	2.2995	2.2688	2.2388	2.2096	2.1532	2.0993
82	12	3.6799	3.6129	3.5480	3.4852	3.3655	3.2529

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{3}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
82	17	3.6573	3.5909	3.5266	3.4644	3.3457	3.2342
	22	3.3693	3.5743	3.5104	3.4486	3.3308	3.2200
	27	3.6160	3.5508	3.4876	3.4264	3.3098	3.2000
	32	3.5932	3.5286	3.4659	3.4053	3.2898	3.1810
	37	3.5741	3.5100	3.4481	3.3878	3.2731	3.1652
	42	3.5522	3.4888	3.4273	3.3678	3.2543	3.1473
	47	3.5066	3.4446	3.3845	3.3262	3.2151	3.1104
	52	3.4289	3.3689	3.3108	3.2545	3.1469	3.0456
	57	3.3525	3.2946	3.2385	3.1842	3.0803	2.9823
	62	3.2536	3.1986	3.1452	3.0934	2.9944	2.9010
	67	3.0996	3.0488	2.9995	2.9516	2.8599	2.7732
	72	2.8867	2.8416	2.7977	2.7551	2.6733	2.5958
	77	2.5707	2.5339	2.4978	2.4628	2.3954	2.3313
	82	2.0744	2.0481	2.0225	1.9974	1.9490	1.9026
83	13	3.4016	3.3426	3.2853	3.2298	3.1238	3.0238
	18	3.3815	3.3229	3.2662	3.2111	3.1059	3.0067
	23	3.3657	3.3085	3.2521	3.1975	3.0930	2.9945
	28	3.3440	3.2864	3.2306	3.1765	3.0731	2.9754
	33	3.3256	3.2685	3.2131	3.1594	3.0568	2.9600
	38	3.3089	3.2523	3.1973	3.1441	3.0422	2.9461
	43	3.2859	3.2299	3.1756	3.1229	3.0222	2.9271
	48	3.2412	3.1865	3.1334	3.0819	2.9835	2.8905
	53	3.1711	3.1182	3.0668	3.0169	2.9216	2.8315
	58	3.1017	3.0506	3.0018	2.9528	2.8606	2.7735
	63	3.0067	2.9581	2 9109	2.8652	2.7775	2.6945
	68	2.8610	2.8162	2.7727	2.7303	2.6492	2.5723
	73	2.6571	2.6175	2.5789	2.5414	2.4693	2.4008
	78	2.3431	2.3112	2.2800	2.2496	2.1911	2.1352
	83	1.8710	1.8484	1.8264	1.8048	1.7631	1.7232
84	14	3.1476	3.0955	3.0449	2.9958	2.9018	2.8129
	19	3.1311	3.0794	3.0292	2.9804	2.8870	2.7988
	24	3.1163	3.0657	3.0157	2.9673	2.8746	2.7870
	29	3.0969	3.0461	2.9967	2.9487	2.8569	2.7700
	34	3.0814	3.0309	2.9818	2.9342	2.8430	2.7568
	39	3.0671	3.0170	2.9683	2.9210	2.8305	2.7448
	44	3.0440	2.9944	2.9464	2.8995	2.8102	2.7255
	49	2.9991	2.9508	2.9038	2.8582	2.7709	2.6882
	54	2.9385	2.8917	2.8461	2.8019	2.7171	2.6368
	59	2.8738	2.8285	2.7845	2.7417	2.6598	2.5821
	64	2.7815	2.7386	2.6969	2.6563	2.5784	2.5046
	69	2.6436	2.6042	2.5657	2.5282	2.4563	2.3879
	74	2.4496	2.4148	2.3807	2.3476	2.2840	2.2233
	79	2.1325	2.1048	2.0778	2.0515	2.0006	1.9520
	84	1.6907	1.6713	1.6522	1.6335	1.5974	1.5627

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
85	15	2.9215	2.8754	2.8306	2.7870	2.7034	2.6241
	20	2.9086	2.8628	2.8183	2.7749	2.6919	2.6131
	25	2.8936	2.8492	2.8052	2.7622	2.6797	2.6015
	30	2.8773	2.8322	2.7884	2.7457	2.6640	2.5864
	35	2.8629	2.8182	2.7746	2.7323	2.6511	2.5741
	40	2.8511	2.8066	2.7634	2.7213	2.6406	2.5641
	45	2.8287	2.7848	2.7422	2.7006	2.6210	2.5454
	50	2.7824	2.7397	2.6981	2.6576	2.5799	2.5060
	55	2.7296	2.6881	2.6476	2.6082	2.5327	2.4609
	60	2.6712	2.6311	2.5919	2.5539	2.4808	2.4113
	65	2.5804	2.5424	2.5054	2.4694	2.4000	2.3341
	70	2.4506	2.4156	2.3814	2.3481	2.2842	2.2232
	75	2.2639	2.2332	2.2031	2.1738	2.1174	2.0636
	80	1.9419	1.9180	1.8945	1.8716	1.8274	1.7850
	85	1.5366	1.5196	1.5029	1.4866	1.4550	1.4246
86	16	2.7247	2.6838	2.6439	2.6052	2.5306	2.4598
	21	2.7144	2.6737	2.6341	2.5955	2.5214	2.4509
	26	2.6989	2.6602	2.6208	2.5826	2.5090	2.4390
	31	2.6852	2.6452	2.6062	2.5682	2.4952	2.4258
	36	2.6735	2.6337	2.5949	2.5572	2.4846	2.4156
	41	2.6625	2.6230	2.5844	2.5470	2.4749	2.4063
	46	2.6404	2.6015	2.5634	2.5265	2.4554	2.3877
	51	2.5945	2.5565	2.5195	2.4834	2.4141	2.3481
	56	2.5480	2.5110	2.4749	2.4399	2.3723	2.3080
	61	2.4939	2.4582	2.4233	2.3893	2.3240	2.2617
	66	2.4056	2.3717	2.3389	2.3066	2.2447	2.1857
	71	2.2809	2.2498	2.2194	2.1898	2.1327	2.0781
	76	2.0984	2.0712	2.0446	2.0187	1.9686	1.9207
	81	1.7756	1.7547	1.7343	1.7142	1.6756	1.6384
	86	1.4122	1.3972	1.3824	1.3680	1.3399	1.3128
87	17	2.5542	2.5179	2.4825	2.4480	2.3816	2.3182
	22	2.5453	2.5092	2.4740	2.4397	2.3735	2.3105
	27	2.5301	2.4964	2.4614	2.4274	2.3617	2.2991
	32	2.5193	2.4837	2.4490	2.4152	2.3500	2.2878
	37	2.5089	2.4735	2.4390	2.4053	2.3405	2.2787
	42	2.4983	2.4635	2.4292	2.3958	2.3314	2.2700
	47	2.4762	2.4416	2.4078	2.3749	2.3113	2.2508
	52	2.4328	2.3991	2.3661	2.3339	2.2720	2.2129
	57	2.3914	2.3585	2.3263	2.2950	2.2346	2.1769
	62	2.3395	2.3077	2.2766	2.2463	2.1878	2.1320
	67	2.2549	2.2249	2.1955	2.1668	2.1114	2.0585
	72	2.1364	2.1086	2.0815	2.0551	2.0040	1.9551
	77	1.9556	1.9316	1.9080	1.8850	1.8405	1.7978
	82	1.6309	1.6127	1.5947	1.5773	1.5433	1.5106
	87	1.3191	1.3056	1.2923	1.2793	1.2540	1.2295

TABLE XXVII.

Showing the Values of Annuities on Two Joint Lives, according to the rate of mortality among the members of the Equitable.

Age.		$2\frac{1}{2}$ per Cent.	3 per Cent.	$3\frac{1}{2}$ per Cent.	4 per Cent.	5 per Cent.	6 per Cent.
Older.	Younger.						
88	18	2.3925	2.3606	2.3295	2.2991	2.2404	2.1842
	23	2.3852	2.3534	2.3224	2.2922	2.2337	2.1778
	28	2.3699	2.3412	2.3103	2.2803	2.2223	2.1668
	33	2.3624	2.3311	2.3005	2.2706	2.2129	2.1578
	38	2.3534	2.3222	2.2915	2.2621	2.2048	2.1498
	43	2.3424	2.3115	2.2812	2.2517	2.1948	2.1402
	48	2.3205	2.2900	2.2602	2.2312	2.1750	2.1212
	53	2.2811	2.2514	2.2223	2.1939	2.1390	2.0865
	58	2.2440	2.2148	2.1864	2.1587	2.1051	2.0537
	63	2.1942	2.1661	2.1386	2.1118	2.0599	2.0102
	68	2.1142	2.0876	2.0615	2.0361	1.9869	1.9398
	73	2.0006	1.9762	1.9521	1.9288	1.8833	1.8398
	78	1.8196	1.7985	1.7777	1.7575	1.7182	1.6805
	83	1.5025	1.4866	1.4709	1.4556	1.4258	1.3970
	88	1.2441	1.2319	1.2199	1.2081	1.1852	1.1631
89	19	2.2225	2.1950	2.1680	2.1419	2.0911	2.0424
	24	2.2157	2.1884	2.1617	2.1355	2.0850	2.0364
	29	2.2010	2.1777	2.1511	2.1252	2.0749	2.0267
	34	2.1964	2.1694	2.1430	2.1172	2.0672	2.0192
	39	2.1891	2.1622	2.1359	2.1102	2.0605	2.0127
	44	2.1782	2.1515	2.1254	2.0999	2.0505	2.0031
	49	2.1565	2.1302	2.1045	2.0794	2.0307	1.9840
	54	2.1235	2.0978	2.0726	2.0480	2.0004	1.9546
	59	2.0896	2.0644	2.0399	2.0158	1.9692	1.9244
	64	2.0425	2.0181	1.9944	1.9711	1.9259	1.8826
	69	1.9685	1.9454	1.9229	1.9008	1.8579	1.8167
	74	1.8635	1.8423	1.8214	1.8010	1.7614	1.7232
	79	1.6819	1.6637	1.6458	1.6283	1.5942	1.5614
	84	1.3823	1.3685	1.3549	1.3416	1.3157	1.2907
	89	1.1747	1.1638	1.1532	1.1427	1.1222	1.1025
90	20	1.9987	1.9762	1.9541	1.9324	1.8904	1.8499
	25	1.9926	1.9702	1.9481	1.9266	1.8847	1.8444
	30	1.9785	1.9613	1.9394	1.9180	1.8764	1.8363
	35	1.9762	1.9540	1.9323	1.9109	1.8695	1.8296
	40	1.9709	1.9488	1.9266	1.9058	1.8646	1.8249
	45	1.9614	1.9395	1.9180	1.8969	1.8559	1.8164
	50	1.9398	1.9182	1.8970	1.8762	1.8358	1.7970
	55	1.9132	1.8920	1.8712	1.8507	1.8112	1.7730
	60	1.8854	1.8646	1.8442	1.8242	1.7854	1.7480
	65	1.8417	1.8217	1.8020	1.7826	1.7450	1.7088
	70	1.7771	1.7580	1.7392	1.7208	1.6851	1.6506
	75	1.6839	1.6662	1.6488	1.6317	1.5986	1.5666
	80	1.5080	1.4930	1.4782	1.4636	1.4353	1.4079
	85	1.2416	1.2300	1.2186	1.2074	1.1856	1.1644
	90	1.0590	1.0500	1.0411	1.0322	1.0150	.9984

TABLE XXVIII.

Showing the Present Value of a Deferred Annuity of £1 on a Single Life, according to the rate of mortality among the members of the Equitable, reckoning Interest at 3½ per Cent.—*See page* 277.

Present Age.	When the Annuity is to commence at the Age of						Present Age.
	50	55	60	65	70	75	
21	3.596	2.448	1.591	.971	.542	.265	21
22	3.747	2.551	1.658	1.012	.564	.276	22
23	3.906	2.659	1.728	1.055	.588	.288	23
24	4.072	2.772	1.802	1.099	.613	.300	24
25	4.247	2.891	1.879	1.147	.640	.313	25
26	4.430	3.015	1.960	1.196	.667	.326	26
27	4.622	3.146	2.045	1.248	.696	.340	27
28	4.825	3.284	2.135	1.303	.727	.355	28
29	5.039	3.430	2.230	1.361	.759	.371	29
30	5.266	3.585	2.330	1.422	.793	.388	30
31	5.503	3.746	2.435	1.486	.829	.405	31
32	5.754	3.917	2.546	1.553	.867	.424	32
33	6.019	4.097	2.663	1.625	.907	.443	33
34	6.297	4.287	2.786	1.700	.948	.464	34
35	6.589	4.485	2.915	1.779	.992	.485	35
36	6.898	4.695	3.052	1.863	1.039	.508	36
37	7.222	4.916	3.195	1.950	1.088	.532	37
38	7.566	5.151	3.348	2.043	1.140	.557	38
39	7.928	5.397	3.508	2.141	1.194	.584	39
40	8.308	5.656	3.676	2.243	1.251	.612	40
41		5.928	3.853	2.351	1.312	.641	41
42		6.214	4.039	2.465	1.375	.672	42
43		6.515	4.235	2.584	1.442	.705	43
44		6.835	4.443	2.711	1.512	.739	44
45		7.176	4.664	2.846	1.588	.776	45
46			4.897	2.989	1.667	.815	46
47			5.144	3.139	1.751	.856	47
48			5.406	3.299	1.840	.900	48
49			5.686	3.470	1.936	.946	49
50			5.986	3.653	2.038	.996	50
51				3.850	2.148	1.050	51
52				4.064	2.267	1.108	52
53				4.296	2.397	1.172	53
54				4.549	2.537	1.240	54
55				4.819	2.688	1.314	55
56					2.851	1.394	56
57					3.027	1.480	57
58					3.218	1.573	58
59					3.426	1.675	59
60					3.653	1.786	60
61						1.906	61
62						2.036	62
63						2.179	63
64						2.336	64
65						2.510	65

TABLE XXIX.

Showing the Annual Premium, payable at the beginning of the year, equivalent to a Deferred Annuity of £1 on a Single Life, by the Equitable Table, reckoning Interest at 3½ per Cent.—*See page* 278.

Present Age.	When the Annuity is to commence at the Age of						Present Age.
	50	55	60	65	70	75	
21	.2140	.1360	.0842	.0497	.0271	.0130	21
22	.2275	.1440	.0889	.0523	.0285	.0137	22
23	.2422	.1526	.0939	.0552	.0300	.0144	23
24	.2582	.1619	.0993	.0582	.0316	.0152	24
25	.2757	.1719	.1051	.0615	.0333	.0160	25
26	.2949	.1828	.1114	.0650	.0352	.0168	26
27	.3159	.1946	.1181	.0687	.0371	.0177	27
28	.3391	.2074	.1253	.0727	.0392	.0187	28
29	.3647	.2214	.1331	.0770	.0415	.0198	29
30	.3932	.2367	.1416	.0817	.0439	.0209	30
31	.4249	.2534	.1507	.0866	.0464	.0221	31
32	.4604	.2718	.1607	.0920	.0492	.0233	32
33	.5005	.2921	.1715	.0979	.0522	.0247	33
34	.5459	.3144	.1833	.1040	.0553	.0262	34
35	.5977	.3394	.1962	.1108	.0588	.0278	35
36	.6573	.3671	.2103	.1182	.0625	.0295	36
37	.7266	.3983	.2259	.1262	.0665	.0313	37
38	.8078	.4334	.2431	.1349	.0708	.0333	38
39	.9043	.4732	.2621	.1444	.0755	.0354	39
40	1.0208	.5187	.2833	.1549	.0806	.0377	40
41		.5711	.3070	.1664	.0861	.0402	41
42		.6319	.3335	.1791	.0922	.0429	42
43		.7035	.3635	.1931	.0988	.0458	43
44		.7887	.3977	.2086	.1060	.0490	44
45		.8915	.4368	.2261	.1140	.0525	45
46			.4819	.2456	.1229	.0563	46
47			.5345	.2676	.1326	.0604	47
48			.5965	.2925	.1435	.0650	48
49			.6705	.3209	.1556	.0701	49
50			.7601	.3537	.1692	.0757	50
51				.3916	.1846	.0819	51
52				.4359	.2019	.0885	52
53				.4883	.2216	.0966	53
54				.5509	.2442	.1053	54
55				.6268	.2702	.1150	55
56					.3004	.1261	56
57					.3358	.1386	57
58					.3778	.1529	58
59					.4280	.1694	59
60					.4891	.1884	60
61						.2107	61
62						.2369	62
63						.2681	63
64						.3058	64
65						.3518	65

TABLE XXX.

Showing the Premium, Single or Annual, equivalent to a Survivorship Annuity of £1 on an Assigned Life *A*, after the extinction of another Life *B*, according to the rate of mortality among the members of the Equitable, reckoning Interest at 3½ per Cent.—*See pages* 264—6.

Age of *A*.	Age of *B*.	Single Premium.	Annual Premium	Age of *A*.	Age of *B*.	Single Premium.	Annual Premium.
10	10	2.9159	.14922	25	10	2.0474	.11478
	15	3.4569	.18194		15	2.4559	.14090
	20	4.0089	.21730		20	2.8712	.16875
	25	4.6191	.25895		25	3.3388	.20178
	30	5.2870	.30793		30	3.8628	.24109
	35	6.0082	.36528		35	4.4424	.28767
	40	6.8358	.43760		40	5.1297	.34765
	45	7.8426	.53664		45	5.9974	.43185
	50	9.0150	.67067		50	7.0388	.54792
	55	10.2069	.83322		55	8.1146	.68940
	60	11.4595	1.0420		60	9.2630	.87205
	65	12.8454	1.3365		65	10.5536	1.1309
	70	14.2949	1.7514		70	11.9199	1.4965
	75	15.7831	2.3649		75	13.3383	2.0374
	80	17.3289	3.3793		80	14.8299	2.9335
15	10	2.6125	.13750	30	10	1.7819	.10378
	15	3.1097	.16806		15	2.1466	.12773
	20	3.6176	.20103		20	2.5144	.15297
	25	4.1832	.24001		25	2.9294	.18283
	30	4.8073	.28606		30	3.3968	.21837
	35	5.4857	.34016		35	3.9175	.26057
	40	6.2709	.40875		40	4.5452	.31549
	45	7.2354	.50326		45	5.3536	.39370
	50	8.3674	.63174		50	6.3381	.50248
	55	9.5228	.78768		55	7.3646	.63558
	60	10.7415	.98810		60	8.4702	.80811
	65	12.0956	1.2709		65	9.7241	1.0536
	70	13.5173	1.6698		70	11.0604	1.4016
	75	14.9820	2.2596		75	12.4544	1.9168
	80	16 5097	3.2355		80	13.9262	2.7711
20	10	2.3250	.12603	35	10	1.5294	.09298
	15	2.7781	.15438		15	1.8513	.11479
	20	3.2405	.18483		20	2.1728	.13747
	25	3.7590	.22093		25	2.5353	.16417
	30	4.3356	.26376		30	2.9438	.19580
	35	4.9677	.31430		35	3.4007	.23328
	40	5.7073	.37883		40	3.9608	.28256
	45	6.6270	.46847		45	4.7001	.35398
	50	7.7167	.59103		50	5.6182	.45455
	55	8.8351	.74009		55	6.5861	.57814
	60	10.0206	.93102		60	7.6407	.73914
	65	11.3442	1.2031		65	8.8512	.96980
	70	12.7391	1.5857		70	10.1539	1.2978
	75	14.1818	2.1516		75	11.5231	1.7852
	80	15.6928	3.0891		80	12.9771	2.5949

TABLE XXX.

Showing the Premium, Single or Annual, equivalent to a Survivorship Annuity of £1 on an Assigned Life *A*, after the extinction of another Life *B*, according to the rate of mortality among the members of the Equitable, reckoning Interest at 3½ per Cent.

Age of *A*.	Age of *B*.	Single Premium.	Annual Premium.	Age of *A*.	Age of *B*.	Single Premium.	Annual Premium.
40	10	1.2872	.08240	55	10	.6482	.05291
	15	1.5667	.10212		15	.8085	.06688
	20	1.8426	.12230		20	.9603	.08045
	25	2.1528	.14589		25	1.1276	.09580
	30	2.5017	.17365		30	1.3110	.11315
	35	2.8910	.20624		35	1.5062	.13222
	40	3.3752	.24940		40	1.7543	.15742
	45	4.0331	.31325		45	2.1254	.19730
	50	4.8700	.40454		50	2.6408	.25745
	55	5.7644	.51727		55	3.2152	.33205
	60	6.7532	.66501		60	3.8920	.43215
	65	7.9081	.87866		65	4.7492	.58280
	70	9.1799	1.1878		70	5.7469	.80363
	75	10.4983	1.6378		75	6.8688	1.1392
	80	11.9303	2.3967		80	8.1365	1.7088
45	10	1.0540	.07212	60	10	.4840	.04401
	15	1.2912	.08981		15	.6104	.05615
	20	1.5223	.10761		20	.7290	.06780
	25	1.7805	.12820		25	.8592	.08089
	30	2.0701	.15223		30	.9998	.09539
	35	2.3903	.18002		35	1.1440	.11067
	40	2.7931	.21694		40	1.3263	.13061
	45	3.3568	.27265		45	1.6104	.16315
	50	4.0946	.35379		50	2.0185	.21331
	55	4.8955	.45443		55	2.4752	.27483
	60	5.7973	.58731		60	3.0229	.35738
	65	6.8708	.78100		65	3.7460	.48427
	70	8.0595	1.0593		70	4.6200	.67334
	75	9.3407	1.4756		75	5.6372	.96460
	80	10.7337	2.1752		80	6.8242	1.4653
50	10	.8369	.06226	65	10	.3422	.03560
	15	1.0337	.07805		15	.4368	.04590
	20	1.2225	.09363		20	.5249	.05567
	25	1.4324	.11155		25	.6221	.06667
	30	1.6651	.13201		30	.7260	.07868
	35	1.9189	.15525		35	.8268	.09059
	40	2.2405	.18611		40	.9535	.10594
	45	2.7051	.23373		45	1.1562	.13143
	50	3.3311	.30428		50	1.4627	.17227
	55	4.0214	.39205		55	1.8047	.22146
	60	4.8159	.50893		60	2.2183	.28678
	65	5.7878	.68165		65	2.7873	.38895
	70	6.8869	.93170		70	3.5063	.54384
	75	8.0929	1.3083		75	4.3832	.78687
	80	9.4258	1.9424		80	5.4556	1.2129

TABLE XXX.

Showing the Premium, Single or Annual, equivalent to a Survivorship Annuity of £1 on an Assigned Life *A*, after the extinction of another Life *B*, according to the rate of mortality among the members of the Equitable, reckoning Interest at 3½ per Cent.

Age of *A*.	Age of *B*.	Single Premium.	Annual Premium.
70	10	.2275	.02787
	15	.2943	.03636
	20	.3556	.04426
	25	.4242	.05326
	30	.4981	.06312
	35	.5653	.07225
	40	.6611	.08554
	45	.7807	.10261
	50	.9976	.13496
	55	1.2382	.17314
	60	1.5281	.22271
	65	1.9421	.30123
	70	2.4875	.42148
	75	3.1921	.61419
	80	4.1141	.96230
75	10	.1396	.02092
	15	.1829	.02759
	20	.2222	.03371
	25	.2665	.04071
	30	.3160	.04864
	35	.3584	.05552
	40	.4034	.06293
	45	.4858	.07678
	50	.6275	.10144
	55	.7840	.13003
	60	.9692	.16584
	65	1.2429	.22313
	70	1.6160	.31093
	75	2.1234	.45276
	80	2.8523	.72010
80	10	.0762	.01486
	15	.1014	.01987
	20	.1240	.02441
	25	.1489	.02945
	30	.1786	.03554
	35	.2032	.04063
	40	.2262	.04544
	45	.2696	.05464
	50	.3512	.07237
	55	.4425	.09293
	60	.5470	.11746
	65	.7061	.15699
	70	.9288	.21725
	75	1.2431	.31384
	80	1.7393	.50200

TABLE XXXI.

Showing the Average Value of £1 to be received at the end of the year in which an Assigned Life may fail, estimating the rate of mortality by the experience of the Equitable.—*See page* 284.

Age.	4 per Cent.	5 per Cent.	6 per Cent.	Age.	4 per Cent.	5 per Cent.	6 per Cent.
10	.20590	.15562	.12211	54	.51535	.44860	.39424
11	.21109	.16014	.12605	55	.52505	.45860	.40421
12	.21649	.16491	.13024	56	.53485	.46873	.41435
13	.22184	.16963	.13437	57	.54474	.47900	.42468
14	.22741	.17459	.13876	58	.55472	.48941	.43518
15	.23294	.17950	.14310	59	.56478	.49995	.44585
16	.23843	.18438	.14740	60	.57492	.51060	.45668
17	.24387	.18921	.15165	61	.58540	.52170	.46802
18	.24927	.19399	.15585	62	.59625	.53326	.47992
19	.25462	.19872	.15999	63	.60721	.54499	.49204
20	.25990	.20339	.16406	64	.61827	.55688	.50439
21	.26542	.20829	.16838	65	.62942	.56892	.51695
22	.27115	.21343	.17294	66	.64064	.58110	.52971
23	.27685	.21854	.17746	67	.65193	.59341	.54265
24	.28278	.22390	.18224	68	.66325	.60581	.55566
25	.28869	.22924	.18699	69	.67459	.61828	.56896
26	.29483	.23484	.19203	70	.68590	.63077	.58225
27	.30096	.24043	.19704	71	.69744	.64358	.59597
28	.30707	.24600	.20203	72	.70890	.65636	.60968
29	.31315	.25154	.20699	73	.72053	.66939	.62374
30	.31920	.25705	.21192	74	.73198	.68227	.63769
31	.32550	.26283	.21713	75	.74348	.69525	.65182
32	.33177	.26859	.22231	76	.75532	.70871	.66653
33	.33803	.27432	.22745	77	.76717	.72224	.68139
34	.34452	.28033	.23288	78	.77929	.73615	.69677
35	.35128	.28662	.23860	79	.79129	.75000	.71214
36	.35802	.29291	.24432	80	.80302	.76360	.72729
37	.36504	.29950	.25036	81	.81425	.77665	.74188
38	.37207	.30610	.25641	82	.82505	.78927	.75605
39	.37937	.31300	.26279	83	.83506	.80099	.76925
40	.38696	.32025	.26952	84	.84419	.81170	.78134
41	.39486	.32785	.27663	85	.85235	.82130	.79219
42	.40309	.33581	.28415	86	.85947	.82966	.80164
43	.41165	.34418	.29211	87	.86564	.83690	.80981
44	.42030	.35267	.30021	88	.87151	.84337	.81757
45	.42905	.36128	.30845	89	.87773	.85110	.82583
46	.43816	.37031	.31717	90	.88602	.86093	.83703
47	.44765	.37981	.32640	91	.89582	.87262	.85044
48	.45727	.38949	.33585	92	.90697	.88601	.86588
49	.46705	.39936	.34555	93	.91829	.89966	.88168
50	.47698	.40944	.35548	94	.92971	.91350	.89779
51	.48679	.41942	.36534	95	.94115	.92743	.91406
52	.49646	.42929	.37511	96	.95229	.94104	.93005
53	.50597	.43902	.38475	97	.96154	.95238	.94340

TABLE XXXII.

Showing the Rate of Mortality, and the Average Duration of Human Life, at the Town of Northampton.—*See pages* 95 *and* 119.

Age.	Number living. D	Decrement.	Sum of the living at all higher ages. N	Expectation of life.	Age.	Number living. D	Decrement.	Sum of the living at all higher ages. N	Expectation of life.
- -	- - -	- -	299198	- - -	48	3014	78	55778	19.00
0	11650	3000	287548	25.18	49	2936	79	52842	18.49
1	8650	1367	278898	32.74	50	2857	81	49985	17.99
2	7283	502	271615	37.79	51	2776	82	47209	17.50
3	6781	335	264834	39.55	52	2694	82	44515	17.02
4	6446	197	258388	40.58	53	2612	82	41903	16.54
5	6249	184	252139	40.84	54	2530	82	39373	16.06
6	6065	140	246074	41.07	55	2448	82	36925	15.58
7	5925	110	240149	41.03	56	2366	82	34559	15.10
8	5815	80	234334	40.79	57	2284	82	32275	14.63
9	5735	60	228599	40.36	58	2202	82	30073	14.15
10	5675	52	222924	39.78	59	2120	82	27953	13.68
11	5623	50	217301	39.14	60	2038	82	25915	13.21
12	5573	50	211728	38.49	61	1956	82	23959	12.75
13	5523	50	206205	37.83	62	1874	81	22085	12.28
14	5473	50	200732	37.17	63	1793	81	20292	11.81
15	5423	50	195309	36.51	64	1712	80	18580	11.35
16	5373	53	189936	35.85	65	1632	80	16948	10.88
17	5320	58	184616	35.20	66	1552	80	15396	10.42
18	5262	63	179354	34.58	67	1472	80	13924	9.96
19	5199	67	174155	33.99	68	1392	80	12532	9.50
20	5132	72	169023	33.43	69	1312	80	11220	9.05
21	5060	75	163963	32.90	70	1232	80	9988	8.60
22	4985	75	158978	32.39	71	1152	80	8836	8.17
23	4910	75	154068	31.88	72	1072	80	7764	7.74
24	4835	75	149233	31.36	73	992	80	6772	7.33
25	4760	75	144473	30.85	74	912	80	5860	6.92
26	4685	75	139788	30.33	75	832	80	5028	6.54
27	4610	75	135178	29.82	76	752	77	4276	6.18
28	4535	75	130643	29.30	77	675	73	3601	5.83
29	4460	75	126183	28.79	78	602	68	2999	5.48
30	4385	75	121798	28.27	79	534	65	2465	5.11
31	4310	75	117488	27.76	80	469	63	1996	4.75
32	4235	75	113253	27.24	81	406	60	1590	4.41
33	4160	75	109093	26.72	82	346	57	1244	4.09
34	4085	75	105008	26.20	83	289	55	955	3.80
35	4010	75	100998	25.68	84	234	48	721	3.58
36	3935	75	97063	25.16	85	186	41	535	3.37
37	3860	75	93203	24.64	86	145	34	390	3.19
38	3785	75	89418	24.12	87	111	28	279	3.01
39	3710	75	85708	23.60	88	83	21	196	2.86
40	3635	76	82073	23.08	89	62	16	134	2.66
41	3559	77	78514	22.56	90	46	12	88	2.41
42	3482	78	75032	22.04	91	34	10	54	2.09
43	3404	78	71628	21.54	92	24	8	30	1.75
44	3326	78	68302	21.03	93	16	7	14	1.37
45	3248	78	65054	20.52	94	9	5	5	1.05
46	3170	78	61884	20.02	95	4	3	1	0.75
47	3092	78	58792	19.51	96	1	1	0	0.50

TABLE XXXIII.

Showing the Logarithms of the number living at each age in the Northampton Table, and of the proportion of that number which may be expected to survive one year.—*See pages* 104, 5.

Age.	Log. N° living.	λ, a.	Age.	Log. N° living.	λ, a.
0	4.0663259	$\bar{1}$.8706902	48	3.4791432	$\bar{1}$.9886129
1	3.9370161	.9252942	49	3.4677561	.9881541
2	3.8623103	.9639834	50	3.4559102	.9875093
3	3.8312937	.9779966	51	3.4434195	.9869781
4	3.8092903	.9865202	52	3.4303976	.9865756
5	3.7958105	.9870203	53	3.4169732	.9861473
6	3.7828308	.9898575	54	3.4031205	.9856909
7	3.7726883	.9918614	55	3.3888114	.9852033
8	3.7645497	.9939837	56	3.3740147	.9846814
9	3.7585334	.9954325	57	3.3586961	.9841212
10	3.7539659	.9960022	58	3.3428173	.9835186
11	3.7499681	.9961209	59	3.3263359	.9828683
12	3.7460890	.9960860	60	3.3092042	.9821647
13	3.7421750	.9960504	61	3.2913689	.9814007
14	3.7382254	.9960142	62	3.2727696	.9808107
15	3.7342396	.9959772	63	3.2535803	.9799235
16	3.7302168	.9956948	64	3.2335038	.9792164
17	3.7259116	.9952392	65	3.2127202	.9781715
18	3.7211508	.9947690	66	3.1908917	.9770161
19	3.7159198	.9943668	67	3.1679078	.9757314
20	3.7102866	.9938639	68	3.1436392	.9742946
21	3.7041505	.9935147	69	3.1179338	.9726769
22	3.6976652	.9934163	70	3.0906107	.9708418
23	3.6910815	.9933150	71	3.0614525	.9687423
24	3.6843965	.9932105	72	3 0301948	.9663169
25	3.6776070	.9931026	73	2.9965117	.9634831
26	3.6707096	.9929913	74	2.9599948	.9601285
27	3.6637009	.9928764	75	2.9201233	.9560945
28	3.6565773	.9927576	76	2.8762178	.9530860
29	3.6493349	.9926347	77	2.8293038	.9502927
30	3.6419696	.9925077	78	2.7795965	.9479448
31	3.6344773	.9923761	79	2.7275413	.9436315
32	3.6268534	.9922399	80	2.6711728	.9373532
33	3.6190933	.9920988	81	2.6085260	.9305501
34	3.6111921	.9919523	82	2.5390761	.9218217
35	3.6031444	.9918003	83	2.4608978	.9083181
36	3.5949447	.9916426	84	2.3692159	.9002970
37	3.5865873	.9914786	85	2.2695129	.8918551
38	3.5780659	.9913080	86	2.1613680	.8839550
39	3.5693739	.9911305	87	2.0453230	.8737551
40	3.5605044	.9908236	88	1.9190781	.8733136
41	3.5513280	.9905008	89	1.7923917	.8703661
42	3.5418288	.9901608	90	1.6627578	.8687211
43	3.5319896	.9899326	91	1.5314789	.8487323
44	3.5219222	.9896938	92	1.3802112	.8239088
45	3.5116160	.9894433	93	1.2041200	.7501225
46	3.5010593	.9891802	94	0.9542425	.6478175
47	3.4902395	.9889037	95	0.6020600	.3979400

TABLE XXXIV.

Showing, out of the number entering upon any year, the proportion which die within that year, or survive it, according to the Northampton rate of mortality.—*See pages* 104, 5.

Age.	Proportion which die.	Proportion which survive.	Reciprocal of ditto.	Age.	Proportion which die.	Proportion which survive.	Reciprocal of ditto.
0	.257511	.742489	1.34682	48	.025879	.974121	1.02656
1	.158035	.841965	1.18770	49	.026908	.973092	1.02765
2	.068928	.931072	1.07402	50	.028351	.971649	1.02918
3	.049403	.950597	1.05197	51	.029539	.970461	1.03044
4	.030562	.969438	1.03152	52	.030438	.969562	1.03139
5	.029445	.970555	1.03034	53	.031394	.968606	1.03241
6	.023084	.976916	1.02363	54	.032411	.967589	1.03350
7	.018565	.981435	1.01891	55	.033497	.966503	1.03466
8	.013757	.986243	1.01395	56	.034658	.965342	1.03590
9	.010462	.989538	1.01057	57	.035902	.964098	1.03723
10	.009163	.990837	1.00925	58	.037239	.962761	1.03868
11	.008892	.991108	1.00897	59	.038679	.961321	1.04024
12	.008972	.991028	1.00905	60	.040235	.959765	1.04192
13	.009053	.990947	1.00914	61	.041922	.958078	1.04375
14	.009136	.990864	1.00921	62	.043223	.956777	1.04518
15	.009220	.990780	1.00930	63	.045176	.954824	1.04731
16	.009864	.990136	1.00996	64	.046729	.953271	1.04902
17	.010902	.989098	1.01102	65	.049020	.950980	1.05155
18	.011972	.988028	1.01212	66	.051546	.948454	1.05434
19	.012887	.987113	1.01305	67	.054348	.945652	1.05747
20	.014030	.985970	1.01423	68	.057471	.942529	1.06097
21	.014822	.985178	1.01505	69	.060975	.939025	1.06493
22	.015045	.984955	1.01527	70	.064935	.935065	1.06944
23	.015275	.984725	1.01551	71	.069444	.930556	1.07463
24	.015512	.984488	1.01576	72	.074627	.925373	1.08064
25	.015756	.984244	1.01601	73	.080645	.919355	1.08772
26	.016009	.983991	1.01627	74	.087719	.912281	1.09615
27	.016269	.983731	1.01654	75	.096154	.903846	1.10638
28	.016538	.983462	1.01682	76	.102393	.897607	1.11407
29	.016816	.983184	1.01710	77	.108148	.891852	1.12126
30	.017104	.982896	1.01740	78	.112957	.887043	1.12734
31	.017401	.982599	1.01771	79	.121723	.878277	1.13859
32	.017710	.982290	1.01803	80	.134328	.865672	1.15517
33	.018029	.981971	1.01836	81	.147783	.852217	1.17341
34	.018360	.981640	1.01870	82	.164740	.835260	1.19723
35	.018704	.981296	1.01906	83	.190311	.809689	1.23504
36	.019060	.980940	1.01943	84	.205128	.794872	1.25806
37	.019430	.980570	1.01981	85	.220430	.779570	1.28276
38	.019815	.980185	1.02022	86	.234483	.765517	1.30634
39	.020216	.979784	1.02063	87	.252252	.747748	1.33735
40	.020908	.979092	1.02135	88	.253012	.746988	1.33871
41	.021635	.978365	1.02211	89	.258065	.741935	1.34783
42	.022401	.977599	1.02291	90	.260869	.739131	1.35294
43	.022914	.977086	1.02345	91	.294118	.705882	1.41667
44	.023452	.976548	1.02401	92	.333333	.666667	1.50000
45	.024015	.975985	1.02461	93	.437500	.562500	1.77778
46	.024606	.975394	1.02523	94	.555556	.444444	2.25000
47	.025227	.974773	1.02588	95	.750000	.250000	4.00000

TABLE XXXV.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the Northampton rate of mortality.

(4 per Cent.) [*See page* 219.

Age.	D	N	Age.	D	N
	- - -	131970.4425	48	458.7150	5360.3419
0	11650.0000	120320.4425	49	429.6574	4930.6845
1	8317.3076	112003.1349	50	402.0159	4528.6686
2	6733.5429	105269.5920	51	375.5944	4153.0742
3	6028.2843	99241.3077	52	350.4804	3802.5938
4	5510.0677	93731.2400	53	326.7429	3475.8509
5	5136.2225	88595.0175	54	304.3128	3171.5381
6	4793.2576	83801.7599	55	283.1246	2888.4135
7	4502.5130	79299.2469	56	263.1162	2625.2973
8	4248.9636	75050.2833	57	244.2281	2381.0692
9	4029.3349	71020.9484	58	226.4038	2154.6654
10	3833.8267	67187.1217	59	209.5891	1945.0763
11	3652.5935	63534.5282	60	193.7331	1751.3432
12	3480.8794	60053.6488	61	178.7866	1572.5566
13	3316.9707	56736.6781	62	164.7033	1407.8533
14	3160.5211	53576.1570	63	151.5234	1256.3299
15	3011.1994	50564.9576	64	139.1137	1117.2162
16	2868.6886	47696.2690	65	127.5126	989.7036
17	2731.1458	44965.1232	66	116.5980	873.1056
18	2597.4711	42367.6521	67	106.3345	766.7711
19	2467.6659	39899.9862	68	96.6878	670.0833
20	2342.1779	37557.8083	69	87.6261	582.4572
21	2220.4980	35337.3103	70	79.1183	503.3389
22	2103.4476	33233.8627	71	71.1353	432.2036
23	1992.1163	31241.7464	72	63.6494	368.5543
24	1886.2372	29355.5092	73	56.6341	311.9202
25	1785.5560	27569.9532	74	50.0642	261.8559
26	1689.8289	25880.1243	75	43.9160	217.9399
27	1598.8244	24281.2999	76	38.1667	179.7733
28	1512.3203	22768.9796	77	32.9410	146.8323
29	1430.1053	21338.8743	78	28.2485	118.5837
30	1351.9773	19986.8970	79	24.0939	94.48981
31	1277.7437	18709.1533	80	20.3472	74.14256
32	1207.2204	17501.9329	81	16.9366	57.20599
33	1140.2317	16361.7012	82	13.8785	43.32749
34	1076.6104	15285.0908	83	11.1463	32.18120
35	1016.1961	14268.8947	84	8.67791	23.50329
36	958.8363	13310.0584	85	6.63253	16.87076
37	904.3858	12405.6726	86	4.97166	11.89910
38	852.7053	11552.9673	87	3.65951	8.23960
39	803.6624	10749.3049	88	2.63114	5.60846
40	757.1306	9992.1743	89	1.88984	3.71862
41	712.7891	9279.3852	90	1.34821	2.37041
42	670.5458	8608.8394	91	.958175	1.41223
43	630.3125	7978.5269	92	.650345	.761890
44	592.1821	7386.3448	93	.416888	.345003
45	556.0523	6830.2925	94	.225480	.119522
46	521.8259	6308.4666	95	.096359	.023163
47	489.4097	5819.0569	96	.023163	.000000

TABLE XXXV.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the Northampton rate of mortality.

(3 per Cent.) [*See page* 219.

Age.	D	N	S	M	R
		154597.351	2874184.7		
0	11650.000	142947.351	2719587.3	7147.166	70883.26
1	8398.058	134549.293	2576639.9	4234.544	63736.09
2	6864.926	127684.367	2442090.7	2946.015	59501.55
3	6205.576	121478.791	2314406.3	2486.614	56555.53
4	5727.188	115751.604	2192927.5	2188.971	54068.92
5	5390.442	110361.161	2077175.9	2019.037	51879.95
6	5079.342	105281.819	1966814.7	1864.940	49860.91
7	4817.567	100464.252	1861532.9	1751.107	47995.97
8	4590.415	95873.837	1761068.7	1664.272	46244.87
9	4395.400	91478.437	1665194.8	1602.958	44580.59
10	4222.733	87255.705	1573716.4	1558.313	42977.64
11	4062.175	83193.530	1486460.7	1520.747	41419.32
12	3908.790	79284.740	1403267.2	1485.678	39898.58
13	3760.894	75523.846	1323982.4	1451.630	38412.90
14	3618.298	71905.548	1248458.6	1418.574	36961.27
15	3480.817	68424.731	1176553.0	1386.481	35542.69
16	3348.276	65076.455	1108128.3	1355.323	34156.21
17	3218.688	61857.767	1043051.8	1323.257	32800.89
18	3090.870	58766.897	981194.1	1289.188	31477.63
19	2964.917	55801.980	922427.2	1253.260	30188.44
20	2841.464	52960.516	866625.2	1216.164	28935.18
21	2719.999	50240.516	813664.7	1177.460	27719.02
22	2601.634	47638.882	763424.2	1138.318	26541.56
23	2487.856	45151.026	715785.3	1100.317	25403.24
24	2378.500	42772.526	670634.3	1063.422	24302.93
25	2273.402	40499.124	627861.7	1027.601	23239.50
26	2172.410	38326.714	587362.6	992.8241	22211.90
27	2075.372	36251.343	549035.9	959.0599	21219.08
28	1982.143	34269.199	512784.5	926.2791	20260.02
29	1892.585	32376.615	478515.3	894.4531	19333.74
30	1806.562	30570.053	446138.7	863.5541	18439.29
31	1723.945	28846.108	415568.7	833.5551	17575.73
32	1644.607	27201.501	386722.6	804.4298	16742.18
33	1568.429	25633.072	359521.1	776.1528	15937.75
34	1495.293	24137.779	333888.0	748.6994	15161.59
35	1425.087	22712.691	309750.2	722.0456	14412.90
36	1357.703	21354.988	287037.5	696.1682	13690.85
37	1293.034	20061.954	265682.5	671.0445	12994.68
38	1230.981	18830.973	245620.6	646.6525	12323.64
39	1171.446	17659.528	226789.6	622.9710	11676.98
40	1114.334	16545.194	209130.1	599.9792	11054.01
41	1059.258	15485.936	192584.9	577.3595	10454.03
42	1006.156	14479.780	177099.0	555.1096	9876.67
43	954.968	13524.811	162619.2	533.2273	9321.57
44	905.908	12618.903	149094.4	511.9823	8788.34
45	858.897	11760.007	136475.5	491.3561	8276.36
46	813.855	10946.152	124715.5	471.3307	7785.00
47	770.708	10175.444	113769.3	451.8885	7313.67

TABLE XXXV.

Being a preparatory Table for determining the Values of Annuities, &c. on Single Lives, according to the Northampton rate of mortality.

(3 per Cent.)

Age.	D	N	S	M	R
48	729.384	9446.059	103593.9	433.0126	6861.780
49	689.814	8756.245	94147.80	414.6865	6428.768
50	651.702	8104.543	85391.56	396 6660	6014.081
51	614.782	7489.762	77287.01	378.7275	5617.415
52	579.244	6910.517	69797.25	361.0965	5238.688
53	545.256	6365.262	62886.73	343.9790	4877.591
54	512.756	5852.506	56521.47	327.3600	4533.612
55	481.686	5370.820	50668.97	311.2251	4206.252
56	451.991	4918.829	45298.15	295.5601	3895.027
57	423.618	4495.211	40379.32	280.3512	3599.467
58	396.514	4098.697	35884.11	265.5855	3319.116
59	370.629	3728.068	31785.41	251.2498	3053.530
60	345.916	3382.152	28057.34	237.3317	2802.281
61	322.328	3059.824	24675.19	223.8190	2564.949
62	299.821	2760.004	21615.37	210.6998	2341.130
63	278.506	2481.497	18855.36	198.1180	2130.430
64	258.179	2223.317	16373.87	185.9027	1932.312
65	238.946	1984.371	14150.55	174.1896	1746.409
66	220.615	1763.756	12166.18	162.8177	1572.220
67	203.149	1560.608	10402.42	151.7770	1409.402
68	186.512	1374.095	8841.81	141.0579	1257.625
69	170.673	1203.422	7467.72	130.6510	1116.567
70	155.598	1047.824	6264.30	120.5472	985.916
71	141.257	906.5667	5216.47	110.7377	865.369
72	127.619	778.9479	4309.91	101.2139	754.631
73	114.655	664.2927	3530.96	91.9675	653.417
74	102.339	561.9540	2866.67	82.9904	561.450
75	90.6425	471.3115	2304.71	74.2748	478.460
76	79.5405	391.7710	1833.40	65.8130	404.185
77	69.3167	322.4543	1441.63	57.9058	338.372
78	60.0196	262.4347	1119.18	50.6277	280.466
79	51.6893	210.7454	856.74	44.0455	229.838
80	44.0751	166.6703	646.00	37.9370	185.793
81	37.0434	129.6269	479.33	32.1889	147.856
82	30.6495	98.9774	349.70	26.8740	115.667
83	24.8547	74.1227	250.72	21.9718	88.793
84	19.5384	54.5843	176.60	17.3795	66.821
85	15.0781	39.5062	122.01	13.4883	49.442
86	11.4121	28.0941	82.51	10.2614	35.953
87	8.4817	19.6124	54.41	7.66344	25.692
88	6.1574	13.4550	34.80	5.58622	18.029
89	4.4655	8.9895	21.35	4.07368	12.442
90	3.2166	5.7729	12.36	2.95484	8.369
91	2.3083	3.4846	6.584	2.14015	5.414
92	1.5819	1.8827	3.100	1.48101	3.274
93	1.0238	.8589	1.217	.969059	1.793
94	.5591	.2998	.3583	.534150	.8236
95	.2413	.0585	.0585	.232548	.2894
96	.0585	.000	.0000	.056858	.0569

TABLE XXXVI.

Showing the Values of Annuities on Single Lives, according to the Northampton rate of mortality.—*See pages* 225, 6.

Age.	3 per Cent.	4 per Cent.	Age.	3 per Cent.	4 per Cent.
0	12.2702	10.3279	48	12.9508	11.6866
1	16.0215	13.4663	49	12.6937	11.4758
2	18.5995	15.6336	50	12.4360	11.2649
3	19.5758	16.4626	51	12.1828	11.0586
4	20.2109	17.0109	52	11.9303	10.8497
5	20.4735	17.2500	53	11.6740	10.6379
6	20.7275	17.4832	54	11.4138	10.4220
7	20.8537	17.6122	55	11.1500	10.2011
8	20.8857	17.6632	56	10.8826	9.9777
9	20.8123	17.6260	57	10.6115	9.7494
10	20.6633	17.5248	58	10.3369	9.5169
11	20.4800	17.3944	59	10.0588	9.2804
12	20.2838	17.2524	60	9.7774	9.0400
13	20.0814	17.1050	61	9.4929	8.7957
14	19.8728	16.9517	62	9.2055	8.5478
15	19.6577	16.7923	63	8.9100	8.2913
16	19.4358	16.6265	64	8.6115	8.0310
17	19.2183	16.4638	65	8.3047	7.7616
18	19.0131	16.3111	66	7.9948	7.4882
19	18.8208	16.1691	67	7.6821	7.2109
20	18.6385	16.0354	68	7.3673	6.9301
21	18.4708	15.9141	69	7.0510	6.6473
22	18.3112	15.7997	70	6.7342	6.3619
23	18.1486	15.6827	71	6.4179	6.0758
24	17.9830	15.5630	72	6.1037	5.7904
25	17.8144	15.4405	73	5.7939	5.5076
26	17.6425	15.3152	74	5.4912	5.2304
27	17.4674	15.1870	75	5.1997	4.9626
28	17.2890	15.0557	76	4.9254	4.7102
29	17.1070	14.9212	77	4.6520	4.4574
30	16.9217	14.7835	78	4.3725	4.1979
31	16.7326	14.6423	79	4.0772	3.9217
32	16.5398	14.4977	80	3.7815	3.6439
33	16.3432	14.3494	81	3.4994	3.3777
34	16.1425	14.1953	82	3.2294	3.1219
35	15.9378	14.0415	83	2.9823	2.8874
36	15.7288	13.8815	84	2.7938	2.7084
37	15.5154	13.7172	85	2.6202	2.5436
38	15.2976	13.5486	86	2.4619	2.3934
39	15.0750	13.3754	87	2.3124	2.2516
40	14.8476	13.1974	88	2.1852	2.1316
41	14.6196	13.0184	89	2.0131	1.9677
42	14.3912	12.8385	90	1.7948	1.7582
43	14.1626	12.6580	91	1.5010	1.4739
44	13.9296	12.4691	92	1.1903	1.1715
45	13.6920	12.2835	93	.8390	.8276
46	13.4498	12.0892	94	.5362	.5301
47	13.2028	11.8899	95	.2427	.2404

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.—*See pages* 227—233.

Age.		3 per Cent.	4 per Cent.	Age.		3 per Cent.	4 per Cent.
Older.	Younger.			Older.	Younger.		
0	0	5.6150	4.904	17	2	13.6591	11.981
1	1	9.4908	8.252		7	15.4906	13.599
2	2	12.7897	11.107		12	15.3088	13.480
3	3	14.1960	12.325		17	14.7378	13.019
4	4	15.1812	13.185	18	3	14.2769	12.531
5	0	9.3380	8.136		8	15.4363	13.569
	5	15.6381	13.591		13	15.0862	13.303
6	1	12.3469	10.741		18	14.5164	12.841
	6	16.0993	14.005	19	4	14.6569	12.876
7	2	14.4612	12.581		9	15.3165	13.482
	7	16.3752	14.224		14	14.8708	13.130
8	3	15.3003	13.319		19	14.3164	12.679
	8	16.5106	14.399	20	0	8.8227	7.780
9	4	15.8096	13.775		5	14.7759	12.993
	9	16.4837	14.396		10	15.1510	13.355
10	0	9.5331	8.335		15	14.6599	12.961
	5	15.9748	13.933		20	14.1335	12.535
	10	16.3391	14.277	21	1	11.4132	10.053
11	1	12.3468	10.782		6	14.9040	13.121
	6	16.1100	14.068		11	14.9739	13.217
	11	16.1420	14.133		16	14.4571	12.799
12	2	14.2397	12.438		21	13.9747	12.409
	7	16.1378	14.111	22	2	13.1722	11.605
	12	15.9259	13.966		7	14.9503	13.178
13	3	14.8953	13.019		12	14.7956	13.078
	8	16.0897	14.089		17	14.2654	12.646
	13	15.7021	13.789		22	13.8303	12.293
14	4	15.2870	13.374	23	3	13.7944	12.161
	9	15.9571	13.992		8	14.9297	13.168
	14	15.4700	13.604		13	14.6124	12.934
15	0	9.1882	8.068		18	14.0822	12.500
	5	15.3917	13.479		23	13.6838	12.179
	10	15.7627	13.841	24	4	14.1784	12.511
	15	15.2292	13.411		9	14.8340	13.112
16	1	11.8648	10.406		14	14.4238	12.784
	6	15.4864	13.578		19	13.9082	12.361
	11	15.5382	13.664		24	13.5349	12.062
	16	14.9794	13.212	25	0	8.5408	7.568
					5	14.3015	12.633
					10	14.6838	12.998
					15	14.2298	12.630
					20	13.7411	12.229

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.

Age.		3 per Cent.	4 per Cent.	Age.		3 per Cent.	4 per Cent.
Older.	Younger.			Older.	Younger.		
25	25	13.3837	11.944	32	17	13.3209	11.911
26	1	11.0378	9.770		22	12.9609	11.615
	6	14.4204	12.754		27	12.6411	11.359
	11	14.5086	12.861		32	12.2526	11.042
	16	14.0299	12.470	33	3	12.7431	11.355
	21	13.5845	12.105		8	13.8206	12.323
	26	13.2301	11.822		13	13.5699	12.125
27	2	12.7225	11.264		18	13.1218	11.750
	7	14.4514	12.798		23	12.7980	11.485
	12	14.3232	12.715		28	12.4743	11.225
	17	13.8322	12.311		33	12.0793	10.902
	22	13.4336	11.987	34	4	13.0610	11.651
	27	13.0740	11.699		9	13.6988	12.234
28	3	13.3070	11.790		14	13.3636	11.959
	8	14.4173	12.786		19	12.9304	11.595
	13	14.1327	12.564		24	12.6322	11.352
	18	13.6424	12.158		29	12.3044	11.088
	23	13.2803	11.866		34	11.9028	10.759
	28	12.9153	11.573	35	0	7.8547	7.039
29	4	13.6610	12.116		5	13.1365	11.732
	9	14.3102	12.710		10	13.5256	12.098
	14	13.9366	12.408		15	13.1513	11.787
	19	13.4611	12.013		20	12.7445	11.445
	24	13.1245	11.743		25	12.4634	11.217
	29	12.7540	11.445		30	12.1314	10.948
30	0	8.2225	7.325		35	11.7227	10.612
	5	13.7627	12.220	36	1	10.1039	9.047
	10	14.1501	12.586		6	13.2068	11.812
	15	13.7349	12.246		11	13.3282	11.941
	20	13.2862	11.873		16	12.9324	11.609
	25	12.9661	11.618		21	12.5674	11.302
	30	12.5898	11.313		26	12.2914	11.078
31	1	10.6050	9.438		31	11.9551	10.805
	6	13.8598	12.322		36	11.5391	10.462
	11	13.9653	12.441	37	2	11.6006	10.392
	16	13.5270	12.078		7	13.1950	11.819
	21	13.1210	11.742		12	13.1203	11.773
	26	12.8050	11.489		17	12.7145	11.430
	31	12.4227	11.179		22	12.3945	11.163
32	2	12.2031	10.865		27	12.1160	10.936
	7	13.8717	12.350		32	11.7753	10.659
	12	13.7704	12.286		37	11.3516	10.307

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.

Age.		3 per Cent.	4 per Cent.	Age.		3 per Cent.	4 per Cent.
Older.	Younger.			Older.	Younger.		
38	3	12.0875	10.838	43	3	11.3429	10.242
	8	13.1223	11.772		8	12.3252	11.130
	13	12.9065	11.600		13	12.1442	10.985
	18	12.5027	11.257		18	11.7857	10.677
	23	12.2181	11.020		23	11.5403	10.470
	28	11.9373	10.791		28	11.3023	10.272
	33	11.5919	10.508		33	11.0076	10.027
	38	11.1601	10.149		38	10.6349	9.716
39	4	12.3619	11.097		43	10.1753	9.326
	9	12.9816	11.665	44	4	11.5786	10.468
	14	12.6863	11.420		9	12.1739	11.012
	19	12.2973	11.089		14	11.9188	10.799
	24	12.0382	10.874		19	11.5745	10.502
	29	11.7549	10.642		24	11.3540	10.317
	34	11.4047	10.354		29	11.1147	10.117
	39	10.9644	9.986		34	10.8168	9.869
40	0	7.4271	6.700		39	10.4375	9.550
	5	12.4051	11.150		44	9.9779	9.160
	10	12.7912	11.513	45	0	6.9567	6.321
	15	12.4595	11.234		5	11.5973	10.500
	20	12.0963	10.924		10	11.9760	10.851
	25	11.8546	10.725		15	11.6871	10.607
	30	11.5687	10.490		20	11.3674	10.330
	35	11.2134	10.196		25	11.1642	10.160
	40	10.7641	9.820		30	10.9236	9.959
41	1	9.5231	8.585		35	10.6222	9.706
	6	12.4460	11.203		40	10.2359	9.381
	11	12.5807	11.342		45	9.7768	8.990
	16	12.2293	11.044	46	1	8.8879	8.071
	21	11.9063	10.768		6	11.6105	10.528
	26	11.6706	10.574		11	11.7557	10.697
	31	11.3820	10.336		16	11.4487	10.408
	36	11.0213	10.037		21	11.1673	10.165
	41	10.5656	9.654		26	10.9706	10.000
42	2	10.9075	9.839		31	10.7288	9.797
	7	12.4125	11.190		36	10.4238	9.540
	12	12.3635	11.165		41	10.0331	9.210
	17	12.0030	10.856		46	9.5718	8.815
	22	11.7233	10.619	47	2	10.1471	9.221
	27	11.4865	10.423		7	11.5502	10.491
	32	11.1949	10.182		12	11.5252	10.481
	37	10.8284	9.877		17	11.2100	10.208
	42	10.3692	9.491		22	10.9699	10.001

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.

Age.		3 per Cent.	4 per Cent.	Age.		3 per Cent.	4 per Cent.
Older.	Younger.			Older.	Younger.		
47	27	10.7733	9.836	51	36	9.7078	8.937
	32	10.5300	9.631		41	9.3832	8.658
	37	10.2212	9.370		46	8.9973	8.326
	42	9.8290	9.037		51	8.5075	7.900
	47	9.3626	8.637	52	2	9.2999	8.520
48	3	10.5158	9.566		7	10.5858	9.690
	8	11.4354	10.404		12	10.5827	9.698
	13	11.2884	10.284		17	10.3129	9.461
	18	10.9757	10.011		22	10.1110	9.284
	23	10.7684	9.833		27	9.9527	9.148
	28	10.5719	9.667		32	9.7559	8.980
	33	10.3272	9.461		37	9.5036	8.763
	38	10.0143	9.195		42	9.1791	8.483
	43	9.6239	8.862		47	8.7902	8.147
	48	9.1491	8.453		52	8.3043	7.723
49	4	10.6968	9.744	53	3	9.6110	8.815
	9	11.2601	10.263		8	10.4584	9.591
	14	11.0450	10.080		13	10.3441	9.497
	19	10.7459	9.818		18	10.0765	9.260
	24	10.5628	9.661		23	9.9053	9.111
	29	10.3663	9.495		28	9.7479	8.975
	34	10.1201	9.286		33	9.5509	8.806
	39	9.8028	9.015		38	9.2961	8.586
	44	9.4146	8.683		43	8.9747	8.308
	49	8.9309	8.266		48	8.5798	7.965
50	0	6.4235	5.882		53	8.0989	7.544
	5	10.6793	9.742	54	4	9.7513	8.957
	10	11.0446	10.085		9	10.2764	9.442
	15	10.7987	9.872		14	10.1003	9.290
	20	10.5229	9.630		19	9.8450	9.063
	25	10.3566	9.488		24	9.6965	8.934
	30	10.1602	9.321		29	9.5401	8.799
	35	9.9123	9.110		34	9.3427	8.629
	40	9.5902	8.834		39	9.0851	8.406
	45	9.2045	8.503		44	8.7674	8.130
	50	8.7146	8.081		49	8.3660	7.780
51	1	8.1708	7.479		54	7.8913	7.362
	6	10.6641	9.745	55	0	5.8608	5.412
	11	10.8161	9.894		5	9.7075	8.931
	16	10.5538	9.665		10	10.0549	9.256
	21	10.3135	9.454		15	9.8509	9.077
	26	10.1543	9.318		20	9.6168	8.869
	31	9.9578	9.151		25	9.4846	8.754

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.

Age.		3 per Cent.	4 per Cent.	Age.		3 per Cent.	4 per Cent.
Older.	Younger.			Older.	Younger.		
55	30	9.3291	8.619	59	14	9.0537	8.399
	35	9.1314	8.448		19	8.8415	8.207
	40	8.8707	8.221		24	8.7251	8.104
	45	8.5570	7.948		29	8.6055	7.999
	50	8.1519	7.593		34	8.4539	7.866
	55	7.6817	7.179		39	8.2531	7.689
56	1	7.4120	6.843		44	8.0033	7.469
	6	9.6591	8.902		49	7.6842	7.186
	11	9.8146	9.052		54	7.3043	6.850
	16	9.5958	8.858		59	6.8245	6.421
	21	9.3945	8.679	60	0	5.2384	4.881
	26	9.2695	8.570		5	8.6296	8.011
	31	9.1150	8.436		10	8.9526	8.314
	36	8.9168	8.264		15	8.7900	8.170
	41	8.6553	8.035		20	8.5969	7.995
	46	8.3436	7.763		25	8.4957	7.906
	51	7.9410	7.409		30	8.3780	7.802
	56	7.4701	6.993		35	8.2272	7.669
57	2	8.3928	7.756		40	8.0251	7.490
	7	9.5496	8.817		45	7.7810	7.274
	12	9.5659	8.839		50	7.4607	6.989
	17	9.3405	8.639		55	7.0882	6.659
	22	9.1745	8.491		60	6.6062	6.226
	27	9.0513	8.383	61	1	6.5715	6.123
	32	8.8978	8.250		6	8.5421	7.944
	37	8.6989	8.076		11	8.6965	8.092
	42	8.4393	7.848		16	8.5210	7.935
	47	8.1270	7.574		21	8.3573	7.787
	52	7.7307	7.225		26	8.2634	7.704
	57	7.2566	6.805		31	8.1475	7.601
58	3	8.6303	7.986		36	7.9976	7.469
	8	9.3954	8.691		41	7.7963	7.290
	13	9.3123	8.622		46	7.5559	7.076
	18	9.0889	8.422		51	7.2405	6.795
	23	8.9514	8.299		56	6.8706	6.465
	28	8.8299	8.193		61	6.3869	6.030
	33	8.6774	8.060	62	2	7.3909	6.894
	38	8.4776	7.884		7	8.4003	7.828
	43	8.2227	7.660		12	8.4333	7.863
	48	7.9072	7.382		17	8.2520	7.700
	53	7.5185	7.039		22	8.1198	7.580
	58	7.0413	6.614		27	8.0282	7.499
59	4	8.7129	8.075		32	7.9143	7.397
	9	9.1917	8.519		37	7.7650	7.265

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.

Age.		3 per Cent.	4 per Cent.
Older.	Younger.		
62	42	7.5669	7.088
	47	7.3280	6.875
	52	7.0209	6.600
	57	6.6515	6.270
	62	6.1668	5.831
63	3	7.5456	7.048
	8	8.2143	7.669
	13	8.1609	7.625
	18	7.9814	7.462
	23	7.8748	7.365
	28	7.7856	7.286
	33	7.6735	7.186
	38	7.5249	7.053
	43	7.3327	6.881
	48	7.0930	6.667
	53	6.7955	6.399
	58	6.4273	6.070
	63	5.9387	5.626
64	4	7.5627	7.076
	9	7.9846	7.470
	14	7.8838	7.381
	19	7.7141	7.226
	24	7.6265	7.147
	29	7.5398	7.069
	34	7.4297	6.971
	39	7.2815	6.838
	44	7.0955	6.671
	49	6.8548	6.454
	54	6.5682	6.196
	59	6.2015	5.867
	64	5.7093	5.417
65	0	4.5473	4.274
	5	7.4290	6.963
	10	7.7185	7.236
	15	7.5969	7.127
	20	7.4439	6.986
	25	7.3702	6.920
	30	7.2860	6.844
	35	7.1778	6.747
	40	7.0299	6.614
	45	6.8507	6.453
	50	6.6113	6.236
	55	6 3346	5.986
	60	5.9702	5.658
65	65	5.4713	5.201
66	1	5.6333	5.295
	6	7.2904	6.846
	11	7.4372	6.987
	16	7.3047	6.866
	21	7.1771	6.749
	26	7.1104	6.689
	31	7.0287	6.615
	36	6.9224	6.520
	41	6.7766	6.388
	46	6.6025	6.230
	51	6.3696	6.019
	56	6.0987	5.774
	61	5.7374	5.447
	66	5.2314	4.982
67	2	6.2659	5.896
	7	7.1043	6.684
	12	7.1491	6.730
	17	7.0118	6.604
	22	6.9115	6.512
	27	6.8474	6.454
	32	6.7682	6.382
	37	6.6637	6.288
	42	6.5220	6.159
	47	6.3511	6.004
	52	6.1278	5.801
	57	5.8608	5.559
	62	5.5033	5.285
	67	4.9899	4.760
68	3	6.3301	5.965
	8	6.8843	6.490
	13	6.8572	6.468
	18	6.7214	6.343
	23	6.6429	6.271
	28	6.5815	6.215
	33	6.5048	6.146
	38	6.4019	6.052
	43	6.2665	5.929
	48	6.0966	5.774
	53	5.8839	5.580
	58	5.6213	5.341
	63	5.2650	5.017
	68	4.7473	4.537

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.

Age.		3 per Cent.	4 per Cent.	Age.		3 per Cent.	4 per Cent.
Older.	Younger.			Older.	Younger.		
69	4	6.2770	5.924	72	2	5.0618	4.814
	9	6.6282	6.262		7	5.7139	5.418
	14	6.5621	6.202		12	5.7636	5.478
	19	6.4342	6.084		17	5.6676	5.389
	24	6.3720	6.027		22	5.5956	5.321
	29	6.3133	5.973		27	5.5538	5.283
	34	6.2390	5.906		32	5.5028	5.236
	39	6.1374	5.813		37	5.4352	5.174
	44	6.0087	5.696		42	5.3413	5.087
	49	5.8394	5.541		47	5.2284	4.983
	54	5.6383	5.357		52	5.0773	4.845
	59	5.3806	5.121		57	4.8993	4.679
	64	5.0258	4.798		62	4.6592	4.458
	69	4.5042	4.312		67	4.2984	4.124
70	0	3.7821	3.592		72	3.7817	3.639
	5	6.1022	5.768	73	3	5.0512	4.811
	10	6.3472	6.008		8	5.4803	5.204
	15	6.2642	5.933		13	5.4733	5.212
	20	6.1497	5.826		18	5.3779	5.123
	25	6.0995	5.780		23	5.3234	5.072
	30	6.0433	5.729		28	5.2840	5.036
	35	5.9714	5.663		33	5.2354	4.991
	40	5.8709	5.571		38	5.1696	4.930
	45	5.7491	5.460		43	5.0814	4.848
	50	5.5822	5.306		48	4.9701	4.746
	55	5.3917	5.132		53	4.8288	4.614
	60	5.1393	4.900		58	4.6563	4.455
	65	4.7829	4.573		63	4.4202	4.236
	70	4.2614	4.087		68	4.0594	3.901
71	1	4.6110	4.380		73	3.5488	3.421
	6	5.9257	5.610	74	4	4.9532	4.726
	11	6.0563	5.744		9	5.2255	4.969
	16	5.9644	5.660		14	5.1881	4.950
	21	5.8705	5.572		19	5.0981	4.866
	26	5.8263	5.532		24	5.0566	4.827
	31	5.7727	5.483		29	5.0195	4.792
	36	5.7031	5.419		34	4.9732	4.749
	41	5.6051	5.329		39	4.9088	4.690
	46	5.4886	5.222		44	4.8265	4.613
	51	5.3284	5.074		49	4.7162	4.511
	56	5.1450	4.905		54	4.5853	4.389
	61	4.8985	4.679		59	4.4184	4.234
	66	4.5401	4.349		64	4.1865	4.019
	71	4.0201	3.862		69	3.8252	3.683

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.

Age.		3 per Cent.	4 per Cent.	Age.		3 per Cent.	4 per Cent.
Older.	Younger.			Older.	Younger.		
74	74	3.3246	3.211	77	72	3.1760	3.070
75	5	4.7686	4.557		77	2.7417	2.656
	10	4.9622	4.725	78	8	4.1802	4.016
	15	4.9116	4.695		13	4.1856	4.022
	20	4.8311	4.619		18	4.1236	3.964
	25	4.7990	4.589		23	4.0877	3.930
	30	4.7642	4.557		28	4.0640	3.908
	35	4.7199	4.516		33	4.0353	3.881
	40	4.6566	4.457		38	3.9963	3.844
	45	4.5802	4.386		43	3.9428	3.794
	50	4.4720	4.285		48	3.8754	3.731
	55	4.3504	4.171		53	3.7876	3.648
	60	4.1893	4.021		58	3.6821	3.549
	65	3.9585	3.806		63	3.5385	3.414
	70	3.5993	3.471		68	3.3100	3.199
	75	3.1146	3.015		73	2.9637	2.869
76	6	4.5991	4.403		78	2.5503	2.470
	11	4.7071	4.487	79	9	3.9215	3.775
	16	4.6492	4.452		14	3.9045	3.759
	21	4.5838	4.391		19	3.8462	3.704
	26	4.5564	4.365		24	3.8201	3.679
	31	4.5236	4.335		29	3.7984	3.659
	36	4.4812	4.295		34	3.7716	3.633
	41	4.4199	4.238		39	3.7341	3.598
	46	4.3479	4.171		44	3.6856	3.552
	51	4.2449	4.074		49	3.6195	3.490
	56	4.1294	3.966		54	3.5406	3.416
	61	3.9742	3.821		59	3.4409	3.322
	66	3.7435	3.606		64	3.3031	3.192
	71	3.3865	3.270		69	3.0778	2.979
	76	2.9269	2.833		74	2.7432	2.659
77	7	4.4021	4.222		79	2.3385	2.271
	12	4.4498	4.268	80	10	3.6476	3.517
	17	4.3881	4.210		15	3.6213	3.492
	22	4.3390	4.164		20	3.5695	3.443
	27	4.3135	4.140		25	3.5506	3.425
	32	4.2827	4.111		30	3.5307	3.406
	37	4.2421	4.073		35	3.5059	3.383
	42	4.1839	4.019		40	3.4696	3.349
	47	4.1150	3.954		45	3.4260	3.308
	52	4.0193	3.864		50	3.3622	3.247
	57	3.9086	3.761		55	3.2913	3.180
	62	3.7599	3.621		60	3.1977	3.092
	67	3.5291	3.405		65	3.0636	2.965

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.

Age.		3 per Cent.	4 per Cent.	Age.		3 per Cent.	4 per Cent.
Older.	Younger.			Older.	Younger.		
80	70	2.8438	2.757*	83	68	2.4037	2.336
	75	2.5265	2.448		73	2.1997	2.141
	80	2.1225	2.068		78	1.9475	1.899
81	11	3.3802	3.264		83	1.5380	1.510
	16	3.3488	3.235	84	14	2.7031	2.622
	21	3.3076	3.195		19	2.6688	2.589
	26	3.2922	3.181		24	2.6535	2.574
	31	3.2740	3.164		29	2.6417	2.563
	36	3.2509	3.142		34	2.6275	2.549
	41	3.2164	3.109		39	2.6074	2.530
	46	3.1767	3.072		44	2.5810	2.505
	51	3.1172	3.015		49	2.5449	2.470
	56	3.0518	2.953		54	2.5009	2.428
	61	2.9642	2.870		59	2.4460	2.376
	66	2.8331	2.746		64	2.3714	2.305
	71	2.6186	2.542		69	2.2442	2.183
	76	2.3258	2.258		74	2.0437	1.991
	81	1.9173	1.869		79	1.7928	1.751
82	12	3.1220	3.020		84	1.4164	1.387
	17	3.0877	2.987	85	15	2.5350	2.462
	22	3.0577	2.958		20	2.5034	2.431
	27	3.0437	2.945		25	2.4926	2.421
	32	3.0271	2.929		30	2.4817	2.411
	37	3.0055	2.909		35	2.4684	2.398
	42	2.9733	2.878		40	2.4485	2.379
	47	2.9362	2.843		45	2.4248	2.356
	52	2.8821	2.792		50	2.3888	2.322
	57	2.8208	2.733		55	2.3492	2.284
	62	2.7393	2.656		60	2.2970	2.234
	67	2.6102	2.533		65	2.2235	2.163
	72	2.4011	2.334		70	2.0969	2.042
	77	2.1317	2.077		75	1.9029	1.856
	82	1.7191	1.681		80	1.6451	1.608
83	13	2.8847	2.794		85	1.3090	1.339
	18	2.8495	2.760	86	16	2.3805	2.315
	23	2.8282	2.740		21	2.3547	2.290
	28	2.8155	2.728		26	2.3461	2.282
	33	2.8002	2.713		31	2.3360	2.272
	38	2.7796	2.694		36	2.3236	2.260
	43	2.7505	2.666		41	2.3041	2.241
	48	2.7145	2.632		46	2.2825	2.221
	53	2.6657	2.585		51	2.2483	2.188
	58	2.6080	2.530		56	2.2114	2.153
	63	2.5305	2.457		61	2.1622	2.105

TABLE XXXVII.

Showing the Values of Annuities on Two Joint Lives, according to the Northampton rate of mortality.

Age.		3 per Cent.	4 per Cent.	Age.		3 per Cent.	4 per Cent.
Older	Younger.			Older.	Younger.		
86	66	2.0892	2.035	89	64	1.7900	1.751
	71	1.9628	1.914		69	1.7218	1.685
	76	1.7816	1.739		74	1.6038	1.570
	81	1.5109	1.478		79	1.4565	1.427
	86	1.2185	1.195		84	1.1879	1.164
87	17	2.2349	2.177		89	1.0361	1.015
	22	2.2159	2.158	90	20	1.7397	1.704
	27	2.2080	2.151		25	1.7343	1.699
	32	2.1988	2.142		30	1.7291	1.694
	37	2.1871	2.130		35	1.7230	1.688
	42	2.1687	2.113		40	1.7137	1.679
	47	2.1486	2.093		45	1.7026	1.668
	52	2.1172	2.063		50	1.6853	1.651
	57	2.0823	2.030		55	1.6663	1.633
	62	2.0365	1.985		60	1.6409	1.608
	67	1.9638	1.915		65	1.6067	1.575
	72	1.8381	1.794		70	1.5455	1.515
	77	1.6706	1.633		75	1.4406	1.413
	82	1.3854	1.356		80	1.3023	1.278
	87	1.1416	1.124		85	1.0748	1.054
88	18	2.1124	2.061		90	.9386	.922
	23	2.0989	2.048	91	21	1.4589	1.432
	28	2.0918	2.041		26	1.4555	1.429
	33	2.0834	2.033		31	1.4516	1.425
	38	2.0723	2.022		36	1.4468	1.420
	43	2.0558	2.006		41	1.4391	1.413
	48	2.0362	1.987		46	1.4310	1.405
	53	2.0079	1.960		51	1.4170	1.391
	58	1.9751	1.928		56	1.4025	1.377
	63	1.9320	1.886		61	1.3826	1.358
	68	1.8605	1.817		66	1.3544	1.330
	73	1.7361	1.697		71	1.3032	1.280
	78	1.5803	1.546		76	1.2212	1.200
	83	1.2848	1.259		81	1.0964	1.078
	88	1.1030	1.030		86	.9212	.902
89	19	1.9481	1.904		91	.7697	.756
	24	1.9390	1.895	92	22	1.1608	1.142
	29	1.9328	1.889		27	1.1584	1.140
	34	1.9254	1.882		32	1.1556	1.137
	39	1.9153	1.872		37	1.1522	1.134
	44	1.9011	1.859		42	1.1464	1.128
	49	1.8822	1.840		47	1.1407	1.122
	54	1.8583	1.817		52	1.1306	1.113
	59	1.8288	1.788		57	1.1199	1.102

TABLE XXXVIII.

Showing the Single, or Annual, Premium for the Assurance of £1 on a Single Life, according to the Northampton rate of mortality, reckoning Interest at 3 per Cent.

Age.	Single Premium.	Annual Premium.	Age.	Single Premium.	Annual Premium.
8	.362554	.016566	53	.630857	.049776
9	.364690	.016719	54	.638432	.051429
10	.369029	.017035	55	.646115	.053178
11	.374368	.017429	56	.653906	.055031
12	.380086	.017858	57	.661801	.056996
13	.385980	.018309	58	.669801	.059082
14	.392056	.018783	59	.677901	.061300
15	.398320	.019282	60	.686096	.063661
16	.404782	.019808	61	.694382	.066176
17	.411116	.020334	62	.702752	.068860
18	.417095	.020841	63	.711359	.071782
19	.422696	.021326	64	.720052	.074916
20	.428006	.021794	65	.728990	.078347
21	.432890	.022233	66	.738017	.082050
22	.437540	.022657	67	.747123	.086053
23	.442275	.023097	68	.756292	.090387
24	.447097	.023553	69	.765504	.095081
25	.452010	.024025	70	.774733	.100170
26	.457016	.024515	71	.783946	.105684
27	.462115	.025023	72	.793096	.111645
28	.467312	.025552	73	.802121	.118066
29	.472609	.026101	74	.810938	.124930
30	.478009	.026672	75	.819426	.132172
31	.483516	.027267	76	.827415	.139638
32	.489132	.027887	77	.835381	.147805
33	.494860	.028533	78	.843519	.157007
34	.500704	.029208	79	.852121	.167834
35	.506667	.029914	80	.860733	.180013
36	.512754	.030651	81	.868951	.193128
37	.518969	.031423	82	.876815	.207317
38	.525314	.032233	83	.884012	.221987
39	.531796	.033082	84	.889503	.234467
40	.538419	.033975	85	.894559	.247107
41	.545060	.034896	86	.899170	.259739
42	.551713	.035846	87	.903523	.272773
43	.558371	.036826	88	.907227	.284824
44	.565158	.037855	89	.912239	.302754
45	.572077	.038938	90	.918599	.328687
46	.579133	.040079	91	.927154	.370708
47	.586328	.041283	92	.936206	.427439
48	.593668	.042555	93	.946438	.514659
49	.601156	.043900	94	.955255	.621817
50	.608661	.045301	95	.963804	.775562
51	.616035	.046730	96	.970874	.970874
52	.623391	.048212			

TABLE XXXIX.

Showing the Annual Premium for the Assurance of £100 on a Single Life for 1, 4, 7 or 10 years, or for the whole period of Life, according to the Northampton rate of mortality, at 3 per Cent.

Age.	1 Year.			4 Years.			7 Years.			10 Years.			Life.			Age.
	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	
14	0	17	9	0	18	11	1	1	4	1	3	5	1	17	7	14
15	0	17	11	1	0	3	1	2	11	1	4	7	1	18	7	15
16	0	19	2	1	2	1	1	4	6	1	6	0	1	19	7	16
17	1	1	2	1	4	1	1	6	1	1	6	11	2	0	8	17
18	1	3	3	1	6	0	1	7	5	1	8	4	2	1	8	18
19	1	5	0	1	7	6	1	8	6	1	9	3	2	2	8	19
20	1	7	3	1	8	8	1	9	5	1	10	1	2	3	7	20
21	1	8	9	1	9	5	1	10	1	1	10	9	2	4	6	21
22	1	9	3	1	9	10	1	10	6	1	11	3	2	5	4	22
23	1	9	8	1	10	4	1	11	0	1	11	9	2	6	2	23
24	1	10	2	1	10	10	1	11	6	1	12	3	2	7	1	24
25	1	10	7	1	11	4	1	12	1	1	12	9	2	8	1	25
26	1	11	1	1	11	10	1	12	7	1	13	4	2	9	0	26
27	1	11	7	1	12	4	1	13	2	1	13	11	2	10	1	27
28	1	12	1	1	12	11	1	13	9	1	14	7	2	11	1	28
29	1	12	8	1	13	6	1	14	4	1	15	2	2	12	3	29
30	1	13	3	1	14	1	1	14	11	1	15	10	2	13	4	30
31	1	13	9	1	14	8	1	15	7	1	16	6	2	14	6	31
32	1	14	5	1	15	4	1	16	3	1	17	4	2	15	9	32
33	1	15	0	1	15	11	1	16	11	1	18	2	2	17	1	33
34	1	15	8	1	16	8	1	17	8	1	19	1	2	18	5	34
35	1	16	4	1	17	4	1	18	7	2	0	1	2	19	10	35
36	1	17	0	1	18	1	1	19	7	2	1	1	3	1	4	36
37	1	17	9	1	18	11	2	0	8	2	2	1	3	2	10	37
38	1	18	6	2	0	0	2	1	9	2	3	2	3	4	6	38
39	1	19	3	2	1	3	2	2	11	2	4	4	3	6	2	39
40	2	0	7	2	2	7	2	4	1	2	5	8	3	7	11	40
41	2	2	0	2	3	10	2	5	4	2	7	1	3	9	10	41
42	2	3	6	2	5	0	2	6	6	2	8	6	3	11	8	42
43	2	4	6	2	6	0	2	7	9	2	10	0	3	13	8	43
44	2	5	6	2	7	2	2	9	2	2	11	7	3	15	9	44
45	2	6	8	2	8	4	2	10	10	2	13	3	3	17	11	45
46	2	7	9	2	9	9	2	12	6	2	15	0	4	0	2	46
47	2	9	0	2	11	6	2	14	4	2	16	11	4	2	7	47
48	2	10	3	2	13	7	2	16	4	2	18	11	4	5	1	48
49	2	12	3	2	15	9	2	18	6	3	1	1	4	7	10	49
50	2	15	1	2	18	0	3	0	8	3	3	5	4	10	7	50
51	2	17	4	2	19	11	3	2	8	3	5	7	4	13	6	51
52	2	19	1	3	1	10	3	4	9	3	7	11	4	16	5	52
53	3	1	0	3	3	11	3	7	0	3	10	3	4	19	7	53
54	3	2	11	3	6	1	3	9	5	3	12	10	5	2	10	54
55	3	5	1	3	8	5	3	12	0	3	15	6	5	6	4	55
56	3	7	3	3	10	11	3	14	8	3	18	5	5	10	1	56
57	3	9	9	3	13	7	3	17	6	4	1	6	5	14	0	57
58	3	12	4	3	16	6	4	0	6	4	4	10	5	18	2	58
59	3	15	1	3	19	5	4	3	8	4	8	6	6	2	7	59
60	3	18	2	4	2	6	4	7	1	4	12	6	6	7	4	60
61	4	1	5	4	5	8	4	10	10	4	16	11	6	12	4	61
62	4	3	11	4	9	1	4	15	0	5	1	8	6	17	9	62

TABLE XL.

Showing the Premium required, for a given number of Payments, to secure £100 at the extinction of a Single Life, according to the Northampton rate of mortality, at 3 per Cent.

Age.	1 Payment.			5 Payments.			7 Payments.			10 Payments.			15 Payments.			20 Payments.		
	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
14	39	4	1	8	9	2	6	5	9	4	13	5	3	8	10	2	16	10
15	39	16	8	8	12	2	6	8	0	4	15	2	3	10	2	2	18	0
16	40	9	7	8	15	3	6	10	5	4	17	1	3	11	7	2	19	3
17	41	2	3	8	18	5	6	12	10	4	18	11	3	13	0	3	0	6
18	41	14	2	9	1	4	6	15	1	5	0	8	3	14	5	3	1	8
19	42	5	5	9	4	1	6	17	3	5	2	3	3	15	8	3	2	10
20	42	16	0	9	6	8	6	19	2	5	3	10	3	16	10	3	3	9
21	43	5	9	9	9	0	7	1	0	5	5	2	3	17	6	3	4	8
22	43	15	1	9	11	1	7	2	6	5	6	5	3	18	10	3	5	6
23	44	4	7	9	13	3	7	4	2	5	7	8	3	19	10	3	6	5
24	44	14	2	9	15	6	7	5	10	5	9	0	4	0	10	3	7	3
25	45	4	0	9	17	8	7	7	7	5	10	3	4	1	10	3	8	2
26	45	14	0	10	0	0	7	9	4	5	11	8	4	2	11	3	9	1
27	46	4	3	10	2	4	7	11	1	5	13	0	4	4	0	3	10	1
28	46	14	8	10	4	8	7	13	0	5	14	5	4	5	1	3	11	0
29	47	5	3	10	7	2	7	14	10	5	15	11	4	6	3	3	12	1
30	47	16	0	10	9	6	7	16	8	5	17	4	4	7	5	3	13	1
31	48	7	0	10	12	3	7	18	8	5	18	11	4	8	8	3	14	3
32	48	18	3	10	14	9	8	0	8	6	0	5	4	9	11	3	15	4
33	49	9	9	10	17	5	8	2	8	6	2	0	4	11	2	3	16	6
34	50	1	5	11	0	2	8	4	9	6	3	11	4	12	6	3	17	9
35	50	13	4	11	2	10	8	6	10	6	5	4	4	13	11	3	19	0
36	51	5	6	11	5	9	8	9	1	6	7	2	4	15	4	4	0	4
37	51	17	11	11	8	9	8	11	5	6	8	11	4	16	10	4	1	8
38	52	10	8	11	11	8	8	13	9	6	10	10	4	18	4	4	3	1
39	53	3	7	11	14	9	8	16	2	6	12	9	4	19	11	4	4	7
40	53	16	10	11	18	0	8	18	10	6	14	9	5	1	8	4	6	2
41	54	10	2	12	1	3	9	1	4	6	16	10	5	3	4	4	7	9
42	55	3	5	12	4	7	9	3	10	6	18	10	5	5	1	4	9	4
43	55	16	9	12	7	9	9	6	4	7	0	11	5	6	10	4	11	0
44	56	10	4	12	11	1	9	8	11	7	3	0	5	8	8	4	12	9
45	57	4	2	12	14	5	9	11	7	7	5	3	5	10	6	4	14	6
46	57	18	3	12	17	10	9	14	5	7	7	6	5	12	6	4	16	5
47	58	12	8	13	1	5	9	17	4	7	9	11	5	14	7	4	18	5
48	59	7	4	13	5	3	10	0	3	7	12	6	5	16	9	5	0	6
49	60	2	4	13	9	3	10	3	6	7	15	1	5	19	1	5	2	9
50	60	17	4	13	13	2	10	6	8	7	17	9	6	1	5	5	5	0
51	61	12	1	13	17	1	10	9	11	8	0	5	6	3	9	5	7	4
52	62	6	9	14	0	10	10	13	0	8	3	1	6	6	2	5	9	8
53	63	1	9	14	4	10	10	16	2	8	5	9	6	8	7	5	12	2
54	63	16	10	14	8	7	10	19	6	8	8	5	6	11	1	5	14	9
55	64	12	3	14	12	11	11	2	11	8	11	6	6	13	11	5	17	8
56	65	7	10	14	17	3	11	7	6	8	14	7	6	16	9	6	0	8
57	66	3	7	15	1	8	11	10	0	8	17	9	6	19	9	6	3	10
58	66	19	7	15	6	1	11	13	10	9	1	0	7	2	11	6	7	4
59	67	15	10	15	10	9	11	17	9	9	4	5	7	6	4	6	11	0
60	68	12	2	15	15	5	12	1	7	9	8	0	7	9	11	6	14	11
61	69	8	9	16	0	3	12	5	7	9	11	9	7	13	9	6	19	3
62	70	5	6	16	5	2	12	10	0	9	15	9	7	18	0	7	3	10

TABLE XLI.

Assurances on Two Joint Lives.

Showing the Premium required for securing a Sum payable on the extinction of the *first* of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.	Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.
Older.	Younger.				Older.	Younger.			
		£. s. d.					£. s. d.		
14	14	3 3 2	.52030	.03159	27	22	4 0 4	.57961	.04015
						27	4 3 10	.59008	.04193
15	10	3 1 1	.51177	.03053					
	15	3 5 0	.52731	.03249	28	13	3 13 11	.55925	.03696
						18	3 18 4	.57353	.03917
16	11	3 2 8	.51831	.03134		23	4 1 10	.58407	.04090
	16	3 6 11	.53458	.03345		28	4 5 6	.59470	.04274
17	12	3 4 5	.52499	.03219	29	14	3 15 8	.56496	.03782
	17	3 8 10	.54162	.03442		19	4 0 1	.57881	.04003
18	13	3 6 1	.53147	.03304		24	4 3 4	.58861	.04167
	18	3 10 8	.54807	.03532		29	4 7 2	.59940	.04358
19	14	3 7 9	.53775	.03388	30	10	3 13 9	.55874	.03688
	19	3 12 4	.55389	.03616		15	3 17 6	.57083	.03874
						20	4 1 9	.58390	.04087
20	10	3 5 7	.52959	.03279		25	4 4 11	.59322	.04248
	15	3 9 6	.54389	.03473		30	4 8 11	.60419	.04446
	20	3 13 11	.55922	.03695					
21	11	3 6 11	.53475	.03348	31	11	3 15 5	.56412	.03770
	16	3 11 2	.54980	.03557		16	3 19 5	.57689	.03971
	21	3 15 4	.56385	.03765		21	4 3 5	.58871	.04169
						26	4 6 7	.59792	.04331
22	12	3 8 4	.53994	.03418		31	4 10 9	.60905	.04537
	17	3 12 9	.55538	.03638	32	12	3 17 2	.56980	.03858
	22	3 16 7	.56805	.03830		17	4 1 5	.58289	.04070
23	13	3 9 10	.54528	.03493		22	4 5 0	.59338	.04250
	18	3 14 4	.56072	.03718		27	4 8 4	.60269	.04418
	23	3 18 0	.57232	.03898		32	4 12 8	.61401	.04633
24	14	3 11 5	.55077	.03571	33	13	3 19 0	.57564	.03951
	19	3 15 11	.56579	.03795		18	4 3 4	.58869	.04169
	24	3 19 4	.57666	.03967		23	4 6 8	.59812	.04335
						28	4 10 2	.60755	.04509
25	10	3 9 3	.54319	.03463		33	4 14 8	.61905	.04733
	15	3 13 1	.55642	.03654					
	20	3 17 5	.57065	.03871	34	14	4 1 0	.58165	.04050
	25	4 0 10	.58106	.04040		19	4 5 4	.59427	.04266
						24	4 8 6	.60295	.04423
26	11	3 10 9	.54830	.03536		29	4 12 1	.61250	.04604
	16	3 14 10	.56224	.03741		34	4 16 9	.62420	.04838
	21	3 18 11	.57521	.03944					
	26	4 2 4	.58553	.04115	35	10	3 19 5	.57693	.03972
						15	4 3 1	.58783	.04154
27	12	3 12 3	.55370	.03614		20	4 7 3	.59968	.04363
	17	3 16 7	.56800	.03830					

TABLE XLI.

Assurances on Two Joint Lives.

Showing the Premium required for securing a Sum payable on the extinction of the *first* of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.	Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.
Older.	Younger.				Older.	Younger.			
		£. s. d.					£. s. d.		
35	25	4 10 4	.60787	.04515	41	41	5 14 8	.66314	.05734
	30	4 14 1	.61753	.04703	42	12	4 11 5	.61078	.04571
	35	4 18 11	.62944	.04947		17	4 15 7	.62128	.04778
36	11	4 1 4	.58268	.04067		22	4 18 11	.62942	.04947
	16	4 5 4	.59420	.04265		27	5 1 11	.63632	.05096
	21	4 9 2	.60484	.04458		32	5 5 9	.64481	.05288
	26	4 12 3	.61288	.04611		37	5 10 10	.65549	.05542
	31	4 16 2	.62267	.04806		42	5 17 8	.66886	.05883
	36	5 1 3	.63479	.05063	43	13	4 13 11	.61716	.04695
37	12	4 3 5	.58873	.04169		18	4 18 2	.62760	.04909
	17	4 7 7	.60055	.04379		23	5 1 3	.63475	.05062
	22	4 11 1	.60987	.04553		28	5 4 4	.64168	.05216
	27	4 14 3	.61798	.04712		33	5 8 4	.65027	.05415
	32	4 18 4	.62791	.04915		38	5 13 8	.66112	.05682
	37	5 3 8	.64025	.05184		43	6 0 9	.67451	.06036
38	13	4 5 7	.59496	.04278	44	14	4 16 7	.62373	.04828
	18	4 9 10	.60672	.04493		19	5 0 10	.63376	.05040
	23	4 13 1	.61501	.04653		24	5 3 8	.64018	.05182
	28	4 16 4	.62319	.04817		29	5 6 10	.64715	.05342
	33	5 0 7	.63325	.05029		34	5 11 0	.65582	.05550
	38	5 6 3	.64583	.05311		39	5 16 7	.66687	.05831
39	14	4 7 11	.60137	.04394		44	6 3 11	.68026	.06197
	19	4 12 2	.61271	.04608	45	10	4 15 11	.62206	.04794
	24	4 15 2	.62025	.04757		15	4 19 5	.63048	.04969
	29	4 18 7	.62850	.04928		20	5 3 6	.63979	.05173
	34	5 3 0	.63871	.05149		25	5 6 2	.64571	.05308
	39	5 8 11	.65153	.05446		30	5 9 6	.65272	.05474
40	10	4 6 9	.59832	.04338		35	5 13 10	.66149	.05692
	15	4 10 4	.60798	.04517		40	5 19 9	.67274	.05987
	20	4 14 6	.61856	.04723		45	6 7 4	.68612	.06367
	25	4 17 4	.62560	.04867	46	11	4 18 6	.62848	.04927
	30	5 0 11	.63393	.05044		16	5 2 5	.63742	.05120
	35	5 5 6	.64427	.05275		21	5 6 2	.64562	.05306
	40	5 11 9	.65736	.05588		26	5 8 10	.65135	.05441
41	11	4 9 0	.60445	.04451		31	5 12 3	.65839	.05614
	16	4 12 11	.61469	.04646		36	5 16 10	.66727	.05841
	21	4 16 9	.62409	.04836		41	6 3 0	.67865	.06151
	26	4 19 7	.63096	.04980		46	6 10 11	.69209	.06547
	31	5 3 3	.63936	.05164	47	12	5 1 5	.63519	.05071
	36	5 8 1	.64987	.05406		17	5 5 7	.64437	.05277

TABLE XLI.

Assurances on Two Joint Lives.

Showing the Premium required for securing a Sum payable on the extinction of the *first* of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.	Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.
Older.	Younger.	£. s. d.			Older.	Younger.	£. s. d.		
47	22	5 8 10	.65137	.05442	52	17	5 18 7	.67050	.05927
	27	5 11 7	.65709	.05581		22	6 1 9	.67638	.06088
	32	5 15 2	.66418	.05760		27	6 4 4	.68099	.06218
	37	6 0 0	.67317	.05999		32	6 7 8	.68672	.06385
	42	6 6 5	.68459	.06322		37	6 12 2	.69407	.06608
	47	6 14 9	.69818	.06738		42	6 18 3	.70352	.06911
48	13	5 4 6	.64209	.05225		47	7 6 1	.71485	.07302
	18	5 8 9	.65120	.05438		52	7 16 8	.72901	.07835
	23	5 11 8	.65723	.05585	53	13	5 18 1	.66959	.05903
	28	5 14 7	.66296	.05729		18	6 2 4	.67739	.06116
	33	5 18 4	.67008	.05916		23	6 5 2	.68237	.06257
	38	6 3 4	.67920	.06167		28	6 7 10	.68696	.06392
	43	6 10 0	.69057	.06500		33	6 11 4	.69270	.06565
	48	6 18 10	.70440	.06941		38	6 16 0	.70012	.06800
49	14	5 7 10	.64918	.05390		43	7 2 3	.70948	.07113
	19	5 12 0	.65789	.05601		48	7 10 6	.72098	.07526
	24	5 14 9	.66323	.05736		53	8 1 7	.73499	.08078
	29	5 17 8	.66894	.05885	54	14	6 1 11	.67669	.06096
	34	6 1 7	.67612	.06080		19	6 6 2	.68413	.06308
	39	6 6 11	.68536	.06344		24	6 8 9	.68845	.06436
	44	6 13 9	.69667	.06689		29	6 11 6	.69301	.06575
	49	7 3 2	.71075	.07157		34	6 15 1	.69876	.06756
50	10	5 7 10	.64919	.05390		39	7 0 1	.70626	.07003
	15	5 11 3	.65635	.05563		44	7 6 6	.71552	.07326
	20	5 15 4	.66439	.05766		49	7 15 3	.72721	.07764
	25	5 17 10	.66923	.05893		54	8 6 8	.74103	.08334
	30	6 0 11	.67495	.06048	55	10	6 2 8	.67802	.06133
	35	6 5 0	.68217	.06251		15	6 6 1	.68396	.06303
	40	6 10 7	.69155	.06530		20	6 10 2	.69078	.06506
	45	6 17 9	.70279	.06887		25	6 12 6	.69463	.06625
	50	7 7 7	.71705	.07381		30	6 15 5	.69915	.06769
51	11	5 11 0	.65584	.05550		35	6 19 2	.70491	.06958
	16	5 14 10	.66349	.05743		40	7 4 4	.71251	.07218
	21	5 18 6	.67048	.05926		45	7 11 0	.72164	.07551
	26	6 1 1	.67512	.06053		50	8 0 3	.73344	.08014
	31	6 4 3	.68084	.06213		55	8 12 1	.74714	.08606
	36	6 8 6	.68813	.06426	56	11	6 6 8	.68502	.06334
	41	6 14 4	.69758	.06718		16	6 10 6	.69139	.06525
	46	7 1 10	.70882	.07090		21	6 14 2	.69725	.06708
	51	7 12 1	.72309	.07605		26	6 16 6	.70089	.06825
52	12	5 14 5	.66264	.05721		31	6 19 6	.70539	.06974

TABLE XLI.

Assurances on Two Joint Lives.

Showing the Premium required for securing a Sum payable on the extinction of the *first* of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age.		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
Older.	Younger.	£.	s.	d.			Older.	Younger.	£.	s.	d.		
56	36	7	3	5	.71117	.07171	60	40	8	3	4	.73714	.08168
	41	7	8	11	.71878	.07444		45	8	9	6	.74424	.08476
	46	7	15	10	.72786	.07790		50	8	18	2	.75357	.08907
	51	8	5	5	.73959	.08272		55	9	9	0	.76442	.09451
	56	8	17	11	.75330	.08894		60	10	4	8	.77846	.10235
57	12	6	11	0	.69226	.06552	61	11	7	8	0	.71758	.07400
	17	6	15	2	.69882	.06758		16	7	11	10	.72269	.07591
	22	6	18	4	.70366	.06916		21	7	15	6	.72746	.07774
	27	7	0	9	.70725	.07036		26	7	17	8	.73020	.07883
	32	7	3	10	.71172	.07191		31	8	0	5	.73357	.08019
	37	7	7	11	.71751	.07398		36	8	4	0	.73794	.08202
	42	7	13	7	.72507	.07681		41	8	9	1	.74380	.08456
	47	8	0	11	.73417	.08044		46	8	15	6	.75080	.08775
	52	8	10	10	.74571	.08541		51	9	4	6	.75999	.09223
	57	9	4	0	.75952	.09199		56	9	15	10	.77076	.09793
58	13	6	15	8	.69964	.06785		61	10	12	6	.78485	.10625
	18	7	0	0	.70615	.06999	62	12	7	13	9	.72525	.07688
	23	7	2	9	.71016	.07136		17	7	17	11	.73053	.07896
	28	7	5	2	.71370	.07260		22	8	1	1	.73438	.08053
	33	7	8	5	.71814	.07421		27	8	3	3	.73704	.08164
	38	7	12	9	.72396	.07639		32	8	6	1	.74037	.08305
	43	7	18	7	.73138	.07930		37	8	9	11	.74471	.08496
	48	8	6	3	.74057	.08314		42	8	15	3	.75048	.08760
	53	8	16	6	.75189	.08827		47	9	1	11	.75744	.09095
	58	9	10	6	.76579	.09523		52	9	11	1	.76638	.09555
59	14	7	0	8	.70718	.07034		57	10	3	2	.77714	.10157
	19	7	5	0	.71336	.07248		62	11	0	10	.79126	.11041
	24	7	7	5	.71675	.07370	63	13	8	0	1	.73318	.08003
	29	7	10	0	.72023	.07498		18	8	4	5	.73841	.08222
	34	7	13	4	.72465	.07665		23	8	7	1	.74152	.08355
	39	7	17	11	.73050	.07895		28	8	9	5	.74411	.08470
	44	8	3	11	.73777	.08194		33	8	12	4	.74738	.08617
	49	8	12	1	.74706	.08603		38	8	16	4	.75171	.08818
	54	9	2	7	.75813	.09130		43	9	1	9	.75730	.09088
	59	9	17	4	.77211	.09868		48	9	8	11	.76428	.09444
60	10	7	2	8	.71012	.07135		53	9	18	4	.77295	.09915
	15	7	6	1	.71486	.07302		58	10	11	0	.78368	.10551
	20	7	10	2	.72048	.07507		63	11	10	0	.79791	.11499
	25	7	12	4	.72343	.07619	64	14	8	6	11	.74125	.08344
	30	7	15	0	.72686	.07751		19	8	11	3	.74619	.08563
	35	7	18	6	.73125	.07925		24	8	13	7	.74874	.08680

TABLE XLI.

Assurances on Two Joint Lives.

Showing the Premium required for securing a Sum payable on the extinction of the *first* of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
Older.	Younger.	£.	s.	d.		
64	29	8	15	11	.75127	.08797
	34	8	19	0	.75448	.08950
	39	9	3	3	.75880	.09163
	44	9	8	10	.76421	.09440
	49	9	16	5	.77122	.09819
	54	10	6	0	.77957	.10301
	59	10	19	6	.79025	.10974
	64	11	19	10	.80458	.11992
65	10	8	11	2	.74606	.08557
	15	8	14	5	.74961	.08720
	20	8	18	7	.75407	.08930
	25	9	0	8	.75621	.09035
	30	9	3	2	.75866	.09156
	35	9	6	4	.76181	.09316
	40	9	10	10	.76612	.09541
	45	9	16	6	.77134	.09825
	50	10	4	6	.77831	.10226
	55	10	14	5	.78637	.10722
	60	11	8	8	.79699	.11434
	65	12	10	10	.81152	.12540
66	11	8	18	10	.75426	.08940
	16	9	2	7	.75812	.09129
	21	9	6	4	.76183	.09317
	26	9	8	4	.76378	.09417
	31	9	10	10	.76616	.09543
	36	9	14	2	.76925	.09710
	41	9	18	11	.77350	.09946
	46	10	4	10	.77857	.10241
	51	10	13	2	.78535	.10657
	56	11	3	6	.79325	.11175
	61	11	18	7	.80377	.11930
	66	13	2	8	.81851	.13135
67	12	9	7	2	.76265	.09359
	17	9	11	5	.76665	.09569
	22	9	14	6	.76957	.09727
	27	9	16	7	.77144	.09831
	32	9	19	3	.77374	.09960
	37	10	2	9	.77679	.10136
	42	10	7	8	.78092	.10382
	47	10	13	10	.78589	.10691
	52	11	2	4	.79240	.11117
67	57	11	13	3	.80017	.11663
	62	12	9	3	.81059	.12464
	67	13	15	8	.82554	.13782
68	13	9	16	4	.77115	.09815
	18	10	0	9	.77511	.10038
	23	10	3	5	.77739	.10171
	28	10	5	6	.77918	.10277
	33	10	8	3	.78142	10412
	38	10	11	11	.78441	.10598
	43	10	17	0	.78836	.10849
	48	11	3	7	.79331	.11179
	53	11	12	3	.79950	.11614
	58	12	3	10	.80715	.12190
	63	13	1	0	.81753	.13050
	68	14	9	9	.83260	.14487
69	14	10	6	3	.77975	.10311
	19	10	10	9	.78347	.10539
	24	10	13	1	.78528	.10652
	29	10	15	3	.78699	.10761
	34	10	18	0	.78916	.10902
	39	11	2	0	.79211	.11098
	44	11	7	1	.79587	.11356
	49	11	14	2	.80080	.11709
	54	12	3	0	.80665	.12152
	59	12	15	2	.81416	.12760
	64	13	13	8	.82449	.13683
	69	15	5	1	.83968	.15255
70	10	10	14	0	.78601	.10698
	15	10	17	1	.78842	.10854
	20	11	1	6	.79176	.11074
	25	11	3	6	.79322	.11173
	30	11	5	8	.79486	.11285
	35	11	8	8	.79695	.11432
	40	11	12	10	.79988	.11642
	45	11	18	1	.80343	.11904
	50	12	5	7	.80829	.12280
	55	12	14	8	.81384	.12733
	60	13	7	6	.82119	.13376
	65	14	7	7	.83157	.14380
	70	16	1	11	.84676	.16094

TABLE XLII.

Assurances on Last Survivors.

Showing the Premium required for securing a Sum payable on the extinction of the last Survivor of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
Older.	Younger.	£.	s.	d.		
14	14	1	0	10	.26381	.01043
15	10	1	0	0	.25558	.01000
	15	1	1	6	.26933	.01073
16	11	1	0	6	.26084	.01027
	16	1	2	1	.27498	.01104
17	12	1	1	2	.26621	.01056
	17	1	2	9	.28061	.01136
18	13	1	1	9	.27161	.01086
	18	1	3	4	.28612	.01167
19	14	1	2	4	.27700	.01116
	19	1	4	0	.29150	.01198
20	10	1	1	3	.26745	.01063
	15	1	2	11	.28244	.01146
	20	1	4	7	.29679	.01229
21	11	1	1	10	.27251	.01091
	16	1	3	6	.28787	.01177
	21	1	5	2	.30193	.01259
22	12	1	2	5	.27769	.01119
	17	1	4	2	.29328	.01208
	22	1	5	10	.30703	.01290
23	13	1	3	0	.28298	.01149
	18	1	4	10	.29865	.01240
	23	1	6	5	.31223	.01322
24	14	1	3	7	.28838	.01180
	19	1	5	5	.30400	.01272
	24	1	7	1	.31753	.01355
25	10	1	2	5	.27785	.01120
	15	1	4	3	.29391	.01212
	20	1	6	1	.30937	.01304
	25	1	7	9	.32296	.01389
26	11	1	3	0	.28308	.01150
	16	1	4	11	.29956	.01245
	21	1	6	9	.31470	.01337
	26	1	8	6	.32850	.01424
27	12	1	3	7	.28850	.01181
	17	1	5	7	.30523	.01279
27	22	1	7	5	.32005	.01371
	27	1	9	3	.33415	.01461
28	13	1	4	3	.29404	.01213
	18	1	6	3	.31088	.01313
	23	1	8	1	.32552	.01405
	28	1	10	0	.33992	.01500
29	14	1	4	11	.29971	.01246
	19	1	7	0	.31650	.01348
	24	1	8	10	.33110	.01441
	29	1	10	9	.34582	.01539
30	10	1	3	7	.28830	.01180
	15	1	5	7	.30551	.01281
	20	1	7	8	.32212	.01384
	25	1	9	7	.33680	.01479
	30	1	11	7	.35183	.01581
31	11	1	4	3	.29376	.01211
	16	1	6	4	.31141	.01316
	21	1	8	5	.32770	.01419
	26	1	10	4	.34261	.01517
	31	1	12	6	.35798	.01623
32	12	1	4	11	.29942	.01244
	17	1	7	1	.31736	.01354
	22	1	9	1	.33329	.01456
	27	1	11	2	.34856	.01558
	32	1	13	4	.36425	.01668
33	13	1	5	7	.30520	.01279
	18	1	7	10	.32327	.01391
	23	1	9	10	.33902	.01493
	28	1	12	0	.35462	.01600
	33	1	14	4	.37067	.01715
34	14	1	6	4	.31111	.01315
	19	1	8	7	.32913	.01429
	24	1	10	8	.34485	.01532
	29	1	12	11	.36081	.01644
	34	1	15	3	.37721	.01764
35	10	1	4	10	.29877	.01241
	15	1	7	1	.31716	.01352
	20	1	9	4	.33499	.01467

TABLE XLII.

Assurances on Last Survivors.

Showing the Premium required for securing a Sum payable on the extinction of the Last Survivor of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
Older,	Younger.	£.	s.	d.		
35	25	1	11	6	.35081	.01573
	30	1	13	9	.36715	.01689
	35	1	16	4	.38389	.01815
36	11	1	5	6	.30444	.01275
	16	1	7	10	.32334	.01391
	21	1	10	2	.34080	.01506
	26	1	12	4	.35689	.01616
	31	1	14	9	.37360	.01737
	36	1	17	4	.39072	.01867
37	12	1	6	3	.31033	.01310
	17	1	8	7	.32954	.01431
	22	1	10	11	.34664	.01545
	27	1	13	2	.36310	.01660
	32	1	15	9	.38019	.01786
	37	1	18	5	.39769	.01922
38	13	1	6	11	.31633	.01347
	18	1	9	5	.33569	.01471
	23	1	11	9	.35258	·01586
	28	1	14	2	.36944	.01706
	33	1	16	9	.38692	.01837
	38	1	19	7	.40480	.01981
39	14	1	7	9	.32248	.01386
	19	1	10	3	.34178	.01512
	24	1	12	7	.35864	·01628
	29	1	15	1	.37591	.01754
	34	1	17	10	.39379	.01891
	39	2	0	10	.41206	.02041
40	10	1	6	1	.30913	.01303
	15	1	8	6	.32876	.01426
	20	1	11	1	.34787	.01553
	25	1	13	6	.36483	.01673
	30	1	16	1	.38250	·01304
	35	1	19	0	.40082	.01948
	40	2	2	1	.41948	.02104
41	11	1	6	9	.31498	.01339
	16	1	9	4	.33515	.01468
	21	1	11	11	.35386	.01595
	26	1	14	4	.37112	.01718
	31	1	17	2	.38922	.01856
	36	2	0	2	.40794	.02006
41	41	2	3	5	.42698	.02170
42	12	1	7	6	.32102	.01376
	17	1	10	3	.34155	.01511
	22	1	12	9	.35983	.01636
	27	1	15	4	.37751	.01766
	32	1	18	2	.39604	.01910
	37	2	1	4	.41519	.02068
	42	2	4	9	.43457	.02238
43	13	1	8	4	.32719	.01416
	18	1	11	1	.34787	.01553
	23	1	13	7	.36590	.01680
	28	1	16	4	.38400	.01815
	33	1	19	4	.40296	.01966
	38	2	2	7	.42257	.02131
	43	2	6	2	.44223	.02309
44	14	1	9	2	.33348	.01457
	19	1	11	11	.35409	.01596
	24	1	14	6	.37208	.01726
	29	1	17	4	.39062	.01866
	34	2	0	6	.41004	.02024
	39	2	4	0	.43008	.02198
	44	2	7	9	.45006	.02386
45	10	1	7	3	.31905	.01364
	15	1	10	0	.33992	.01500
	20	1	12	10	.36029	.01640
	25	1	15	5	.37838	.01772
	30	1	18	5	.39737	.01920
	35	2	1	8	.41725	.02085
	40	2	5	4	.43776	.02267
	45	2	9	3	.45803	.02461
46	11	1	8	1	.32502	.01402
	16	1	10	11	.34650	.01544
	21	1	13	8	.36640	.01684
	26	1	16	5	.38480	.01821
	31	1	19	6	.40426	.01976
	36	2	3	0	.42462	.02149
	41	2	6	10	.44554	.02340
	46	2	10	10	.46618	.02543
47	12	1	8	10	.33122	.01442
	17	1	11	9	.35307	.01589

TABLE XLII.

Assurances on Last Survivors.

Showing the Premium required for securing a Sum payable on the extinction of the last Survivor of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age.		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
Older.	Younger.	£.	s.	d.			Older.	Younger.	£.	s.	d.		
47	22	1	14	7	.37250	.01729	52	17	1	13	4	.36401	.01666
	27	1	17	5	.39135	.01872		22	1	16	5	.38455	.01819
	32	2	0	8	.41128	.02034		27	1	19	7	.40452	.01978
	37	2	4	4	.43213	.02216		32	2	3	2	.42580	.02160
	42	2	8	4	.45345	.02416		37	2	7	4	.44829	.02366
	47	2	12	7	.47448	.02629		42	2	12	0	.47158	.02599
48	13	1	9	8	.33756	.01484		47	2	17	1	.49487	.02853
	18	1	12	8	.35956	.01635		52	3	2	7	.51777	.03127
	23	1	15	6	.37871	.01775	53	13	1	11	0	.34725	.01550
	28	1	18	6	.39802	.01925		18	1	14	4	.37056	.01715
	33	2	1	11	.41845	.02096		23	1	17	4	.39076	.01868
	38	2	5	9	.43978	.02286		28	2	0	8	.41121	.02034
	43	2	9	11	.46147	.02495		33	2	4	6	.43302	.02224
	48	2	14	5	.48294	.02720		38	2	8	10	.45605	.02441
49	14	1	10	7	.34403	.01527		43	2	13	9	.47975	.02686
	19	1	13	7	.36596	.01681		48	2	19	1	.50355	.02954
	24	1	16	6	.38502	.01823		53	3	4	10	.52672	.03241
	29	1	19	7	.40483	.01981	54	14	1	11	11	.35380	.01594
	34	2	3	2	.42574	.02159		19	1	15	3	.37700	.01762
	39	2	7	2	.44759	.02359		24	1	18	4	.39708	.01918
	44	2	11	7	.46964	.02579		29	2	1	10	.41803	.02092
	49	2	16	4	.49156	.02815		34	2	5	10	.44038	.02292
50	10	1	8	6	.32850	.01425		39	2	10	5	.46397	.02521
	15	1	11	5	.35063	.01572		44	2	15	6	.48807	.02776
	20	1	14	7	.37228	.01727		49	3	1	2	.51238	.03060
	25	1	17	6	.39144	.01873		54	3	7	3	.53583	.03363
	30	2	0	9	.41172	.02038	55	10	1	9	7	.33712	.01481
	35	2	4	6	.43316	.02225		15	1	12	10	.36048	.01641
	40	2	8	9	.45553	.02436		20	1	16	2	.38334	.01810
	45	2	13	4	.47795	.02666		25	1	19	5	.40350	.01969
	50	2	18	4	.50027	.02916		30	2	3	1	.42497	.02152
51	11	1	9	3	.33456	.01464		35	2	7	3	.44787	.02362
	16	1	12	5	.35733	.01619		40	2	12	1	.47202	.02604
	21	1	15	6	.37845	.01773		45	2	17	5	.49655	.02872
	26	1	18	6	.39793	.01925		50	3	3	5	.52134	.03172
	31	2	2	0	.41871	.02098		55	3	9	10	.54509	.03490
	36	2	5	11	.44066	.02294	56	11	1	10	5	.34325	.01522
	41	2	10	4	.46352	.02516		16	1	13	10	.36730	.01692
	46	2	15	2	.48635	.02757		21	1	17	2	.38955	.01858
	51	3	0	5	.50898	.03019		26	2	0	6	.41003	.02023
52	12	1	10	1	.34084	.01506		31	2	4	4	.43203	.02215

TABLE XLII.

Assurances on Last Survivors.

Showing the Premium required for securing a Sum payable on the extinction of the last Survivor of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.	Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.
Older.	Younger.	£. s. d.			Older.	Younger.	£. s. d.		
56	36	2 8 9	.45549	.02436	60	40	2 15 5	.48738	.02769
	41	2 13 10	.48019	.02690		45	3 1 7	.51393	.03079
	46	2 19 6	.50518	.02973		50	3 8 8	.54119	.03435
	51	3 5 9	.53035	.03289		55	3 16 6	.56779	.03826
	56	3 12 6	.55451	.03625		60	4 5 1	.59373	.04256
57	12	1 11 4	.34963	.01565	61	11	1 11 6	.35117	.01576
	17	1 14 10	.37410	.01740		16	1 15 2	.37647	.01758
	22	1 18 1	.39568	.01906		21	1 18 9	.39981	.01939
	27	2 1 7	.41667	.02080		26	2 2 5	.42120	.02119
	32	2 5 7	.43921	.02281		31	2 6 7	.44433	.02329
	37	2 10 3	.46326	.02514		36	2 11 6	.46920	.02574
	42	2 15 7	.48844	.02781		41	2 17 3	.49564	.02862
	47	3 1 7	.51396	.03079		46	3 3 10	.52272	.03190
	52	3 8 3	.53948	.03412		51	3 11 4	.55043	.03566
	57	3 15 5	.56408	.03769		56	3 19 7	.57753	.03981
						61	4 8 10	.60391	.04441
58	13	1 12 3	.35614	.01611					
	18	1 15 10	.38075	.01790	62	12	1 12 5	.35759	.01621
	23	1 19 1	.40192	.01956		17	1 16 2	.38334	.01810
	28	2 2 9	.42341	.02139		22	1 19 9	.40591	.01989
	33	2 7 0	.44652	.02349		27	2 3 6	.42783	.02177
	38	2 11 11	.47116	.02594		32	2 7 11	.45151	.02397
	43	2 17 6	.49679	.02875		37	2 13 1	.47701	.02656
	48	3 3 10	.52290	.03191		42	2 19 2	.50399	.02959
	53	3 10 10	.54877	.03542		47	3 6 1	.53164	.03306
	58	3 18 5	.57381	.03921		52	3 14 1	.55976	.03703
						57	4 2 11	.58741	.04146
59	14	1 13 2	.36278	.01658		62	4 12 9	.61424	.04637
	19	1 16 10	.38724	.01840					
	24	2 0 2	.40825	.02009	63	13	1 13 4	.36416	.01668
	29	2 4 1	.43028	.02202		18	1 17 3	.39004	.01862
	34	2 8 5	.45396	.02421		23	2 0 10	.41211	.02041
	39	2 13 7	.47920	.02679		28	2 4 9	.43456	.02238
	44	2 19 6	.50529	.02974		33	2 9 5	.45884	.02469
	49	3 6 2	.53200	.03310		38	2 14 10	.48496	.02742
	54	3 13 7	.55820	.03680		43	3 1 3	.51243	.03061
	59	4 1 8	.58369	.04083		48	3 8 7	.54075	.03429
						53	3 17 0	.56927	.03849
60	10	1 10 8	.34501	.01533		58	4 6 6	.59748	.04323
	15	1 14 2	.36956	.01707		63	4 17 0	.62481	.04850
	20	1 17 10	.39362	.01890					
	25	2 1 3	.41468	.02063	64	14	1 10 4	.37086	.01516
	30	2 5 3	.43725	.02263		19	1 18 3	.39656	.01913
	35	2 9 11	.46151	.02496		24	2 1 11	.41841	.02095

TABLE XLII.

Assurances on Last Survivors.

Showing the Premium required for securing a Sum payable on the extinction of the last Survivor of Two Assigned Lives, according to the Northampton Table, at 3 per Cent.

Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.	Age.		Annual Premium per Cent.	Single Premium for £1.	Annual Premium for £1.
Older.	Younger.	£. s. d.			Older.	Younger.	£. s. d.		
64	29	2 6 0	.44139	.02301	67	57	4 10 7	.60875	.04531
	34	2 10 11	.46628	.02544		62	5 3 3	.63929	.05162
	39	2 16 8	.49305	.02832		67	5 17 7	.66871	.05879
	44	3 3 4	.52100	.03168	68	13	1 14 4	.37112	.01718
	49	3 11 2	.54999	.03560		18	1 18 7	.39828	.01928
	54	4 0 1	.57891	.04004		23	2 2 5	.42118	.02119
	59	4 10 3	.60770	.04512		28	2 6 7	.44442	.02330
	64	5 1 7	.63552	.05078		33	2 11 7	.46973	.02580
65	10	1 11 7	.35196	.01581		38	2 17 7	.49720	.02880
	15	1 15 4	.37770	.01767		43	3 4 9	.52630	.03236
	20	1 19 4	.40293	.01965		48	3 13 2	.55665	.03657
	25	2 3 0	.42479	.02151		53	4 3 0	.58765	.04150
	30	2 7 4	.44834	.02367		58	4 14 7	.61894	.04731
	35	2 12 6	.47385	.02623		63	5 8 3	.65012	.05412
	40	2 18 7	.50129	.02927		68	6 3 9	.67998	.06188
	45	3 5 7	.52973	.03280	69	14	1 15 4	.37781	.01768
	50	3 13 11	.55934	.03696		19	1 19 7	.40473	.01980
	55	4 3 5	.58874	.04169		24	2 3 6	.42732	.02173
	60	4 14 3	.61810	.04714		29	2 7 11	.45112	.02394
	65	5 6 6	.64646	.05326		34	2 13 2	.47705	.02657
66	11	1 12 6	.35813	.01625		39	2 19 6	.50519	.02973
	16	1 16 5	.38468	.01821		44	3 7 0	.53479	.03348
	21	2 0 4	.40908	.02016		49	3 15 11	.56586	.03796
	26	2 4 2	.43125	.02208		54	4 6 5	.59729	.04320
	31	2 8 8	.45537	.02435		59	4 18 10	.62925	.04943
	36	2 14 1	.48152	.02705		64	5 13 7	.66107	.05681
	41	3 0 6	.50958	.03026		69	6 10 6	.69133	.06523
	46	3 8 0	.53858	.03399	70	10	1 12 5	.35775	.01622
	51	3 16 10	.56870	.03840		15	1 16 5	.38463	.01820
	56	4 6 11	.59867	.04345		20	2 0 8	.41098	.02032
	61	4 18 7	.62863	.04930		25	2 4 7	.43352	.02229
	66	5 11 10	.65752	.05592		30	2 9 2	.45788	.02460
67	12	1 13 5	.36456	.01671		35	2 14 9	.48445	.02737
	17	1 17 6	.39159	.01874		40	3 1 5	.51327	.03071
	22	2 1 4	.41509	.02067		45	3 9 4	.54338	.03466
	27	2 5 4	.43780	.02268		50	3 18 10	.57510	.03942
	32	2 10 1	.46252	.02506		55	4 10 0	.60701	.04498
	37	2 15 10	.48930	.02790		60	5 3 5	.63964	.05170
	42	3 2 7	.51792	.03129		65	5 19 5	.67215	.05971
	47	3 10 6	.54756	.03524		70	6 17 8	.70270	.06884
	52	3 19 10	.57811	.03991					

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable on the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
A.	*B.*	£.	*s.*	*d.*			*A.*	*B.*	£.	*s.*	*d.*		
14	14	1	11	7	.26016	.01580	17	12	1	15	0	.28563	.01751
	19	1	11	0	.24582	.01549		17	1	14	5	.27082	.01721
	24	1	10	4	.23418	.01518		22	1	13	10	.25794	.01690
	29	1	9	9	.22191	.01486		27	1	13	2	.24577	.01657
	34	1	9	0	.20843	.01451		32	1	12	5	.23231	.01622
	39	1	8	4	.19361	.01415		37	1	11	9	.21749	.01586
	44	1	7	7	.17783	.01377		42	1	11	0	.20130	.01548
	49	1	6	9	.16097	.01336		47	1	10	2	.18430	.01509
	54	1	5	11	.14369	.01295		52	1	9	5	.16625	.01470
	59	1	5	0	.12566	.01250		57	1	8	7	.14772	.01429
	64	1	3	11	.10635	.01197		62	1	7	8	.12812	.01385
	69	1	2	9	.08591	.01136		67	1	6	9	.10701	.01336
	74	1	1	4	.06595	.01066		72	1	5	7	.08528	.01279
	79	0	19	8	.04824	.00984		77	1	4	3	.06540	.01214
	84	0	18	0	.03325	.00898		82	1	2	7	.04618	.01130
15	10	1	13	1	.27738	.01655	18	13	1	15	11	.28906	.01797
	15	1	12	6	.26366	.01625		18	1	15	4	.27404	.01766
	20	1	11	11	.24959	.01594		23	1	14	8	.26162	.01735
	25	1	11	3	.23796	.01563		28	1	14	0	.24905	.01701
	30	1	10	7	.22531	.01529		33	1	13	4	.23516	.01665
	35	1	9	11	.21141	.01494		38	1	12	7	.21984	.01628
	40	1	9	2	.19609	.01457		43	1	11	10	.20328	.01590
	45	1	8	4	17995	.01418		48	1	11	0	.18572	.01551
	50	1	7	7	.16261	.01378		53	1	10	3	.16738	.01511
	55	1	6	9	.14500	.01336		58	1	9	5	.14833	.01470
	60	1	5	10	.12637	.01291		63	1	8	7	.12817	.01427
	65	1	4	9	.10652	.01239		68	1	7	7	.10650	.01379
	70	1	3	7	.08559	.01178		73	1	6	6	.08449	.01325
	75	1	2	2	.06548	.01108		78	1	5	3	.06467	.01262
	80	1	0	5	.04724	.01022		83	1	3	7	.04546	.01181
16	11	1	14	1	.28166	.01703	19	14	1	16	9	.29193	.01839
	16	1	13	6	.26729	.01673		19	1	16	2	.27695	.01808
	21	1	12	10	.25379	.01642		24	1	15	6	.26474	.01776
	26	1	12	2	.24197	.01610		29	1	14	10	.25177	.01741
	31	1	11	6	.22893	.01576		34	1	14	1	.23742	.01704
	36	1	10	10	.21457	.01540		39	1	13	4	.22157	.01666
	41	1	10	1	.19880	.01503		44	1	12	7	.20463	.01627
	46	1	9	3	.18226	.01464		49	1	11	9	.18648	.01588
	51	1	8	6	.16454	.01424		54	1	11	0	16783	.01548
	56	1	7	8	.14650	.01383		59	1	10	2	.14826	.01507
	61	1	6	9	.12737	.01338		64	1	9	3	.12753	.01464
	66	1	5	9	.10690	.01287		69	1	8	4	.10531	.01417
	71	1	4	7	.08552	.01228		74	1	7	3	.08316	.01364
	76	1	3	2	.06552	.01160		79	1	6	1	.06311	.01302
	81	1	1	6	.04670	.01074		84	1	4	6	.04499	.01226

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable on the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
A.	*B.*	£.	*s.*	*d.*			*A.*	*B.*	£.	*s.*	*d.*		
20	10	1	18	2	.30838	.01909	23	13	2	0	4	.31470	.02016
	15	1	17	7	.29430	.01879		18	1	19	8	.29910	.01983
	20	1	17	0	.27962	.01848		23	1	19	0	.28617	.01949
	25	1	16	3	.26743	.01814		28	1	18	3	.27288	.01911
	30	1	15	7	.25404	.01778		33	1	17	5	.25801	.01870
	35	1	14	10	.23920	.01740		38	1	16	7	.24147	.01827
	40	1	14	0	.22277	.01701		43	1	15	8	.22354	.01783
	45	1	13	3	.20543	.01661		48	1	14	9	.20446	.01737
	50	1	12	5	.18672	.01620		53	1	13	10	.18450	.01692
	55	1	11	7	.16770	.01580		58	1	12	11	.16377	.01646
	60	1	10	9	.14761	.01538		63	1	11	11	.14167	.01596
	65	1	9	11	.12623	.01495		68	1	11	0	.11843	.01550
	70	1	9	0	.10354	.01448		73	1	9	11	.09465	.01497
	75	1	7	11	.08144	.01397		78	1	8	9	.07323	.01439
	80	1	6	8	.06102	.01335		83	1	7	4	.05230	.01366
21	11	1	18	11	.31084	.01946	24	14	2	1	1	.31659	.02053
	16	1	18	4	.29601	.01915		19	2	0	5	.30105	.02019
	21	1	17	8	.28193	.01883		24	1	19	8	.28833	.01984
	26	1	17	0	.26951	.01848		29	1	18	11	.27458	.01944
	31	1	16	2	.25565	.01810		34	1	18	0	.25916	.01901
	36	1	15	5	.24026	.01771		39	1	17	1	.24202	.01856
	41	1	14	7	.22331	.01730		44	1	16	2	.22361	.01810
	46	1	13	9	.20548	.01689		49	1	15	3	.20383	.01763
	51	1	12	11	.18631	.01647		54	1	14	4	.18348	.01715
	56	1	12	1	.16679	.01605		59	1	13	4	.16211	.01667
	61	1	11	3	.14614	.01562		64	1	12	4	.13954	.01618
	66	1	10	4	.12406	.01517		69	1	11	4	.11544	.01566
	71	1	9	5	.10095	.01469		74	1	10	3	.09150	.01511
	76	1	8	4	.07914	.01417		79	1	9	0	.06985	.01449
	81	1	7	1	.05831	.01354		84	1	7	6	.05024	.01375
22	12	1	19	7	.31278	.01980	25	10	2	2	6	.33295	.02123
	17	1	19	0	.29744	.01949		15	2	1	10	.31846	.02091
	22	1	18	4	.28403	.01915		20	2	1	2	.30322	.02057
	27	1	17	7	.27119	.01879		25	2	0	5	.29054	.02020
	32	1	16	10	.25682	.01840		30	1	19	7	.27631	.01978
	37	1	16	0	.24090	.01799		35	1	18	8	.26031	.01934
	42	1	15	1	.22342	.01756		40	1	17	9	.24250	.01887
	47	1	14	3	.20502	.01713		45	1	16	9	.22363	.01838
	52	1	13	5	.18544	.01669		50	1	15	9	.20318	.01789
	57	1	12	6	.16532	.01625		55	1	14	9	.18237	.01739
	62	1	11	7	.14410	.01580		60	1	13	9	.16035	.01689
	67	1	10	8	.12130	.01533		65	1	12	9	.13701	.01637
	72	1	9	8	.09782	.01483		70	1	11	8	.11235	.01583
	77	1	8	7	.07627	.01429		75	1	10	6	.08842	.01525
	82	1	7	2	.05518	.01360		80	1	9	2	.06637	.01458

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable at the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
A.	*B.*	£.	*s.*	*d*			*A.*	*B.*	£.	*s.*	*d.*		
26	11	2	3	3	.33558	.02164	29	14	2	5	11	.34304	.02296
	16	2	2	7	.32028	.02131		19	2	5	3	.32704	.02262
	21	2	1	11	.30571	.02096		24	2	4	6	.31403	.02223
	26	2	1	2	.29277	.02057		29	2	3	7	.29970	.02179
	31	2	0	3	.27803	.02014		34	2	2	7	.28329	.02129
	36	1	19	4	.26145	.01967		39	2	1	6	.26473	.02076
	41	1	18	4	.24302	.01918		44	2	0	5	.24467	.02020
	46	1	17	4	.22366	.01868		49	1	19	3	.22300	.01962
	51	1	16	4	.20258	.01816		54	1	18	1	.20061	.01903
	56	1	15	3	.18117	.01764		59	1	16	10	.17706	.01843
	61	1	14	3	.15851	.01711		64	1	15	8	.15214	.01782
	66	1	13	2	.13435	.01657		69	1	14	4	.12563	.01718
	71	1	12	0	.10917	.01599		74	1	13	0	.09939	.01651
	76	1	10	9	.08551	.01539		79	1	11	7	.07579	.01580
	81	1	9	4	.06297	.01467		84	1	9	11	.05446	.01496
27	12	2	4	1	.33807	.02206	30	10	2	7	7	.36038	.02379
	17	2	3	6	.32223	.02173		15	2	6	11	.34552	.02345
	22	2	2	9	.30842	.02137		20	2	6	2	.32987	.02309
	27	2	1	11	.29505	.02096		25	2	5	5	.31692	.02269
	32	2	1	0	.27977	.02051		30	2	4	6	.30210	.02223
	37	2	0	1	.26256	.02002		35	2	3	5	.28506	.02171
	42	1	19	0	.24356	.01951		40	2	2	4	.26578	.02115
	47	1	17	10	.22287	.01893		45	2	1	1	.24516	.02056
	52	1	16	11	.20200	.01844		50	1	19	11	.22274	.01996
	57	1	15	10	.17989	.01790		55	1	18	8	.19982	.01935
	62	1	14	8	.15655	.01734		60	1	17	5	.17550	.01871
	67	1	13	7	.13156	.01677		65	1	16	1	.14968	.01806
	72	1	12	4	.10593	.01616		70	1	14	9	.12250	.01739
	77	1	11	1	.08252	.01553		75	1	13	5	.09620	.01669
	82	1	9	6	.05967	.01476		80	1	11	10	.07212	.01592
28	13	2	5	0	.34057	.02251	31	11	2	8	7	.36366	.02430
	18	2	4	4	.32448	.02216		16	2	7	11	.34796	.02395
	23	2	3	7	.31119	.02179		21	2	7	2	.33307	.02359
	28	2	2	9	.29736	.02137		26	2	6	4	.31988	.02317
	33	2	1	9	.28152	.02089		31	2	5	5	.30452	.02269
	38	2	0	9	.26366	.02038		36	2	4	3	.28685	.02214
	43	1	19	8	.24413	.01984		41	2	3	1	.26688	.02155
	48	1	18	7	.22327	.01929		46	2	1	11	.24561	.02094
	53	1	17	6	.20134	.01873		51	2	0	7	.22256	.02031
	58	1	16	4	.17852	.01816		56	1	19	4	.19894	.01967
	63	1	15	2	.15441	.01758		61	1	18	0	.17386	.01901
	68	1	13	11	.12862	.01697		66	1	16	8	.14709	.01832
	73	1	12	8	.10265	.01634		71	1	15	3	.11928	.01761
	78	1	11	4	.07934	.01567		76	1	13	9	.09321	.01688
	83	1	9	8	.05663	.01484		81	1	12	1	.06854	.01604

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable on the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
A.	*B.*	£.	*s.*	*d.*			*A.*	*B.*	£.	*s.*	*d.*		
32	12	2	9	8	.36683	.02484	35	10	2	13	11	.39157	.02696
	17	2	9	0	.35058	.02448		15	2	13	2	.37643	.02660
	22	2	8	3	.33655	.02411		20	2	12	6	36048	.02623
	27	2	7	4	.32293	.02367		25	2	11	8	.34755	.02582
	32	2	6	4	.30701	.02317		30	2	10	8	.33248	.02532
	37	2	5	2	.28865	.02259		35	2	9	6	.31472	.02474
	42	2	4	0	.26803	.02198		40	2	8	2	.29411	.02408
	47	2	2	8	.24603	.02134		45	2	6	9	.27169	.02338
	52	2	1	4	.22243	.02068		50	2	5	3	.24705	.02264
	57	2	0	0	.19801	.02001		55	2	3	9	.22172	.02189
	62	1	18	7	.17212	.01931		60	2	2	3	.19474	.02111
	67	1	17	2	.14437	.01859		65	2	0	7	.16594	.02029
	72	1	15	8	.11599	.01784		70	1	18	11	.13556	.01945
	77	1	14	1	.09013	.01706		75	1	17	2	.10622	.01857
	82	1	12	4	.06507	.01616		80	1	15	3	.07936	.01761
33	13	2	10	10	.37001	.02540	36	11	2	15	3	.39565	.02761
	18	2	10	1	.35353	.02503		16	2	14	6	.37963	.02725
	23	2	9	4	.34011	.02465		21	2	13	9	.36457	.02687
	28	2	8	5	.32603	.02420		26	2	12	11	.35142	.02644
	33	2	7	4	.30953	.02367		31	2	11	10	.33582	.02592
	38	2	6	2	.29046	.02307		36	2	10	7	.31740	.02531
	43	2	4	10	.26926	.02242		41	2	9	3	.29604	.02463
	48	2	3	6	.24639	.02175		46	2	7	9	.27289	.02389
	53	2	2	1	.22224	.02106		51	2	6	3	.24754	.02312
	58	2	0	9	.19698	.02036		56	2	4	8	.22141	.02233
	63	1	19	3	.17020	.01962		61	2	3	0	.19351	.02151
	68	1	17	9	.14154	.01886		66	2	1	4	.16361	.02065
	73	1	16	2	.11268	.01807		71	1	19	6	.13244	.01976
	78	1	14	6	.08685	.01725		76	1	17	8	.10328	.01884
	83	1	12	7	.06188	.01628		81	1	15	7	.07569	.01781
34	14	2	12	0	.37322	.02598	37	12	2	16	7	.39965	.02830
	19	2	11	3	.35685	.02562		17	2	15	10	.38307	.02793
	24	2	10	5	.34379	.02522		22	2	15	1	.36898	.02755
	29	2	9	6	.32921	.02474		27	2	14	2	.35543	.02710
	34	2	8	5	.31210	.02419		32	2	13	1	.33926	.02656
	39	2	7	1	.29228	.02356		37	2	11	10	.32013	.02592
	44	2	5	9	.27047	.02289		42	2	10	5	.29808	.02520
	49	2	4	5	.24670	.02219		47	2	8	10	.27410	.02443
	54	2	2	11	.22201	.02147		52	2	7	3	.24811	.02362
	59	2	1	5	.19589	.02072		57	2	5	7	.22106	.02279
	64	1	19	11	.16817	.01995		62	2	3	10	.19223	.02193
	69	1	18	4	.13860	.01915		67	2	2	1	.16118	.02103
	74	1	16	7	.10939	.01831		72	2	0	2	.12929	.02009
	79	1	14	10	.08317	.01743		77	1	18	3	.10028	.01913
	84	1	12	11	.05964	.01644		82	1	16	0	.07214	.01801

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable on the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
A.	*B.*	£.	*s.*	*d.*			*A.*	*B.*	£.	*s.*	*d.*		
38	13	2	18	1	.40369	.02903	41	11	3	3	8	.43208	.03182
	18	2	17	4	.38689	.02865		16	3	2	11	.41589	.03144
	23	2	16	6	.37354	.02826		21	3	2	1	.40079	.03105
	28	2	15	7	.35953	.02779		26	3	1	3	.38794	.03062
	33	2	14	5	.34279	.02722		31	3	0	2	.37248	.03008
	38	2	13	1	.32291	.02656		36	2	18	10	.35383	.02943
	43	2	11	7	.30022	.02580		41	2	17	4	.33157	.02867
	48	2	10	0	.27529	.02499		46	2	15	7	.30680	.02781
	53	2	8	4	.24867	.02415		51	2	13	9	.27908	.02688
	58	2	6	7	.22066	.02328		56	2	11	10	.25018	.02591
	63	2	4	9	.19077	.02238		61	2	9	10	.21912	.02491
	68	2	2	10	.15864	.02143		66	2	7	9	.18560	.02387
	73	2	0	11	.12615	.02045		71	2	5	7	.15047	.02278
	78	1	18	10	.09708	.01943		76	2	3	4	.11742	.02167
	83	1	16	6	.06896	.01824		81	2	0	11	.08619	.02044
39	14	2	19	7	.40777	.02979	42	12	3	5	4	.43676	.03268
	19	2	18	10	.39113	.02942		17	3	4	7	.41998	.03230
	24	2	18	0	.37823	.02901		22	3	3	10	.40600	.03191
	29	2	17	1	.36378	.02852		27	3	2	11	.39276	.03146
	34	2	15	10	.34643	.02793		32	3	1	10	.37678	.03090
	39	2	14	6	.32578	.02723		37	3	0	5	.35741	.03022
	44	2	12	11	.30243	.02644		42	2	18	10	.33444	.02942
	49	2	11	2	.27649	.02559		47	2	17	0	.30875	.02851
	54	2	9	5	.24923	.02471		52	2	15	1	.28030	.02754
	59	2	7	7	.22024	.02380		57	2	13	1	.25033	.02652
	64	2	5	8	.18926	.02285		62	2	10	11	.21817	.02547
	69	2	3	9	.15604	.02186		67	2	8	9	.18330	.02437
	74	2	1	8	.12309	.02083		72	2	6	6	.14729	.02323
	79	1	19	6	.09351	.01975		77	2	4	1	.11435	.02206
	84	1	17	1	.06690	.01855		82	2	1	6	.08244	.02075
40	10	3	1	11	.42717	.03097	43	13	3	7	2	.44131	.03357
	15	3	1	2	.41189	.03060		18	3	6	5	.42433	.03319
	20	3	0	5	.39579	.03022		23	3	5	7	.41121	.03279
	25	2	19	7	.38310	.02980		28	3	4	8	.39756	.03232
	30	2	18	7	.36815	.02929		33	3	3	6	.38101	.03173
	35	2	17	4	.35016	.02867		38	3	2	1	.36090	.03102
	40	2	15	11	.32868	.02794		43	3	0	4	.33726	.03018
	45	2	14	3	.30470	.02712		48	2	18	6	.31050	.02923
	50	2	12	6	.27777	.02623		53	2	16	5	.28132	.02820
	55	2	10	7	.24981	.02531		58	2	14	3	.25023	.02713
	60	2	8	8	.21980	.02435		63	2	12	1	.21682	.02602
	65	2	6	9	.18758	.02336		68	2	9	9	.18065	.02486
	70	2	4	8	.15338	.02232		73	2	7	4	.14386	.02366
	75	2	2	6	.12021	.02125		78	2	4	10	.11078	.02241
	80	2	0	3	.08986	.02011		83	2	2	1	.07882	.02102

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable on the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent			Single Premium for £1.	Annual Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
A.	*B.*	£.	*s.*	*d.*			*A.*	*B.*	£.	*s.*	*d.*		
44	14	3	9	1	.44590	.03452	47	12	3	16	2	.47696	.03808
	19	3	8	3	.42913	.03413		17	3	15	4	.46007	.03768
	24	3	7	5	.41657	.03372		22	3	14	7	.44635	.03729
	29	3	6	5	.40248	.03322		27	3	13	9	.43423	.03688
	34	3	5	3	.38535	.03261		32	3	12	7	.41815	.03627
	39	3	3	9	.36445	.03186		37	3	11	1	.39907	.03556
	44	3	2	0	.34013	.03098		42	3	9	5	.37585	.03471
	49	3	0	0	.31225	.02998		47	3	7	5	.34909	.03369
	54	2	17	10	.28233	.02891		52	3	5	1	.31845	.03253
	59	2	15	7	.25008	.02778		57	3	2	7	.28538	.03127
	64	2	13	2	.21537	.02660		62	2	19	11	.24937	.02994
	69	2	10	9	.17787	.02538		67	2	17	2	.21001	.02857
	74	2	8	3	.14048	.02411		72	2	14	4	.16908	.02715
	79	2	5	7	.10671	.02277		77	2	11	4	.13137	.02568
	84	2	2	9	.07659	.02139		82	2	8	0	.09452	.02401
45	10	3	11	10	.46583	.03590	48	13	3	18	7	.48255	.03927
	15	3	11	0	.45053	.03551		18	3	17	9	.46548	.03886
	20	3	10	3	.43435	.03512		23	3	16	11	.45277	.03847
	25	3	9	5	.42208	.03470		28	3	16	0	.43969	.03800
	30	3	8	4	.40755	.03418		33	3	14	10	.42370	.03741
	35	3	7	1	.38980	.03354		38	3	13	4	.40391	.03667
	40	3	5	6	.36805	.03276		43	3	11	7	.38007	.03577
	45	3	3	8	.34306	.03183		48	3	9	5	.35220	.03470
	50	3	1	7	.31410	.03078		53	3	7	0	.32077	.03348
	55	2	19	4	.28335	.02965		58	3	4	4	.28644	.03216
	60	2	16	11	.24988	.02846		63	3	1	6	.24894	.03076
	65	2	14	5	.21369	.02722		68	2	18	8	.20804	.02932
	70	2	11	10	.17500	.02593		73	2	15	8	.16614	.02783
	75	2	9	2	.13724	.02460		78	2	12	7	.12820	.02630
	80	2	6	4	.10247	.02315		83	2	9	1	.09114	.02454
46	11	3	13	11	.47144	.03696	49	14	4	1	1	.48821	.04053
	16	3	13	1	.45516	.03656		19	4	0	3	.47141	.04013
	21	3	12	4	.44014	.03617		24	3	19	6	.45940	.03973
	26	3	11	6	.42768	.03574		29	3	18	6	.44595	.03923
	31	3	10	5	.41278	.03519		34	3	17	3	.42942	.03862
	36	3	9	1	.39438	.03452		39	3	15	9	.40887	.03785
	41	3	7	5	.37186	.03370		44	3	13	10	.38442	.03691
	46	3	5	6	.34605	.03273		49	3	11	7	.35538	.03579
	51	3	3	3	.31619	.03163		54	3	9	0	.32317	.03450
	56	3	0	10	.28436	.03043		59	3	6	2	.28748	.03310
	61	2	18	4	.24964	.02918		64	3	3	3	.24849	.03164
	66	2	15	9	.21191	.02787		69	3	0	3	.20602	.03012
	71	2	13	1	.17205	.02652		74	2	17	2	.16335	.02858
	76	2	10	3	.13433	.02512		79	2	13	11	.12456	.02696
	81	2	7	1	.09840	.02356		84	2	10	5	.08933	.02520

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable on the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1
A.	*B.*	£.	*s.*	*d.*			*A.*	*B.*	£.	*s.*	*d.*		
50	10	4	4	6	.50891	.04225	53	13	4	12	11	.52703	.04646
	15	4	3	9	.49374	.04185		18	4	12	1	.51001	.04604
	20	4	2	11	.47767	.04145		23	4	11	4	.49788	.04565
	25	4	2	1	.46605	.04104		28	4	10	4	.48562	.04518
	30	4	1	1	.45221	.04052		33	4	9	2	.47045	.04459
	35	3	19	9	.43512	.03987		38	4	7	8	.45145	.04385
	40	3	18	2	.41378	.03907		43	4	5	10	.42816	.04292
	45	3	16	2	.38868	.03809		48	4	3	7	.40021	.04178
	50	3	13	10	.35853	.03691		53	4	0	9	.36749	.04039
	55	3	11	1	.32541	.03556		58	3	17	7	.33030	.03878
	60	3	8	1	.28791	.03403		63	3	14	0	.28843	.03700
	65	3	5	1	.24758	.03253		68	3	10	3	.24189	.03514
	70	3	1	11	.20367	.03094		73	3	6	7	.19389	.03327
	75	2	18	8	.16054	.02934		78	3	2	10	.15032	.03140
	80	2	15	3	.12054	.02763		83	2	18	7	.10745	.02931
51	11	4	7	2	.51514	.04360	54	14	4	16	1	.53300	.04802
	16	4	6	5	.49895	.04319		19	4	15	3	.51630	.04761
	21	4	5	7	.48417	.04280		24	4	14	5	.50498	.04721
	26	4	4	9	.47254	.04236		29	4	13	5	.49240	.04672
	31	4	3	8	.45828	.04182		34	4	12	2	.47675	.04610
	36	4	2	4	.44058	.04115		39	4	10	8	.45703	.04532
	41	4	0	7	.41850	.04031		44	4	8	8	.43318	.04435
	46	3	18	7	.39263	.03927		49	4	6	3	.40104	.04314
	51	3	16	1	.36154	.03803		54	4	3	4	.37052	.04167
	56	3	13	2	.32722	.03660		59	3	19	11	.33179	.03995
	61	3	10	1	.28867	.03503		64	3	16	1	.28804	.03806
	66	3	6	9	.24605	.03339		69	3	12	2	.23951	.03608
	71	3	3	5	.20070	.03172		74	3	8	2	.19043	.03410
	76	3	0	1	.15756	.03004		79	3	4	2	.14569	.03209
	81	2	16	5	.11617	.02822		84	2	19	11	.10490	.02996
52	12	4	10	0	.52108	.04493	55	10	5	0	3	.55391	.05011
	17	4	9	2	.50425	.04457		15	4	19	4	.53896	.04967
	22	4	8	5	.49094	.04419		20	4	18	7	.52307	.04927
	27	4	7	6	.47900	.04373		25	4	17	9	.51226	.04886
	32	4	6	4	.46429	.04317		30	4	16	8	.49934	.04834
	37	4	4	11	.44597	.04246		35	4	15	5	.48319	.04769
	42	4	3	2	.42322	.04158		40	4	13	9	.46270	.04688
	47	4	1	0	.39641	.04049		45	4	11	9	.43830	.04586
	52	3	18	4	.36450	.03918		50	4	9	2	.40804	.04459
	57	3	15	4	.32878	.03766		55	4	6	1	.37357	.04303
	62	3	12	0	.28868	.03599		60	4	2	5	.33323	.04120
	67	3	8	6	.24407	.03424		65	3	18	4	.28734	.03918
	72	3	4	11	.19734	.03247		70	3	14	2	.23693	.03707
	77	3	1	5	.15412	.03071		75	3	9	11	.18710	.03497
	82	2	17	6	.11160	.02875		80	3	5	7	.14077	.03280

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable on the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1
A.	*B.*	£.	*s.*	*d.*			*A.*	*B.*	£.	*s.*	*d.*		
56	11	5	3	9	.56094	.05187	59	14	5	15	8	.58152	.05784
	16	5	2	10	.54489	.05143		19	5	14	10	.56509	.05742
	21	5	2	1	.53046	.05103		24	5	14	1	.55464	.05703
	26	5	1	3	.51972	.05061		29	5	13	1	.54317	.05655
	31	5	0	2	.50645	.05007		34	5	11	10	.52876	.05593
	36	4	18	9	.48976	.04939		39	5	10	4	.51026	.05515
	41	4	17	1	.46860	.04853		44	5	8	4	.48770	.05417
	46	4	14	11	.44350	.04747		49	5	5	10	.45958	.05292
	51	4	12	3	.41237	.04612		54	5	2	8	.42634	.05134
	56	4	8	11	.37665	.04447		59	4	18	8	.38605	.04934
	61	4	5	1	.33462	.04252		64	4	13	11	.33808	.04695
	66	4	0	8	.28645	.04035		69	4	8	7	.28260	.04429
	71	3	16	3	.23417	.03811		74	4	3	2	.22523	.04157
	76	3	11	9	.18411	.03589		79	3	17	9	.17257	.03886
	81	3	7	1	.13595	.03355		84	3	12	3	.12443	.03611
57	12	5	7	6	.56807	.05376	60	10	6	1	2	.60306	.06059
	17	5	6	7	.55110	.05330		15	6	0	3	.58849	.06011
	22	5	5	10	.53834	.05291		20	5	19	5	.57287	.05969
	27	5	4	11	.52736	.05247		25	5	18	7	.56308	.05930
	32	5	3	10	.51371	.05190		30	5	17	7	.55136	.05879
	37	5	2	5	.49646	.05119		35	5	16	3	.53651	.05814
	42	5	0	7	.47474	.05029		40	5	14	8	.51734	.05732
	47	4	18	4	.44879	.04917		45	5	12	7	.49436	.05630
	52	4	15	6	.41693	.04775		50	5	10	1	.46567	.05504
	57	4	12	0	.37976	.04600		55	5	6	7	.43120	.05331
	62	4	7	10	.33596	.04391		60	5	2	4	.38923	.05117
	67	4	3	2	.28537	.04159		65	4	17	2	.33877	.04860
	72	3	18	5	.23126	.03920		70	4	11	6	.28091	.04576
	77	3	13	8	.18090	.03685		75	4	5	8	.22236	.04285
	82	3	8	8	.13119	.03434		80	3	19	10	.16751	.03991
58	13	5	11	6	.57472	.05573	61	11	6	6	1	.61107	.06302
	18	5	10	7	.55782	.05529		16	6	5	1	.59532	.06253
	23	5	9	10	.54639	.05491		21	6	4	3	.58132	.06213
	28	5	8	11	.53518	.05444		26	6	3	5	.57169	.06172
	33	5	7	8	.52115	.05385		31	6	2	5	.55971	.06119
	38	5	6	2	.50330	.05310		36	6	1	0	.54443	.06051
	43	5	4	4	.48115	.05217		41	5	19	4	.52469	.05965
	48	5	2	0	.45413	.05099		46	5	17	2	.50116	.05857
	53	4	19	0	.42159	.04949		51	5	14	5	.47132	.05720
	58	4	15	3	.38290	.04762		56	5	10	10	.43614	.05541
	63	4	10	9	.33707	.04538		61	5	6	3	.39243	.05313
	68	4	5	10	.28409	.04291		66	5	0	9	.33929	.05036
	73	4	0	8	.22825	.04035		71	4	14	7	.27903	.04731
	78	3	15	8	.17723	.03785		76	4	8	5	.21986	.04420
	83	3	10	4	.12689	.03517		81	4	2	0	.16249	.04099

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable on the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent.			Single Premium for £1.	Annnal Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
A.	*B.*	£.	*s.*	*d.*			*A.*	*B.*	£.	*s.*	*d.*		
62	12	6	11	3	.61889	.06561	65	10	7	10	8	.65695	.07535
	17	6	10	3	.60241	.06511		15	7	9	7	.64308	.07480
	22	6	9	5	.59028	.06472		20	7	8	8	.62784	.07435
	27	6	8	7	.58050	.06430		25	7	8	0	.61920	.07398
	32	6	7	6	.56825	.06375		30	7	7	0	.60899	.07350
	37	6	6	1	.55249	.06303		35	7	5	9	.59587	.07286
	42	6	4	3	.53231	.06214		40	7	4	1	.57855	.07205
	47	6	2	0	.50806	.06101		45	7	2	1	.55766	.07103
	52	5	19	1	.47770	.05956		50	6	19	6	.53073	.06973
	57	5	15	4	.44118	.05766		55	6	16	1	.49904	.06804
	62	5	10	5	.39563	.05520		60	6	11	6	.45822	.06574
	67	5	4	6	.33964	.05223		65	6	5	5	.40576	.06270
	72	4	17	11	.27697	.04894		70	5	18	0	.34127	.05901
	77	4	11	3	.21708	.04561		75	5	10	0	.27259	.05498
	82	4	4	2	.15743	.04210		80	5	1	5	.20600	.05070
63	13	6	16	10	.62690	.06843	66	11	7	18	0	.66634	.07898
	18	6	15	11	.61023	.06794		16	7	16	10	.65122	.07842
	23	6	15	2	.59985	.06759		21	7	16	0	.63777	.07800
	28	6	14	3	.58971	.06712		26	7	15	3	.62943	.07761
	33	6	13	1	.57718	.06654		31	7	14	3	.61906	.07711
	38	6	11	7	.56094	.06580		36	7	12	11	.60564	.07645
	43	6	9	9	.54048	.06486		41	7	11	2	.58791	.07560
	48	6	7	4	.51534	.06368		46	7	9	1	.56666	.07454
	53	6	4	4	.48452	.06215		51	7	6	4	.53931	.07318
	58	6	0	3	.44661	.06013		56	7	2	9	.50680	.07139
	63	5	15	0	.39895	.05750		61	6	17	11	.46447	.06894
	68	5	8	7	.34017	.05430		66	6	11	4	.40926	.06568
	73	5	1	7	.27521	.05078		71	6	3	6	.34196	.06173
	78	4	14	4	.21411	.04718		76	5	14	10	.27236	.05742
	83	4	6	10	.15327	.04341		81	5	5	6	.20220	.05275
64	14	7	2	11	.63490	.07147	67	12	8	5	10	.67552	.08290
	19	7	2	0	.61866	.07100		17	8	4	8	.65964	.08233
	24	7	1	3	.60920	.07062		22	8	3	11	.64827	.08194
	29	7	0	4	.59913	.07016		27	8	3	1	.63988	.08154
	34	6	19	1	.58631	.06955		32	8	2	1	.62937	.08102
	39	6	17	7	.56953	.06877		37	8	0	8	.61561	.08033
	44	6	15	7	.54884	.06780		42	7	18	11	.59762	.07945
	49	6	13	1	.52274	.06655		47	7	14	8	.57588	.07734
	54	6	9	11	.49153	.06495		52	7	13	10	.54833	.07693
	59	6	5	7	.45217	.06279		57	7	10	1	.51481	.07504
	64	5	19	11	.40229	.05996		62	7	4	10	.47095	.07242
	69	5	13	1	.34058	.05652		67	6	17	10	.41277	.06891
	74	5	5	6	.27353	.05274		72	6	9	4	.34271	.06468
	79	4	17	8	.21006	.04882		77	6	0	2	.27208	.06007
	84	4	9	9	.15133	.04489		82	5	10	0	.19861	.05501

TABLE XLIII.

Survivorship Assurances.

Showing the Premium required to secure a Sum payable on the death of *A*, provided he dies before *B*, according to the Northampton Table, at 3 per Cent.

Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.	Age of		Annual Premium per Cent.			Single Premium for £1.	Annual Premium for £1.
A.	*B.*	£.	*s.*	*d.*			*A.*	*B.*	£.	*s.*	*d.*		
68	13	8	14	3	.68471	.08714	71	11	10	5	7	.72541	.10280
	18	8	13	2	.66861	.08659		16	10	4	4	.71163	.10218
	23	8	12	5	.65897	.08622		21	10	3	6	.69894	.10173
	28	8	11	7	.65056	.08581		26	10	2	9	.69201	.10137
	33	8	10	6	.63988	.08526		31	10	1	10	.68346	.10091
	38	8	9	1	.62577	.08454		36	10	0	7	.67233	.10030
	43	8	7	3	.60771	.08363		41	9	19	0	.65715	.09949
	48	8	4	11	.58526	.08247		46	9	17	0	.63896	.09848
	53	8	2	0	.55761	.08100		51	9	14	4	.61498	.09718
	58	7	18	0	.52306	.07900		56	9	11	0	.58686	.09550
	63	7	12	5	.47736	.07620		61	9	6	3	.54918	.09311
	68	7	4	10	.41630	.07243		66	8	19	4	.49668	.08965
	73	6	15	10	.34359	.06791		71	8	10	1	.42690	.08504
	78	6	5	11	.27132	.06295		76	7	19	2	.34910	.07959
	83	5	15	1	.21708	.05754		81	7	6	5	.26494	.07322
69	14	9	3	6	.69383	.09175	72	12	10	17	5	.73517	.10870
	19	9	2	5	.67816	.09122		17	10	16	1	.72052	.10806
	24	9	1	9	.66985	.09086		22	10	15	4	.71008	.10766
	29	9	0	10	.66137	.09043		27	10	14	7	.70319	.10729
	34	8	19	9	.65056	.08987		32	10	13	8	.69461	.10682
	39	8	18	3	.63608	.08912		37	10	12	4	.68328	.10618
	44	8	16	4	.61800	.08818		42	10	10	8	.66802	.10534
	49	8	13	11	.59478	.08697		47	10	8	7	.64952	.10428
	54	8	10	11	.56714	.08544		52	10	5	11	.62565	.10295
	59	8	6	7	.53156	.08331		57	10	2	5	.59692	.10119
	64	8	0	7	.48392	.08031		62	9	17	3	.55821	.09864
	69	7	12	7	.41984	.07628		67	9	9	10	.50297	.09493
	74	7	2	11	.34476	.07145		72	9	0	0	.43037	.09000
	79	6	12	2	.26940	.06607		77	8	8	5	.35161	.08420
	84	6	0	11	.21411	.06047		82	7	14	8	.26301	.07733
70	10	9	14	8	.71527	.09735	73	13	11	10	1	.74469	.11504
	15	9	13	6	.70284	.09675		18	11	8	10	.72975	.11442
	20	9	12	6	.68822	.09626		23	11	8	1	.72117	.11405
	25	9	11	10	.68087	.09591		28	11	7	4	.71433	.11367
	30	9	10	11	.67236	.09546		33	11	6	4	.70571	.11318
	35	9	9	9	.66139	.09487		38	11	5	0	.69416	.11251
	40	9	8	2	.64650	.09409		43	11	3	4	.67901	.11165
	45	9	6	3	.62843	.09311		48	11	1	1	.65998	.11055
	50	9	3	9	.60461	.09186		53	10	18	4	.63634	.10917
	55	9	0	6	.57691	.09026		58	10	14	8	.60701	.10732
	60	8	16	0	.54027	.08800		63	10	9	2	.56692	.10459
	65	8	9	7	.49029	.08478		68	10	1	3	.50906	.10062
	70	8	0	11	.42338	.08047		73	9	10	9	.43376	.09536
	75	7	10	8	.34739	.07533		78	8	18	3	.35329	.08913
	80	6	18	11	.26702	.06947		83	8	3	8	.26188	.08185

TABLE XLIV.

Showing the Value of £100 Policy on a Single Life, at the end of any number of years (not exceeding 48) from the date of the Insurance, according to the Northampton Table, at 3 per Cent.

Age when Assured.	1 Year.	2 Years.	3 Years.	4 Years.	5 Years.	6 Years.	Age when Assured.
14	1.0305	2.0934	3.1353	4.1187	5.0400	5.9134	14
15	1.0739	2.1267	3.1204	4.0513	4.9337	5.7454	15
16	1.0642	2.0687	3.0094	3.9017	4.7222	5.5034	16
17	1.0153	1.9664	2.8680	3.6973	4.4870	5.2911	17
18	.9609	1.8718	2.7096	3.5073	4.3196	5.1470	18
19	.9197	1.7657	2.5711	3.3913	4.2267	5.0777	19
20	.8538	1.6667	2.4946	3.3377	4.1966	5.0717	20
21	.8200	1.6549	2.5053	3.3716	4.2542	5.1533	21
22	.8418	1.6993	2.5727	3.4625	4.3692	5.2923	22
23	.8647	1.7455	2.6429	3.5573	4.4891	5.4389	23
24	.8885	1.7937	2.7161	3.6560	4.6141	5.5909	24
25	.9133	1.8440	2.7923	3.7590	4.7445	5.7494	25
26	.9392	1.8963	2.8719	3.8665	4.8806	5.9148	26
27	.9662	1.9511	2.9551	3.9788	5.0228	6.0877	27
28	.9945	2.0083	3.0420	4.0962	5.1715	6.2686	28
29	1.0240	2.0681	3.1329	4.2190	5.3271	6.4579	29
30	1.0549	2.1307	3.2280	4.3476	5.4901	6.6562	30
31	1.0873	2.1963	3.3278	4.4825	5.6610	6.8642	31
32	1.1212	2.2652	3.4325	4.6240	5.8404	7.0826	32
33	1.1569	2.3375	3.5425	4.7727	6.0290	7.3122	33
34	1.1944	2.4135	3.6581	4.9291	6.2273	7.5537	34
35	1.2339	2.4935	3.7798	5.0937	6.4362	7.7823	35
36	1.2754	2.5778	3.9081	5.2673	6.6303	7.9957	36
37	1.3192	2.6667	4.0435	5.4240	6.8071	8.1912	37
38	1.3655	2.7607	4.1597	5.5613	6.9640	8.3938	38
39	1.4145	2.8329	4.2538	5.6760	7.1256	8.6034	39
40	1.4387	2.8801	4.3226	5.7930	7.2920	8.8206	40
41	1.4624	2.9260	4.4178	5.9388	7.4896	9.0712	41
42	1.4853	2.9993	4.5428	6.1166	7.7217	9.3590	42
43	1.5368	3.1036	4.7011	6.3304	7.9924	9.6880	43
44	1.5912	3.2137	4.8684	6.5563	8.2784	10.0042	44
45	1.6487	3.3302	5.0454	6.7953	8.5490	10.2722	45
46	1.7096	3.4536	5.2328	7.0160	8.7680	10.5160	46
47	1.7743	3.5844	5.3986	7.1811	8.9595	10.7642	47
48	1.8429	3.6898	5.5046	7.3150	9.1524	11.0166	48
49	1.8816	3.7305	5.5749	7.4467	9.3460	11.2723	49
50	1.8843	3.7641	5.6718	7.6074	9.5707	11.5615	50
51	1.9159	3.8603	5.8331	7.8341	9.8631	11.9195	51
52	1.9824	3.9937	6.0338	8.1024	10.1990	12.3230	52
53	2.0520	4.1334	6.2438	8.3828	10.5497	12.7439	53
54	2.1250	4.2796	6.4634	8.6758	10.9158	13.1825	54
55	2.2014	4.4326	6.6930	8.9817	11.2976	13.6391	55
56	2.2815	4.5927	6.9330	9.3009	11.6951	14.1136	56
57	2.3653	4.7601	7.1833	9.6335	12.1084	14.6533	57
58	2.4529	4.9348	7.4443	9.9792	12.5857	15.2186	58
59	2.5444	5.1169	7.7155	10.3876	13.0867	15.8615	59
60	2.6397	5.3062	8.0480	10.8176	13.6647	16.5406	60
61	2.7387	5.5549	8.3996	11.3240	14.2778	17.2573	61
62	2.8955	5.8203	8.8270	11.8640	14.9273	18.0118	62

TABLE XLIV.

Showing the Value of £100 Policy on a Single Life, at the end of any number of years (not exceeding 48) from the date of the Insurance, according to the Northampton Table, at 3 per Cent.

Age when Assured.	7 Years.	8 Years.	9 Years.	10 Years.	11 Years.	12 Years.	Age when Assured.
14	6.7167	7.4816	8.2605	9.0537	9.8618	10.6852	14
15	6.5182	7.3052	8.1068	8.9233	9.7552	10.6026	15
16	6.2989	7.1092	7.9344	8.7755	9.6322	10.5053	16
17	6.1100	6.9443	7.7943	8.6601	9.5427	10.4422	17
18	5.9898	6.8485	7.7233	8.6149	9.5236	10.4501	18
19	5.9448	6.8280	7.7283	8.6458	9.5813	10.5351	19
20	5.9631	6.8717	7.7978	8.7420	9.7046	10.6864	20
21	6.0698	7.0038	7.9561	8.9271	9.9173	10.9273	21
22	6.2350	7.1951	8.1741	9.1725	10.1910	11.2300	22
23	6.4072	7.3945	8.4014	9.4285	10.4763	11.5455	23
24	6.5868	7.6025	8.6384	9.6954	10.7740	11.8749	24
25	6.7741	7.8194	8.8859	9.9741	11.0849	12.2189	25
26	6.9697	8.0460	9.1443	10.2654	11.4098	12.5785	26
27	7.1742	8.2829	9.4146	10.5699	11.7497	12.9548	27
28	7.3881	8.5308	9.6974	10.8887	12.1055	13.3488	28
29	7.6120	8.7904	9.9936	11.2227	12.4784	13.7376	29
30	7.8467	9.0624	10.3042	11.5729	12.8451	14.1197	30
31	8.0929	9.3479	10.6302	11.9160	13.2041	14.4933	31
32	8.3514	9.6478	10.9477	12.2500	13.5534	14.8819	32
33	8.6232	9.9379	11.2550	12.5731	13.9167	15.2865	33
34	8.8837	10.2162	11.5498	12.9091	14.2949	15.7080	34
35	9.1309	10.4806	11.8564	13.2589	14.6890	16.1475	35
36	9.3622	10.7552	12.1753	13.6233	15.1000	16.6064	36
37	9.6023	11.0407	12.5074	14.0032	15.5290	17.0857	37
38	9.8514	11.3378	12.8536	14.3998	15.9773	17.5583	38
39	10.1103	11.6471	13.2147	14.8140	16.4169	17.9918	39
40	10.3794	11.9695	13.5918	15.2177	16.8152	18.4090	40
41	10.6845	12.3305	13.9801	15.6009	17.2180	18.8590	41
42	11.0294	12.7035	14.3484	15.9894	17.6548	19.3445	42
43	11.3873	13.0570	14.7228	16.4133	18.1285	19.8682	43
44	11.7000	13.3917	15.1086	16.8506	18.6175	20.4091	44
45	11.9914	13.7360	15.5061	17.3016	19.1221	20.9673	45
46	12.2899	14.0897	15.9153	17.7663	19.6424	21.5431	46
47	12.5954	14.4527	16.3359	18.2447	20.1784	22.1363	47
48	12.9075	14.8247	16.7679	18.7366	20.7299	22.7468	48
49	13.2256	15.2053	17.2109	19.2416	21.2964	23.3740	49
50	13.5792	15.6232	17.6929	19.7871	21.9045	24.0433	50
51	14.0028	16.1122	18.2466	20.4047	22.5846	24.8261	51
52	14.4736	16.6497	18.8499	21.0724	23.3577	25.6662	52
53	14.9640	17.2087	19.4761	21.8077	24.1628	26.5840	53
54	15.4742	17.7892	20.1695	22.5740	25.0459	27.5427	54
55	16.0043	18.4364	20.8930	23.4186	25.9696	28.5426	55
56	16.6004	19.1124	21.6948	24.3032	26.9341	29.5833	56
57	17.2239	19.8666	22.5359	25.2282	27.9393	30.6631	57
58	17.9253	20.6592	23.4168	26.1936	28.9834	31.7785	58
59	18.6642	21.4912	24.3377	27.1977	30.0630	32.9232	59
60	19.4414	22.3623	25.2969	28.2371	31.1719	34.0868	60
61	20.2573	23.2715	26.2914	29.3058	32.2997	35.2530	61
62	21.1109	24.2158	27.3151	30.3934	33.4298	36.3959	62

TABLE XLIV.

Showing the Value of £100 Policy on a Single Life, at the end of any number of years (not exceeding 48) from the date of the Insurance, according to the Northampton Table, at 3 per Cent.

Age when Assured.	13 Years.	14 Years.	15 Years.	16 Years.	17 Years.	18 Years.	Age when Assured
14	11.5239	12.3788	13.2501	14.1384	15.0442	15.9679	14
15	11.4664	12.3468	13.2444	14.1596	15.0929	16.0449	15
16	11.3953	12.3026	13.2277	14.1712	15.1335	16.1153	16
17	11.3593	12.2944	13.2480	14.2207	15.2131	16.2258	17
18	11.3948	12.3582	13.3408	14.3434	15.3665	16.4107	18
19	11.5079	12.5001	13.5123	14.5454	15.5997	16.6762	19
20	11.6878	12.7095	13.7522	14.8163	15.9028	17.0122	20
21	11.9578	13.0094	14.0827	15.1786	16.2975	17.4405	21
22	12.2902	13.3724	14.4773	15.6055	16.7579	17.9354	22
23	12.6369	13.7512	14.8890	16.0512	17.2386	18.4294	23
24	12.9989	14.1466	15.3190	16.5168	17.7179	18.9212	24
25	13.3770	14.5598	15.7684	16.9802	18.1943	19.4094	25
26	13.7723	14.9920	16.2150	17.4403	18.6665	19.9165	26
27	14.1860	15.4206	16.6575	17.8954	19.1572	20.4436	27
28	14.5955	15.8444	17.0944	18.3685	19.6674	20.9919	28
29	14.9991	16.2616	17.5485	18.8605	20.1983	21.5626	29
30	15.3953	16.6955	18.0210	19.3727	20.7511	22.1572	30
31	15.8074	17.1471	18.5131	19.9062	21.3273	22.7772	31
32	16.2363	17.6173	19.0258	20.4625	21.9283	23.3973	32
33	16.6832	18.1076	19.5606	21.0430	22.5287	23.9884	33
34	17.1491	18.6191	20.1188	21.6219	23.0987	24.5721	34
35	17.6353	19.1532	20.6744	22.1691	23.6603	25.1736	35
36	18.1432	19.6835	21.1968	22.7066	24.2389	25.7935	36
37	18.6459	20.1788	21.7081	23.2601	24.8349	26.4321	37
38	19.1117	20.6614	22.2342	23.8300	25.4486	27.0898	38
39	19.5631	21.1576	22.7755	24.4165	26.0804	27.7668	39
40	20.0264	21.6675	23.3320	25.0198	26.7304	28.4634	40
41	20.5240	22.2128	23.9253	25.6609	27.4192	29.1995	41
42	21.0584	22.7963	24.5576	26.3420	28.1487	29.9769	42
43	21.6322	23.4202	25.2315	27.0654	28.9212	30.7975	43
44	22.2249	24.0645	25.9271	27.8118	29.7173	31.6422	44
45	22.8366	24.7293	26.6445	28.5809	30.5369	32.5483	45
46	23.4675	25.4148	27.3836	29.3724	31.4174	33.4831	46
47	24.1175	26.1206	28.1440	30.2245	32.3261	34.4867	47
48	24.7861	26.8460	28.9642	31.1037	33.3033	35.5250	48
49	25.4725	27.6305	29.8102	32.0511	34.3145	36.5975	49
50	26.2426	28.4642	30.7480	33.0548	35.3816	37.7245	50
51	27.0904	29.4181	31.7691	34.1407	36.5286	38.9277	51
52	28.0394	30.4364	32.8542	35.2887	37.7348	40.1854	52
53	29.0295	31.4962	33.9800	36.4755	38.9757	41.4713	53
54	30.0610	32.5969	35.1446	37.6972	40.2452	42.7758	54
55	31.1335	33.7366	36.3446	38.9478	41.5334	44.0839	55
56	32.2450	34.9117	37.5736	40.2173	42.8253	45.3727	56
57	33.3921	36.1161	38.8216	41.4904	44.0973	46.6072	57
58	34.5685	37.3395	40.0730	42.7431	45.3138	47.7329	54
59	35.7639	38.5661	41.3033	43.9386	46.4186	48.8920	59
60	36.9622	39.7709	42.4750	45.0197	47.5576	50.1502	60
61	38.1379	40.9153	43.5291	46.1358	48.7987	51.6132	61
62	39.2516	41.9389	44.6190	47.3569	50.2507	53.1478	62

TABLE XLIV.

Showing the Value of £100 Policy on a Single Life, at the end of any number of years (not exceeding 48) from the date of the Insurance, according to the Northampton Table, at 3 per Cent.

Age when Assured.	19 Years.	20 Years.	21 Years.	22 Years.	23 Years.	24 Years.	Age when Assured.
14	16.9101	17.8714	18.8524	19.8536	20.8757	21.9196	14
15	17.0162	18.0074	19.0190	20.0519	21.1066	22.1839	15
16	17.1173	18.1399	19.1840	20.2501	21.3391	22.4517	16
17	17.2594	18.3147	19.3923	20.4930	21.6176	22.7453	17
18	17.4769	18.5655	19.6775	20.8136	21.9529	23.0943	18
19	17.7754	18.8982	20.0454	21.1957	22.3482	23.5015	19
20	18.1454	19.3032	20.4642	21.6273	22.7914	23.9780	20
21	18.6083	19.7793	20.9524	22.1265	23.3233	24.5434	21
22	19.1161	20.2989	21.4827	22.6894	23.9195	25.1739	22
23	19.6222	20.8161	22.0330	23.2736	24.5387	25.8288	23
24	20.1254	21.3530	22.6044	23.8805	25.1818	26.5093	24
25	20.6479	21.9106	23.1981	24.5111	25.8505	27.2170	25
26	21.1908	22.4902	23.8153	25.1670	26.5461	27.9282	26
27	21.7553	23.0930	24.4575	25.8497	27.2449	28.6158	27
28	22.3427	23.7205	25.1262	26.5351	27.9194	29.3004	28
29	22.9543	24.3742	25.7972	27.1953	28.5902	30.0058	29
30	23.5917	25.0295	26.4421	27.8514	29.2817	30.7328	30
31	24.2302	25.6579	27.0822	28.5277	29.9944	31.4819	31
32	24.8407	26.2807	27.7421	29.2248	30.7288	32.2537	32
33	25.4448	26.9227	28.4223	29.9433	31.4855	33.0486	33
34	26.0674	27.5845	29.1233	30.6836	32.2650	33.8671	34
35	26.7091	28.2665	29.8457	31.4462	33.0677	34.7094	35
36	27.3704	28.9692	30.5898	32.2315	33.8938	35.5758	36
37	28.0516	29.6931	31.3560	33.0397	34.7435	36.4661	37
38	28.7532	30.4383	32.1446	33.8711	35.6167	37.3800	38
39	29.4753	31.2052	32.9556	34.7254	36.5131	38.3513	39
40	30.2181	31.9936	33.7888	35.6022	37.4668	39.3503	40
41	31.0009	32.8323	34.6621	36.5540	38.4650	40.4295	41
42	31.8253	33.6925	35.6124	37.5517	39.5454	41.5592	42
43	32.6927	34.6416	36.6102	38.6340	40.6781	42.7399	43
44	33.6215	35.6208	37.6762	39.7522	41.8462	43.9547	44
45	34.5798	36.6684	38.7780	40.9059	43.0485	45.2012	45
46	35.6067	37.7517	39.9154	42.0938	44.2826	46.4755	46
47	36.6690	38.8701	41.0865	43.3134	45 5445	47.7715	47
48	37.7659	40.0224	42.2895	44.5609	46.8281	49.0800	48
49	38.8963	41.2060	43.5200	45.8298	48.1240	50.3870	49
50	40.0785	42.4369	44.7910	47.1291	49.4355	51.6885	50
51	41.3314	43.7310	46.1136	48.4644	50.7607	52.9714	51
52	42.6316	45.0612	47.4578	49.7989	52.0527	54.1738	52
53	43.9500	46.3951	48.7836	51.0830	53.2470	55.4051	53
54	45.2721	47.7106	50.0582	52.2675	54.4708	56.7217	54
55	46.5753	48.9739	51.2312	53.4824	55.7821	58.2127	55
56	47.8254	50.1334	52.4353	54.7867	57.2721	59.7603	56
57	48.9692	51.3248	53.7310	56.2745	58.8209	61.2509	57
58	50.1456	52.6103	55.2152	57.8233	60.3122	62.6938	58
56	51.4186	54.0891	56.7627	59.3142	61.7557	63.9899	59
60	52.8905	55.6339	58.2520	60.7572	63.0498	64.7992	60
61	54.4310	57.1201	59.6932	62.0479	63.8448	65.4992	61
62	55.9126	58.5582	60.9793	62.8267	64.5277	66.0789	62

TABLE XLIV.

Showing the Value of £100 Policy on a Single Life, at the end of any number of years (not exceeding 48) from the date of the Insurance, according to the Northampton Table, at 3 per Cent.

Age when Assured.	25 Years.	26 Years.	27 Years.	28 Years.	29 Years.	30 Years.	Age when Assured.
14	22.9858	24.0751	25.1675	26.2618	27.3571	28.4735	14
15	23.2846	24.3883	25.4941	26.6007	27.7287	28.8787	15
16	23.5674	24.6852	25.8039	26.9441	28.1066	29.2919	16
17	23.8750	25.0057	26.1582	27.3333	28.5313	29.7532	17
18	24.2366	25.4009	26.5880	27.7983	29.0327	30.2919	18
19	24.6772	25.8757	27.0978	28.3442	29.6155	30.9127	19
20	25.1876	26.4211	27.6790	28.9622	30.2714	31.5834	20
21	25.7875	27.0563	28.3505	29.6709	30.9943	32.2945	21
22	26.4532	27.7581	29.0895	30.4237	31.7347	33.0426	22
23	27.1448	28.4874	29.8330	31.1551	32.4742	33.8128	23
24	27.8637	29.2210	30.5507	31.8852	33.2355	34.6055	24
25	28.5865	29.9321	31.2745	32.6369	34.0192	35.4213	25
26	29.2863	30.6411	32.0160	33.4111	34.8261	36.2608	26
27	29.9835	31.3715	32.7797	34.2081	35.6565	37.1244	27
28	30.7019	32.1239	33.5663	35.0287	36.5110	38.0127	28
29	31.4421	32.8989	34.3761	35.8733	37.3901	38.9258	29
30	32.2047	33.6972	35.2098	36.7423	38.2939	39.8640	30
31	32.9903	34.5191	36.0679	37.6361	39.2228	40.8272	31
32	33.7993	35.3651	36.9505	38.5547	40.1767	41.8152	32
33	34.6322	36.2356	37.8580	39.4984	41.1554	42.8592	33
34	35.4892	37.1306	38.7902	40.4666	42.1904	43.9316	34
35	36.3707	38.0503	39.7470	41.4915	43.2538	45.0655	35
36	37.2764	38.9942	40.7606	42.5449	44.3792	46.2320	36
37	38.2061	39.9953	41.8027	43.6607	45.5374	47.4303	37
38	39.1932	41.0246	42.9075	44.8093	46.7275	48.6590	38
39	40.2082	42.1171	44.0452	45.9900	47.9483	49.9158	39
40	41.2866	43.2423	45.2151	47.2014	49.1972	51.1967	40
41	42.4138	44.4153	46.4307	48.4556	50.4843	52.5093	41
42	43.5904	45.6357	47.6906	49.7494	51.8045	53.8456	42
43	44.8160	46.9019	48.9918	51.0778	53.1497	55.1934	43
44	46.0732	48.1956	50.3142	52.4184	54.4941	56.5217	44
45	47.3580	49.5108	51.6491	53.7583	55.8186	57.8023	45
46	48.6643	50.8385	52.9831	55.0780	57.0949	58.9929	46
47	49.9834	52.1653	54.2966	56.3486	58.2796	60.2054	47
48	51.3013	53.4711	55.5601	57.5260	59.4866	61.4895	48
49	52.5975	54.7258	56.7286	58.7260	60.7665	62.9230	49
50	53.8575	55.8987	57.9345	60.0141	62.2121	64.4126	50
51	55.0518	57.1266	59.2462	61.4864	63.7292	65.8696	51
52	56.2891	58.4501	60.7341	63.0207	65.2029	67.2910	52
53	57.6098	59.9399	62.2728	64.4992	66.6295	68.5790	53
54	59.1007	61.4824	63.7554	65.9304	67.9207	69.4395	54
55	60.6462	62.9685	65.1907	67.2243	68.7760	70.2048	55
56	62.1350	64.4071	66.4865	68.0732	69.5342	70.8664	56
57	63.5761	65.7040	67.3278	68.8229	70.1862	71.4733	57
58	64.8732	66.5363	68.0676	69.4640	70.7823	71.9038	58
59	65.6948	67.2646	68.6961	70.0476	71.1973	72.7534	59
60	66.4100	67.8788	69.2656	70.4454	72.0421	74.0684	60
61	67.0079	68.4328	69.6440	71.2841	73.3653	76.1644	61
62	67.5434	68.7893	70.4755	72.6153	75.4933	78.5384	62

TABLE XLIV.

Showing the Value of £100 Policy on a Single Life, at the end of any number of years (not exceeding 48) from the date of the Insurance, according to the Northampton Table, at 3 per Cent.

Age when Assured.	31 Years.	32 Years.	33 Years.	34 Years.	35 Years.	36 Years.	Age when Assured.
14	29.6116	30.7721	31.9557	33.1630	34.3497	35.6292	14
15	30.0513	31.2472	32.4670	33.7116	34.9589	36.1844	15
16	30.5008	31.7339	32.9920	34.2528	35.4916	36.7276	16
17	30.9996	32.2712	33.5456	34.7977	36.0470	37.3147	17
18	31.5765	32.8640	34.1290	35.3910	36.6718	37.9713	18
19	32.2126	33.4899	34.7642	36.0574	37.3695	38.7004	19
20	32.8725	34.1586	35.4639	36.7882	38.1314	39.4934	20
21	33.5917	34.9081	36.2438	37.5986	38.9723	40.3646	21
22	34.3700	35.7167	37.0827	38.4678	39.8716	41.2938	22
23	35.1709	36.5485	37.9454	39.3611	40.7954	42.2476	23
24	35.9951	37.4041	38.8322	40.2790	41.7438	43.2261	24
25	36.8429	38.2838	39.7436	41.2216	42.7171	44.2292	25
26	37.7150	39.1882	40.6798	42.1891	43.7152	45.2567	26
27	38.6116	40.1174	41.6410	43.1815	44.7377	46.3377	27
28	39.5332	41.0717	42.6272	44.1985	45.8142	47.4463	28
29	40.4798	42.0509	43.6380	45.2699	46.9184	48.6131	29
30	41.4514	43.0549	44.7037	46.3692	48.0813	49.8109	30
31	42.4478	44.1142	45.7975	47.5279	49.2758	51.0388	31
32	43.4999	45.2016	46.9511	48.7182	50.5006	52.2953	32
33	44.5803	46.3496	48.1366	49.9393	51.7544	53.5780	33
34	45.7216	47.5297	49.3533	51.1897	53.0347	54.8831	34
35	46.8954	48.7411	50.5996	52.4669	54.3377	56.2051	35
36	48.1008	49.9825	51.8731	53.7673	55.6580	57.5359	36
37	49.3363	51.2514	53.1700	55.0852	56.9874	58.8637	37
38	50.5997	52.5440	54.4847	56.4123	58.3138	60.1711	38
39	51.8870	53.8546	55.8089	57.7367	59.6197	61.4327	39
40	53.1925	55.1748	57.1303	59.0404	60.8793	62.6099	40
41	54.5204	56.5045	58.4425	60.3083	62.0641	63.8153	41
42	55.8590	57.8257	59.7192	61.5011	63.2781	65.0937	42
43	57.1898	59.1119	60.9207	62.7246	64.5673	66.5151	43
44	58.4737	60.3107	62.1428	64.0144	65.9925	67.9729	44
45	59.6690	61.5307	63.4325	65.4426	67.4550	69.3756	45
46	60.8858	62.8195	64.8633	66.9094	68.8622	70.7307	46
47	62.1728	64.2521	66.3339	68.3206	70.2216	71.9612	47
48	63.6064	65.7258	67.7483	69.6837	71.4548	72.8063	48
49	65.0822	67.1428	69.1145	70.9188	72.2957	73.5634	49
50	66.5127	68.5222	70.3612	71.7644	73.0564	74.2347	50
51	67.9177	69.7919	71.2222	72.5390	73.7399	74.8736	51
52	69.2019	70.6600	72.0026	73.2269	74.3828	75.3661	52
53	70.0666	71.4364	72.6854	73.8647	74.8679	76.2257	53
54	70.8380	72.1132	73.3171	74.3414	75.7276	77.4868	54
55	71.5078	72.7378	73.7843	75.2007	76.9980	79.4154	55
56	72.1242	73.1942	74.6424	76.4802	78.9520	81.5674	56
57	72.5684	74.0504	75.9311	78.4606	81.1371	84.1626	57
58	73.4218	75.3480	77.9388	80.6801	83.7789	86.4492	58
59	74.7282	77.3841	80.1943	83.3710	86.1085	88.7626	59
60	76.7936	79.6772	82.9368	85.7458	88.4692	90.7213	60
61	79.1262	82.4742	85.3593	88.1566	90.4697		61
62	81.9807	84.9470	87.8231	90.2014			62

TABLE XLIV.

Showing the Value of £100 Policy on a Single Life, at the end of any number of years (not exceeding 48) from the date of the Insurance, according to the Northampton Table, at 3 per Cent.

Age when Assured.	37 Years.	38 Years.	39 Years.	40 Years.	41 Years.	42 Years.	Age when Assured.
14	36.8421	38.0521	39.2802	40.5261	41.7899	43.0714	14
15	37.4071	38.6479	39.9069	41.1838	42.4786	43.7909	15
16	37.9819	39.2545	40.5453	41.8542	43.1807	44.5246	16
17	38.6011	39.9057	41.2287	42.5695	43.9279	45.3033	17
18	39.2894	40.6259	41.9805	43.3528	44.7423	46.1482	18
19	40.0499	41.4176	42.8032	44.2062	45.6258	47.0611	19
20	40.8738	42.2723	43.6883	45.1210	46.5697	48.0330	20
21	41.7752	43.2033	44.6485	46.1096	47.5855	49.1032	21
22	42.7338	44.1908	45.6641	47.1522	48.6824	50.2281	22
23	43.7170	45.2027	46.7035	48.2467	49.8055	51.4080	23
24	44.7248	46.2386	47.7953	49.3677	50.9842	52.6169	24
25	45.7567	47.3273	48.9138	50.5447	52.1921	53.8538	25
26	46.8418	48.4429	50.0889	51.7514	53.4284	55.1170	26
27	47.9541	49.6157	51.2940	52.9869	54.6914	56.4041	27
28	49.1214	50.8188	52.5282	54.2494	55.9787	57.7113	28
29	50.3248	52.0513	53.7898	55.5366	57.2866	59.0334	29
30	51.5553	53.3118	55.0765	56.8446	58.6095	60.3624	30
31	52.8140	54.5976	56.3845	58.1682	59.9398	61.6874	31
32	54.0985	55.9051	57.7084	59.4995	61.2662	62.9921	32
33	55.4051	57.2288	59.0402	60.8270	62.5724	64.2528	33
34	56.7282	58.5608	60.3685	62.1343	63.8344	65.4343	34
35	58.0599	59.8894	61.6766	63.3972	65.0164	66.6313	35
36	59.3884	61.1978	62.9400	64.5794	66.2144	67.8847	36
37	60.6966	62.4612	64.1218	65.7779	67.4698	69.2580	37
38	61.9594	63.6422	65.3204	67.0349	68.8470	70.6612	38
39	63.1388	64.8403	66.5785	68.4157	70.2550	72.0103	39
40	64.3359	66.0990	67.9625	69.8282	71.6087	73.3124	40
41	65.6041	67.4949	69.3878	71.1943	72.9228	74.5047	41
42	67.0124	68.9335	70.7668	72.5210	74.1263	75.3513	42
43	68.4651	70.3260	72.1067	73.7362	74.9797	76.1246	43
44	69.8629	71.6713	73.3263	74.5892	75.7519	76.8123	44
45	71.2132	72.8950	74.1783	75.3599	76.4374	77.4546	45
46	72.4406	73.7454	74.9468	76.0424	77.0767	77.9566	46
47	73.2887	74.5110	75.6257	76.6779	77.5732	78.7848	47
48	74.0506	75.1854	76.2567	77.1681	78.4016	79.9670	48
49	74.7195	75.8109	76.7394	77.9961	79.5909	81.7358	49
50	75.3470	76.2934	77.5742	79.1995	81.3855	83.6985	50
51	75.8381	77.1435	78.8000	81.0280	83.3854	86.0503	51
52	76.6970	78.3859	80.6574	83.0609	85.7778	88.1191	52
53	77.9488	80.2662	82.7183	85.4902	87.8788	90.1947	53
54	79.8528	82.3562	85.1862	87.6248	89.9893	91.9445	54
55	81.9732	84.8646	87.3562	89.7719	91.7696		55
56	84.5239	87.0715	89.5417	91.5843			56
57	86.7697	89.2975	91.3878				57
58	89.0382	91.1792					58
59	90.9574						59

TABLE XLIV.

Showing the Value of £100 Policy on a Single Life, at the end of any number of years (not exceeding 48) from the date of the Insurance, according to the Northampton Table, at 3 per Cent.

Age when Assured.	43 Years.	44 Years.	45 Years.	46 Years.	47 Years.	48 Years.	Age when Assured.
14	44.3702	45.6860	47.0182	48.3663	49.7293	51.1061	14
15	45.1204	46.4666	47.8287	49.2058	50.5970	52.0274	15
16	45.8854	47.2623	48.6544	50.0606	51.5066	52.9672	16
17	46.6950	48.1020	49.5234	50.9850	52.4613	53.9790	17
18	47.5698	49.0057	50.4822	51.9737	53.5070	55.0557	18
19	48.5110	50.0018	51.5078	53.0559	54.6196	56.1969	19
20	49.5377	51.0576	52.6202	54.1984	55.7903	57.3932	20
21	50.6362	52.2122	53.8040	55.4096	57.0263	58.6507	21
22	51.8171	53.4221	55.0409	56.6711	58.3089	59.9497	22
23	53.0266	54.6592	56.3032	57.9549	59.6097	61.2615	23
24	54.2638	55.9220	57.5882	59.2574	60.9236	62.5785	24
25	55.5269	57.2079	58.8922	60.5733	62.2430	63.8901	25
26	56.8135	58.5133	60.2099	61.8950	63.5573	65.1810	26
27	58.1199	59.8326	61.5337	63.2118	64.8509	66.4290	27
28	59.4408	61.1585	62.8529	64.5080	66.1015	67.6011	28
29	60.7683	62.4797	64.1515	65.7610	67.2756	68.7861	29
30	62.0915	63.7806	65.4067	66.9371	68.4632	70.0223	30
31	63.3944	65.0379	66.5846	68.1270	69.7029	71.3682	31
32	64.6536	66.2173	67.7767	69.3697	71.0534	72.7391	32
33	65.8342	67.4113	69.0224	70.7252	72.4300	74.0570	33
34	67.0298	68.6598	70.3825	72.1073	73.7533	75.3283	34
35	68.2809	70.0245	71.7701	73.4360	75.0301	76.4888	35
36	69.6500	71.4176	73.1042	74.7181	76.1951	77.3222	36
37	71.0482	72.7567	74.3915	75.8876	77.0292	78.0803	37
38	72.3925	74.0492	75.5652	76.7221	77.7873	78.7586	38
39	73.6899	75.2269	76.3998	77.4798	78.4646	79.3943	39
40	74.3715	76.0612	77.1566	78.1556	79.0986	79.9010	40
41	75.7118	76.8232	77.8367	78.7935	79.6076	80.7093	41
42	76.4792	77.5078	78.4788	79.3049	80.4230	81.8419	42
43	77.1687	78.1543	78.9929	80.1278	81.5681	83.5052	43
44	77.8134	78.6650	79.8177	81.2804	83.2477	85.3293	44
45	78.3201	79.4913	80.9777	82.9769	85.0921	87.4832	45
46	79.1475	80.6588	82.6915	84.8422	87.2734	89.3685	46
47	80.3224	82.3904	84.5785	87.0520	89.1835	91.2501	47
48	82.0723	84.3000	86.8182	88.9882	91.0921	92.8319	48
49	84.0052	86.5707	88.7814	90.9248	92.6973		49
50	86.3131	88.5662	90.7508	92.5573			50
51	88.3467	90.5732	92.4144				51
52	90.3891	92.2662					52
53	92.1098						53

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